## Financial Toolbox ${ }^{\text {™ }} 4$ User's Guide

MathWorks

## How to Contact MathWorks


www. mathworks.com
comp.soft-sys.matlab
Web
www.mathworks.com/contact_TS.html Technical Support
suggest@mathworks.com
Product enhancement suggestions
bugs@mathworks.com
doc@mathworks.com
service@mathworks.com
info@mathworks.com

Bug reports
Documentation error reports
Order status, license renewals, passcodes
Sales, pricing, and general information

508-647-7000 (Phone)

508-647-7001 (Fax)
The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098
For contact information about worldwide offices, see the MathWorks Web site.

## Financial Toolbox ${ }^{\text {TM }}$ User's Guide

© COPYRIGHT 1995-2011 The MathWorks, Inc.
The software described in this document is furnished under a license agreement. The software may be used or copied only under the terms of the license agreement. No part of this manual may be photocopied or reproduced in any form without prior written consent from The MathWorks, Inc.
FEDERAL ACQUISITION: This provision applies to all acquisitions of the Program and Documentation by, for, or through the federal government of the United States. By accepting delivery of the Program or Documentation, the government hereby agrees that this software or documentation qualifies as commercial computer software or commercial computer software documentation as such terms are used or defined in FAR 12.212, DFARS Part 227.72, and DFARS 252.227-7014. Accordingly, the terms and conditions of this Agreement and only those rights specified in this Agreement, shall pertain to and govern the use, modification, reproduction, release, performance, display, and disclosure of the Program and Documentation by the federal government (or other entity acquiring for or through the federal government) and shall supersede any conflicting contractual terms or conditions. If this License fails to meet the government's needs or is inconsistent in any respect with federal procurement law, the government agrees to return the Program and Documentation, unused, to The MathWorks, Inc.

## Trademarks

MATLAB and Simulink are registered trademarks of The MathWorks, Inc. See www.mathworks.com/trademarks for a list of additional trademarks. Other product or brand names may be trademarks or registered trademarks of their respective holders.

## Patents

MathWorks products are protected by one or more U.S. patents. Please see www.mathworks.com/patents for more information.

## Revision History

October 1995 First printing
January 1998 Second printing
January 1999 Third printing
November 2000
May 2003
June 2004
August 2004
September 2005
March 2006
September 2006
March 2007
September 2007
March 2008
October 2008
March 2009
September 2009 March 2010
September 2010
April 2011

Fourth printing Online only Online only Online only Fifth printing Online only Sixth printing Online only Online only Online only Online only Online only Online only Online only Online only Online only

Revised for Version 1.1
Revised for Version 2.0 (Release 11)
Revised for Version 2.1.2 (Release 12)
Revised for Version 2.3 (Release 13)
Revised for Version 2.4 (Release 14)
Revised for Version 2.4.1 (Release 14+)
Revised for Version 2.5 (Release 14SP3)
Revised for Version 3.0 (Release 2006a)
Revised for Version 3.1 (Release 2006b)
Revised for Version 3.2 (Release 2007a)
Revised for Version 3.3 (Release 2007b)
Revised for Version 3.4 (Release 2008a)
Revised for Version 3.5 (Release 2008b)
Revised for Version 3.6 (Release 2009a)
Revised for Version 3.7 (Release 2009b)
Revised for Version 3.7.1 (Release 2010a)
Revised for Version 3.8 (Release 2010b)
Revised for Version 4.0 (Release 2011a)

## Getting Started

## 1

Product Overview ..... 1-2
Expected Background ..... 1-4
Using Matrix Functions for Finance ..... 1-5
Introduction ..... 1-5
Key Definitions ..... 1-5
Referencing Matrix Elements ..... 1-6
Transposing Matrices ..... 1-7
Matrix Algebra Refresher ..... 1-8
Introduction ..... 1-8
Adding and Subtracting Matrices ..... 1-8
Multiplying Matrices ..... 1-9
Dividing Matrices ..... 1-14
Solving Simultaneous Linear Equations ..... 1-15
Operating Element by Element ..... 1-18
Function Input and Output Arguments ..... 1-19
Input Arguments ..... 1-19
Output Arguments ..... 1-21
Interest Rate Arguments ..... 1-22
Performing Common Financial Tasks
2
Introduction ..... 2-2
Handling and Converting Dates ..... 2-4
Date Formats ..... 2-4
Date Conversions ..... 2-5
Current Date and Time ..... 2-8
Determining Dates ..... 2-9
Formatting Currency ..... 2-12
Charting Financial Data ..... 2-13
Introduction ..... 2-13
High-Low-Close Chart Example ..... 2-14
Bollinger Chart Example ..... 2-15
Analyzing and Computing Cash Flows ..... 2-17
Introduction ..... 2-17
Interest Rates/Rates of Return ..... 2-17
Present or Future Values ..... 2-18
Depreciation ..... 2-19
Annuities ..... 2-19
Pricing and Computing Yields for Fixed-Income Securities ..... 2-21
Introduction ..... 2-21
Terminology ..... 2-21
Framework ..... 2-26
Default Parameter Values ..... 2-27
Coupon Date Calculations ..... 2-30
Yield Conventions ..... 2-31
Pricing Functions ..... 2-31
Yield Functions ..... 2-32
Fixed-Income Sensitivities ..... 2-33
Term Structure of Interest Rates ..... 2-36
Introduction ..... 2-36
Deriving an Implied Zero Curve ..... 2-37
Pricing and Analyzing Equity Derivatives ..... 2-39
Introduction ..... 2-39
Sensitivity Measures ..... 2-39
Analysis Models ..... 2-40

## Portfolio Analysis

3
Analyzing Portfolios ..... 3-2
Portfolio Optimization Functions ..... 3-3
Portfolio Construction Examples ..... 3-5
Introduction ..... 3-5
Efficient Frontier Example ..... 3-5
Portfolio Selection and Risk Aversion ..... 3-8
Introduction ..... 3-8
Optimal Risky Portfolio Example ..... 3-9
Constraint Specification ..... 3-12
Example ..... 3-12
Linear Constraint Equations ..... 3-14
Specifying Additional Constraints ..... 3-17
Active Returns and Tracking Error Efficient Frontier ..... 3-20
Portfolio Optimization Tools
4
Portfolio Optimization Theory ..... 4-2
Portfolio Optimization Problems ..... 4-2
Portfolio Problem Specification ..... 4-2
Return Proxy ..... 4-3
Risk Proxy ..... 4-5
Portfolio Set for Mean-Variance Portfolio Optimization ..... 4-5
Default Portfolio Problem ..... 4-11
Portfolio Object ..... 4-12
Portfolio Object Properties and Methods ..... 4-12
Working with Portfolio Objects ..... 4-17
Setting and Getting Properties ..... 4-18
Displaying Portfolio Objects ..... 4-18
Saving and Loading Portfolio Objects ..... 4-19
Estimating Efficient Portfolios and Frontiers ..... 4-19
Arrays of Portfolio Objects ..... 4-19
Subclassing Portfolio Objects ..... 4-20
Conventions for Representation of Data ..... 4-20
Constructing the Portfolio Object ..... 4-22
Syntax ..... 4-22
Portfolio Problem Sufficiency ..... 4-23
Constructor Examples ..... 4-23
Common Operations on the Portfolio Object ..... 4-29
Naming a Portfolio Object ..... 4-29
Setting Up the Number of Assets in the Asset Universe ..... 4-29
Setting Up a List of Asset Identifiers ..... 4-30
Truncating and Padding Asset Lists ..... 4-31
Setting Up an Initial or Current Portfolio ..... 4-32
Working with Asset Returns and Moments of Asset Returns ..... 4-36
Assignment Using the Portfolio Constructor ..... 4-36
Assignment Using the setAssetMoments Method ..... 4-38
Scalar Expansion of Arguments ..... 4-39
Estimating Asset Moments from Asset Prices or Returns ..... 4-40
Estimating Asset Moments from Asset Returns or Prices with Missing Data ..... 4-44
Estimating Asset Moments from Financial Time Series Data ..... 4-46
Working with a Riskless Asset ..... 4-48
Working with Transaction Costs ..... 4-49
Working with Portfolio Constraints ..... 4-52
Setting Default Constraints for Portfolio Weights ..... 4-52
Working with Bound Constraints ..... 4-55
Working with Budget Constraints ..... 4-57
Working with Group Constraints ..... 4-59
Working with Group Ratio Constraints ..... 4-62
Working with Linear Equality Constraints ..... 4-66
Working with Linear Inequality Constraints ..... 4-68
Working with Turnover Constraints ..... 4-70
Validating the Portfolio Problem ..... 4-73
Validating a Portfolio Set ..... 4-73
Validating Portfolios ..... 4-75
Estimate Efficient Portfolios ..... 4-77
Obtaining Portfolios Along the Entire Efficient Frontier ..... 4-77
Obtaining Endpoints of the Efficient Frontier ..... 4-79
Obtaining Efficient Portfolios for Target Returns ..... 4-81
Obtaining Efficient Portfolios for Target Risks ..... 4-83
Choosing and Controlling the Solver ..... 4-86
Estimate Efficient Frontiers ..... 4-88
Obtaining Portfolio Risks and Returns ..... 4-88
Plotting the Efficient Frontier ..... 4-90
Post-Processing ..... 4-95
Setting Up Tradable Portfolios ..... 4-95
Troubleshooting ..... 4-97
Asset Allocation Example ..... 4-100
Defining the Portfolio Problem ..... 4-100
Simulating Asset Prices ..... 4-101
Setting Up the Portfolio Object ..... 4-103
Validating the Portfolio Problem ..... 4-105
Plotting the Efficient Frontier ..... 4-105
Evaluating Gross vs. Net Portfolio Returns ..... 4-106
Analyzing Descriptive Properties of the Portfolio Structures ..... 4-107
Obtaining a Portfolio at the Specified Return Level on the Efficient Frontier ..... 4-108
Obtaining a Portfolio at the Specified Risk Levels on the Efficient Frontier ..... 4-109
Displaying the Final Results ..... 4-112

5
Overview of Performance Metrics ..... 5-2
Performance Metrics Classes ..... 5-2
Performance Metrics Example ..... 5-3
Using the Sharpe Ratio ..... 5-6
Introduction ..... 5-6
Sharpe Ratio Example ..... 5-6
Using the Information Ratio ..... 5-8
Introduction ..... 5-8
Information Ratio Example ..... 5-8
Tracking Error ..... 5-10
Introduction ..... 5-10
Tracking Error Example ..... 5-10
Risk-Adjusted Return ..... 5-11
Introduction ..... 5-11
Risk-Adjusted Return Example ..... 5-11
Sample and Expected Lower Partial Moments ..... 5-14
Introduction ..... 5-14
Sample Lower Partial Moments Example ..... 5-14
Expected Lower Partial Moments Example ..... 5-15
Maximum and Expected Maximum Drawdown ..... 5-17
Introduction ..... 5-17
Maximum Drawdown Example ..... 5-17
Expected Maximum Drawdown Example ..... 5-21
Credit Risk Analysis
6
Credit Rating ..... 6-2
Introduction ..... 6-2
Example ..... 6-2
Estimation of Transition Probabilities ..... 6-3
Introduction ..... 6-3
Estimating Transition Probabilities ..... 6-4
Estimating $t$-year Default Probabilities and Confidence Intervals ..... 6-7
Removing Outliers, Estimating Subgroup Probabilities, and Aggregating Datasets ..... 6-9
Regression with Missing Data
7
Multivariate Normal Regression ..... 7-2
Introduction ..... 7-2
Multivariate Normal Linear Regression ..... 7-3
Maximum Likelihood Estimation ..... 7-4
Special Case of a Multiple Linear Regression Model ..... 7-5
Least-Squares Regression ..... 7-5
Mean and Covariance Estimation ..... 7-5
Convergence ..... 7-6
Fisher Information ..... 7-6
Statistical Tests ..... 7-7
Maximum Likelihood Estimation with Missing Data ..... 7-9
Introduction ..... 7-9
ECM Algorithm ..... 7-10
Standard Errors ..... 7-10
Data Augmentation ..... 7-11
Multivariate Normal Regression Functions ..... 7-12
Multivariate Normal Regression Without Missing Data ..... 7-14
Multivariate Normal Regression With Missing Data ..... 7-14
Least-Squares Regression with Missing Data ..... 7-15
Multivariate Normal Parameter Estimation with Missing Data ..... 7-15
Support Functions ..... 7-16
Multivariate Normal Regression Types ..... 7-17
Regressions ..... 7-17
Multivariate Normal Regression ..... 7-17
Least-Squares Regression ..... 7-18
Covariance-Weighted Least Squares ..... 7-19
Feasible Generalized Least Squares ..... 7-20
Seemingly Unrelated Regression ..... 7-21
Mean and Covariance Parameter Estimation ..... 7-23
Troubleshooting Multivariate Normal Regression ..... 7-23
Slow Convergence ..... 7-24
Nonrandom Residuals ..... 7-24
Nonconvergence ..... 7-25
Example of Portfolios with Missing Data ..... 7-26
Valuation with Missing Data ..... 7-34
Introduction ..... 7-34
Capital Asset Pricing Model ..... 7-34
Estimation of the CAPM ..... 7-35
Estimation with Missing Data ..... 7-36
Estimation of Some Technology Stock Betas ..... 7-36
Grouped Estimation of Some Technology Stock Betas ..... 7-39
References ..... 7-42
Solving Sample Problems
8
Introduction ..... 8-2
Common Problems in Finance ..... 8-3
Sensitivity of Bond Prices to Changes in Interest Rates ..... 8-3
Constructing a Bond Portfolio to Hedge Against Duration and Convexity ..... 8-6
Sensitivity of Bond Prices to Parallel Shifts in the Yield Curve ..... 8-9
Sensitivity of Bond Prices to Nonparallel Shifts in the Yield Curve ..... 8-12
Constructing Greek-Neutral Portfolios of European Stock Options ..... 8-14
Term Structure Analysis and Interest Rate Swap Pricing ..... 8-18
Producing Graphics with the Toolbox ..... 8-21
Introduction ..... 8-21
Plotting an Efficient Frontier ..... 8-21
Plotting Sensitivities of an Option ..... 8-24
Plotting Sensitivities of a Portfolio of Options ..... 8-26
Financial Time Series Analysis
9
Analyzing Financial Time Series ..... 9-2
Creating Financial Time Series Objects ..... 9-3
Introduction ..... 9-3
Using the Constructor ..... 9-3
Transforming a Text File ..... 9-14
Visualizing Financial Time Series Objects ..... 9-18
Introduction ..... 9-18
Using chartfts ..... 9-18
Zoom Tool ..... 9-21
Combine Axes Tool ..... 9-24
Using Financial Time Series
10
Introduction ..... 10-2
Working with Financial Time Series Objects ..... 10-3
Introduction ..... 10-3
Financial Time Series Object Structure ..... 10-3
Data Extraction ..... 10-4
Object-to-Matrix Conversion ..... 10-6
Indexing a Financial Time Series Object ..... 10-8
Operations ..... 10-15
Data Transformation and Frequency Conversion ..... 10-19
Demonstration Program ..... 10-25
Overview ..... 10-25
Loading the Data ..... 10-26
Create Financial Time Series Objects ..... 10-26
Create Closing Prices Adjustment Series ..... 10-27
Adjust Closing Prices and Make Them Spot Prices ..... 10-28
Create Return Series ..... 10-28
Regress Return Series Against Metric Data ..... 10-28
Plot the Results ..... 10-29
Calculate the Dividend Rate ..... 10-30
Financial Time Series Tool (FTSTool)
11
What Is the Financial Time Series Tool? ..... 11-2
Getting Started with FTSTool ..... 11-4
Loading Data with FTSTool ..... 11-5
Overview ..... 11-5
Obtaining External Data ..... 11-5
Obtaining Internal Data ..... 11-7
Viewing the MATLAB Workspace ..... 11-8
Using FTSTool for Supported Tasks ..... 11-10
Creating a Financial Time Series Object ..... 11-10
Merging Financial Time Series Objects ..... 11-11
Converting a Financial Time Series Object to a MATLAB Double-Precision Matrix ..... 11-12
Plotting the Output in Several Formats ..... 11-12
Viewing Data for a Financial Time Series Object in the Data Table ..... 11-13
Modifying Data for a Financial Time Series Object in the Data Table ..... 11-15
Viewing and Modifying the Properties for a FINTS Object ..... 11-17
Using FTSTool with Other Time Series GUIs ..... 11-18
Financial Time Series Graphical User Interface12
Introduction ..... 12-2
Main Window ..... 12-2
Using the Financial Time Series GUI ..... 12-7
Getting Started ..... 12-7
Data Menu ..... 12-9
Analysis Menu ..... 12-13
Graphs Menu ..... 12-15
Saving Time Series Data ..... 12-19
Trading Date Utilities
13 3
Trading Calendars Graphical User Interface ..... 13-2
UICalendar Graphical User Interface ..... 13-4
Using UICalendar in Standalone Mode ..... 13-4
Using UICalendar with an Application ..... 13-5
Technical Analysis
14
Introduction ..... 14-2
Examples ..... 14-4
Overview ..... 14-4
Moving Average Convergence/Divergence (MACD) ..... 14-4
Williams \%R ..... 14-6
Relative Strength Index (RSI) ..... 14-7
On-Balance Volume (OBV) ..... 14-8
Dates ..... 15-2
Current Time and Date ..... 15-2
Date and Time Components ..... 15-2
Date Conversion ..... 15-3
Financial Dates ..... 15-4
Coupon Bond Dates ..... 15-5
Currency and Price ..... 15-6
Financial Data Charts ..... 15-6
Cash Flows ..... 15-7
Annuities ..... 15-7
Amortization and Depreciation ..... 15-8
Present Value ..... 15-8
Future Value ..... 15-8
Payment Calculations ..... 15-8
Rates of Return ..... 15-9
Cash Flow Sensitivities ..... 15-9
Fixed-Income Securities ..... 15-9
Accrued Interest ..... 15-10
Prices ..... 15-10
Term Structure of Interest Rates ..... 15-10
Yields ..... 15-11
Spreads ..... 15-11
Interest Rate Sensitivities ..... 15-11
Portfolio Optimization Objects ..... 15-12
Portfolio Objects ..... 15-12
Get Methods ..... 15-12
Set Methods ..... 15-13
Add Methods ..... 15-14
Preprocessing Methods ..... 15-14
Efficient Portfolio Estimation Methods ..... 15-14
Efficient Frontier Methods ..... 15-15
Utility Methods ..... 15-15
Portfolio Analysis ..... 15-15
Basic Portfolio Optimization ..... 15-16
Performance Metrics ..... 15-16
Portfolio Utilities ..... 15-17
Financial Statistics ..... 15-18
Expectation Conditional Maximization ..... 15-18
Multivariate Normal Regression ..... 15-19
Expectation Conditional Maximization - Multivariate Normal Regression ..... 15-19
Expectation Conditional Maximization - Least-Squares Regression ..... 15-20
Seemingly Unrelated Regression ..... 15-20
Derivatives ..... 15-20
Option Valuation and Sensitivity ..... 15-21
Credit Risk Utilities ..... 15-21
Estimation of Transition Probabilities ..... 15-22
GARCH Processes ..... 15-22
Univariate GARCH Processes ..... 15-22
Financial Time Series Object and File Construction ..... 15-23
Financial Time Series Arithmetic ..... 15-23
Financial Time Series Math ..... 15-24
Financial Time Series Descriptive Statistics ..... 15-24
Financial Time Series Utility ..... 15-25
Financial Time Series Data Transformation ..... 15-26
Financial Time Series Indicator ..... 15-27
Financial Time Series GUI ..... 15-28

## Class Reference

16

## Functions - Alphabetical List

17

## Bibliography

## A

Bond Pricing and Yields ..... A-2
Term Structure of Interest Rates ..... A-3
Derivatives Pricing and Yields ..... A-4
Portfolio Analysis ..... A-5
Investment Performance Metrics ..... A-6
Financial Statistics ..... A-8
Standard References ..... A-9
Credit Risk Analysis ..... A-11
Portfolio Optimization ..... A-12

B
Bond Examples ..... B-2
Portfolio Examples ..... B-2
Portfolio Object Examples ..... B-2
Estimation of Transition Probabilities ..... B-2
Financial Statistics ..... B-2
Sample Programs ..... B-3
Graphics Programs ..... B-3
Charting Financial Time Series ..... B-3
Indexing Financial Time Series ..... B-3
Financial Time Series Demonstration Program ..... B-3
Financial Time Series Graphical User Interface Examples ..... B-4
Technical Analysis ..... B-4

## Getting Started

- "Product Overview" on page 1-2
- "Expected Background" on page 1-4
- "Using Matrix Functions for Finance" on page 1-5
- "Matrix Algebra Refresher" on page 1-8
- "Function Input and Output Arguments" on page 1-19


## Product Overview

Financial Toolbox ${ }^{\mathrm{TM}}$ provides functions for mathematical modeling and statistical analysis of financial data. You can optimize portfolios of financial instruments, optionally taking into account turnover and transaction costs. The toolbox enables you to estimate risk, analyze interest rate levels, price equity and interest rate derivatives, and measure investment performance. Time series analysis capabilities let you perform transformations or regressions with missing data and convert between different trading calendars and day-count conventions.

With Financial Toolbox software, you can do the following:

- Asset allocation, cash flow analysis, object-oriented portfolio optimization, and risk analysis
- Basic SIA-compliant fixed-income security analysis
- Basic Black-Scholes, Black, and binomial option pricing
- Financial time series, date math, and calendar math
- Basic GARCH estimation, simulation, and forecasting
- Regression and estimation with missing data
- Technical indicators and financial charts

This chapter uses MATLAB ${ }^{\circledR}$ to review the fundamentals of matrix algebra you need for financial analysis and engineering applications. It contains these sections:

- "Using Matrix Functions for Finance" on page 1-5

Reviews "Key Definitions" on page 1-5 and some matrix algebra fundamentals, such as "Referencing Matrix Elements" on page 1-6 and "Transposing Matrices" on page 1-7.

- "Matrix Algebra Refresher" on page 1-8

Provides a brief refresher on using matrix functions in financial analysis and engineering

- "Function Input and Output Arguments" on page 1-19

Describes acceptable formats for providing data to MATLAB and the resulting output from computations on the supplied data.

This material explains some MATLAB concepts and operations using financial examples to help get you started.

## Expected Background

In general, this guide assumes experience working with financial derivatives and some familiarity with the underlying models.

In designing Financial Toolbox documentation, we assume that your title is like one of these:

- Analyst, quantitative analyst
- Risk manager
- Portfolio manager
- Asset allocator
- Financial engineer
- Trader
- Student, professor, or other academic

We also assume your background, education, training, and responsibilities match some aspects of this profile:

- Finance, economics, perhaps accounting
- Engineering, mathematics, physics, other quantitative sciences
- Focus on quantitative approaches to financial problems


## Using Matrix Functions for Finance

In this section...<br>"Introduction" on page 1-5<br>"Key Definitions" on page 1-5<br>"Referencing Matrix Elements" on page 1-6<br>"Transposing Matrices" on page 1-7

## Introduction

Many financial analysis procedures involve sets of numbers; for example, a portfolio of securities at various prices and yields. Matrices, matrix functions, and matrix algebra are the most efficient ways to analyze sets of numbers and their relationships. Spreadsheets focus on individual cells and the relationships between cells. While you can think of a set of spreadsheet cells (a range of rows and columns) as a matrix, a matrix-oriented tool like MATLAB software manipulates sets of numbers more quickly, easily, and naturally.

## Key Definitions

Marrix. A rectangular array of numeric or algebraic quantities subject to mathematical operations; the regular formation of elements into rows and columns. Described as a " $m$-by- $n$ " matrix, with $m$ the number of rows and $n$ the number of columns. The description is always "row-by-column." For example, here is a 2 -by- 3 matrix of two bonds (the rows) with different par values, coupon rates, and coupon payment frequencies per year (the columns) entered using MATLAB notation:

$$
\begin{aligned}
& \text { Bonds }=\left[\begin{array}{lll}
1000 & 0.06 & 2
\end{array}\right. \\
& 500 \quad 0.055 \text { 4] }
\end{aligned}
$$

Vector. A matrix with only one row or column. Described as a "1-by-n" or " $m$-by-1" matrix. The description is always "row-by-column." For example, here is a 1-by- 4 vector of cash flows in MATLAB notation:

$$
\text { Cash }=\left[\begin{array}{llll}
1500 & 4470 & 5280 & -1299
\end{array}\right]
$$

Scalar. A 1-by-1 matrix; that is, a single number.

## Referencing Matrix Elements

To reference specific matrix elements, use (row, column) notation. For example:

```
Bonds(1,2)
ans =
                    0.06
Cash(3)
ans =
5280.00
```

You can enlarge matrices using small matrices or vectors as elements. For example,

```
AddBond = [1000 0.065 2];
Bonds = [Bonds; AddBond]
```

adds another row to the matrix and creates

```
Bonds =
    1000 0.06 2
    500 0.055 4
    1000 0.065 2
```

Likewise,

```
Prices = [987.50
    475.00
    995.00]
Bonds = [Prices, Bonds]
```

adds another column and creates

| Bonds $=$ |  |  |  |
| :--- | ---: | :--- | :--- |
|  |  |  |  |
| 987.50 | 1000 | 0.06 | 2 |
| 475.00 | 500 | 0.055 | 4 |
| 995.00 | 1000 | 0.065 | 2 |

Finally, the colon (:) is important in generating and referencing matrix elements. For example, to reference the par value, coupon rate, and coupon frequency of the second bond:

```
BondItems = Bonds(2, 2:4)
BondItems =
```

    \(500.00 \quad 0.055 \quad 4\)
    
## Transposing Matrices

Sometimes matrices are in the wrong configuration for an operation. In MATLAB, the apostrophe or prime character (') transposes a matrix: columns become rows, rows become columns. For example,

Cash = [ $\left.\begin{array}{llll}1500 & 4470 & 5280 & -1299\end{array}\right]^{\prime}$
produces
Cash =

1500
4470
5280
-1299

## Matrix Algebra Refresher

In this section...<br>"Introduction" on page 1-8<br>"Adding and Subtracting Matrices" on page 1-8<br>"Multiplying Matrices" on page 1-9<br>"Dividing Matrices" on page 1-14<br>"Solving Simultaneous Linear Equations" on page 1-15<br>"Operating Element by Element" on page 1-18

## Introduction

The explanations in the sections that follow should help refresh your skills for using matrix algebra and using MATLAB functions.

In addition, William Sharpe's Macro-Investment Analysis also provides an excellent explanation of matrix algebra operations using MATLAB. It is available on the Web at:

```
http://www.stanford.edu/~wfsharpe/mia/mia.htm
```

Tip When you are setting up a problem, it helps to "talk through" the units and dimensions associated with each input and output matrix. In the example under "Multiplying Matrices" on page 1-9, one input matrix has "five days' closing prices for three stocks," the other input matrix has "shares of three stocks in two portfolios," and the output matrix therefore has "five days' closing values for two portfolios." It also helps to name variables using descriptive terms.

## Adding and Subtracting Matrices

Matrix addition and subtraction operate element-by-element. The two input matrices must have the same dimensions. The result is a new matrix of the same dimensions where each element is the sum or difference of each corresponding input element. For example, consider combining portfolios of
different quantities of the same stocks ("shares of stocks A, B, and C [the rows] in portfolios P and Q [the columns] plus shares of $\mathrm{A}, \mathrm{B}$, and C in portfolios $R$ and $S$ ").

```
Portfolios_PQ = [100 200
    500 400
    300 150];
Portfolios_RS = [175 125
    200 200
    100 500];
NewPortfolios = Portfolios_PQ + Portfolios_RS
NewPortfolios =
```

    \(275 \quad 325\)
    700600
    400650
    Adding or subtracting a scalar and a matrix is allowed and also operates element-by-element.

```
SmallerPortf = NewPortfolios-10
SmallerPortf =
    265.00 315.00
    690.00 590.00
    390.00 640.00
```


## Multiplying Matrices

Matrix multiplication does not operate element-by-element. It operates according to the rules of linear algebra. In multiplying matrices, it helps to remember this key rule: the inner dimensions must be the same. That is, if the first matrix is $m$-by- 3 , the second must be 3 -by- $n$. The resulting matrix is $m$-by- $n$. It also helps to "talk through" the units of each matrix, as mentioned in "Using Matrix Functions for Finance" on page 1-5.

Matrix multiplication also is not commutative; that is, it is not independent of order. A*B does not equal B*A. The dimension rule illustrates this property.

If $A$ is 1 -by- 3 matrix and $B$ is 3 -by- 1 matrix, $A * B$ yields a scalar (1-by- 1 ) matrix but $\mathrm{B} * \mathrm{~A}$ yields a 3 -by- 3 matrix.

## Multiplying Vectors

Vector multiplication follows the same rules and helps illustrate the principles. For example, a stock portfolio has three different stocks and their closing prices today are:

```
ClosePrices = [\begin{array}{llll}{42.5 15 78.875]}\end{array}]
```

The portfolio contains these numbers of shares of each stock.

$$
\text { NumShares }=[100
$$

500
300]
To find the value of the portfolio, multiply the vectors

```
PortfValue = ClosePrices * NumShares
```

which yields:

```
PortfValue =
```

$$
3.5413 e+004
$$

The vectors are 1-by-3 and 3-by-1; the resulting vector is 1-by-1, a scalar. Multiplying these vectors thus means multiplying each closing price by its respective number of shares and summing the result.

To illustrate order dependence, switch the order of the vectors

```
Values = NumShares * ClosePrices
Values =
    1.0e+004 *
        0.4250 0.1500 0.7887
        2.1250 0.7500 3.9438
```

$$
\begin{array}{lll}
1.2750 & 0.4500 & 2.3663
\end{array}
$$

which shows the closing values of 100,500 , and 300 shares of each stock, not the portfolio value, and meaningless for this example.

## Computing Dot Products of Vectors

In matrix algebra, if $X$ and $Y$ are vectors of the same length

$$
\begin{aligned}
& Y=\left[y_{1}, y_{2}, \ldots, y_{n}\right] \\
& X=\left[x_{1}, x_{2}, \ldots, x_{n}\right]
\end{aligned}
$$

then the dot product

$$
X \square Y=x_{1} y_{1}+x_{2} y_{2}+\ldots+x_{n} y_{n}
$$

is the scalar product of the two vectors. It is an exception to the commutative rule. To compute the dot product in MATLAB, use sum (X . * Y) or sum (Y .* $X)$. Just be sure the two vectors have the same dimensions. To illustrate, use the previous vectors.

```
Value = sum(NumShares .* ClosePrices')
Value =
    3.5413e+004
Value = sum(ClosePrices .* NumShares')
Value =
    3.5413e+004
```

As expected, the value in these cases matches the PortfValue computed previously.

## Multiplying Vectors and Matrices

Multiplying vectors and matrices follows the matrix multiplication rules and process. For example, a portfolio matrix contains closing prices for a week. A second matrix (vector) contains the stock quantities in the portfolio.

```
WeekClosePr = [\begin{array}{lll}{42.5 15 78.875}\end{array}]
    42.125 15.5 78.75
    42.125 15.125 79
    42.625 15.25 78.875
    43 15.25 78.625];
PortQuan = [100
    500
    300];
```

To see the closing portfolio value for each day, simply multiply

```
WeekPortValue = WeekClosePr * PortQuan
WeekPortValue =
1.0e+004 *
    3.5412
    3.5587
    3.5475
    3.5550
    3.5513
```

The prices matrix is 5 -by-3, the quantity matrix (vector) is 3 -by- 1 , so the resulting matrix (vector) is $5-$ by- 1 .

## Multiplying Two Matrices

Matrix multiplication also follows the rules of matrix algebra. In matrix algebra notation, if $A$ is an $m$-by- $n$ matrix and $B$ is an $n$-by- $p$ matrix

$$
A=\left[\begin{array}{cccc}
a_{11} & a_{12} & \cdots & a_{1 n} \\
\vdots & \vdots & & \vdots \\
a_{i 1} & a_{i 2} & \cdots & a_{i n} \\
\vdots & \vdots & & \vdots \\
a_{m 1} & a_{m 2} & \cdots & a_{m n}
\end{array}\right], \quad B=\left[\begin{array}{ccccc}
b_{11} & \cdots & b_{1 j} & \cdots & b_{1 p} \\
b_{21} & \cdots & b_{2 j} & \cdots & b_{2 p} \\
\vdots & & \vdots & & \vdots \\
b_{n 1} & \cdots & b_{n j} & \cdots & b_{n p}
\end{array}\right]
$$

then $C=A^{*} B$ is an $m$-by- $p$ matrix; and the element $c_{\mathrm{ij}}$ in the $i$ th row and $j$ th column of $C$ is

$$
c_{i j}=a_{i 1} b_{1 j}+a_{i 2} b_{12}+\ldots+a_{i n} b_{n j} .
$$

To illustrate, assume there are two portfolios of the same three stocks above but with different quantities.

```
Portfolios = [100 200
    500 400
    300 150];
```

Multiplying the 5 -by- 3 week's closing prices matrix by the 3 -by- 2 portfolios matrix yields a 5-by-2 matrix showing each day's closing value for both portfolios.
PortfolioValues $=$ WeekClosePr * Portfolios
PortfolioValues $=$
1.0e+004 *

| 3.5412 | 2.6331 |
| :--- | :--- |
| 3.5587 | 2.6437 |
| 3.5475 | 2.6325 |
| 3.5550 | 2.6456 |
| 3.5513 | 2.6494 |

Monday's values result from multiplying each Monday closing price by its respective number of shares and summing the result for the first portfolio, then doing the same for the second portfolio. Tuesday's values result from multiplying each Tuesday closing price by its respective number of shares and
summing the result for the first portfolio, then doing the same for the second portfolio. And so on through the rest of the week. With one simple command, MATLAB quickly performs many calculations.

## Multiplying a Matrix by a Scalar

Multiplying a matrix by a scalar is an exception to the dimension and commutative rules. It just operates element-by-element.

```
Portfolios = [100 200
    500 400
    300 150];
DoublePort = Portfolios * 2
DoublePort =
    200 400
    1000 800
    600 300
```


## Dividing Matrices

Matrix division is useful primarily for solving equations, and especially for solving simultaneous linear equations (see "Solving Simultaneous Linear Equations" on page 1-15). For example, you want to solve for $X$ in $A^{*} X=B$.

In ordinary algebra, you would divide both sides of the equation by $A$, and $X$ would equal $B / A$. However, since matrix algebra is not commutative ( $A^{*} X$ $\neq X^{*} A$ ), different processes apply. In formal matrix algebra, the solution involves matrix inversion. MATLAB, however, simplifies the process by providing two matrix division symbols, left and right ( $\backslash$ and / ). In general,
$X=A \backslash B$ solves for $X$ in $A^{*} X=B$ and
$X=B / A$ solves for $X$ in $X^{*} A=B$.
In general, matrix A must be a nonsingular square matrix; that is, it must be invertible and it must have the same number of rows and columns. (Generally, a matrix is invertible if the matrix times its inverse equals the identity matrix. To understand the theory and proofs, consult a textbook on linear algebra such as Elementary Linear Algebra by Hill listed in Appendix

A, "Bibliography".) MATLAB gives a warning message if the matrix is singular or nearly so.

## Solving Simultaneous Linear Equations

Matrix division is especially useful in solving simultaneous linear equations. Consider this problem: Given two portfolios of mortgage-based instruments, each with certain yields depending on the prime rate, how do you weight the portfolios to achieve certain annual cash flows? The answer involves solving two linear equations.

A linear equation is any equation of the form

$$
a_{1} x+a_{2} y=b,
$$

where $a_{1}, a_{2}$, and $b$ are constants (with $a_{1}$ and $a_{2}$ not both 0 ), and $x$ and $y$ are variables. (It's a linear equation because it describes a line in the $x y$-plane. For example, the equation $2 x+y=8$ describes a line such that if $x=2$, then $y=4$.)

A system of linear equations is a set of linear equations that you usually want to solve at the same time; that is, simultaneously. A basic principle for exact answers in solving simultaneous linear equations requires that there be as many equations as there are unknowns. To get exact answers for $x$ and $y$, there must be two equations. For example, to solve for $x$ and $y$ in the system of linear equations

$$
\begin{aligned}
& 2 x+y=13 \\
& x-3 y=-18,
\end{aligned}
$$

there must be two equations, which there are. Matrix algebra represents this system as an equation involving three matrices: $A$ for the left-side constants, $X$ for the variables, and $B$ for the right-side constants

$$
A=\left[\begin{array}{cc}
2 & 1 \\
1 & -3
\end{array}\right], \quad X=\left[\begin{array}{l}
x \\
y
\end{array}\right], \quad B=\left[\begin{array}{c}
13 \\
-18
\end{array}\right],
$$

where $A^{*} X=B$.

Solving the system simultaneously means solving for $X$. Using MATLAB,

$$
\begin{aligned}
& \left.A=\begin{array}{rc}
2 & 1 \\
1 & -3
\end{array}\right] ; \\
& B=\left[\begin{array}{l}
{[13} \\
-18
\end{array}\right] ; \\
& X=A \backslash B
\end{aligned}
$$

solves for $X$ in $A * X=B$.

$$
X=\left[\begin{array}{ll}
3 & 7
\end{array}\right]
$$

So $x=3$ and $y=7$ in this example. In general, you can use matrix algebra to solve any system of linear equations such as

$$
\begin{gathered}
a_{11} x_{1}+a_{12} x_{2}+\ldots+a_{1 n} x_{n}=b_{1} \\
a_{21} x_{1}+a_{22} x_{2}+\ldots+a_{2 n} x_{n}=b_{2} \\
\vdots \\
a_{m 1} x_{1}+a_{m 2} x_{2}+\ldots+a_{m n} x_{n}=b_{m}
\end{gathered}
$$

by representing them as matrices

$$
A=\left[\begin{array}{cccc}
a_{11} & a_{12} & \cdots & a_{1 n} \\
a_{21} & a_{22} & \cdots & a_{2 n} \\
\vdots & \vdots & & \vdots \\
a_{m 1} & a_{m 2} & \cdots & a_{m n}
\end{array}\right], \quad X=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\vdots \\
x_{n}
\end{array}\right], \quad B=\left[\begin{array}{c}
b_{1} \\
b_{2} \\
\vdots \\
b_{m}
\end{array}\right]
$$

and solving for $X$ in $A^{*} X=B$.
To illustrate, consider this situation. There are two portfolios of mortgage-based instruments, M1 and M2. They have current annual cash payments of $\$ 100$ and $\$ 70$ per unit, respectively, based on today's prime rate. If the prime rate moves down one percentage point, their payments would be $\$ 80$ and $\$ 40$. An investor holds 10 units of M1 and 20 units of M2. The investor's receipts equal cash payments times units, or $\mathrm{R}=\mathrm{C}$ * U , for each prime-rate scenario. As word equations:

|  | M1 | M2 |
| :--- | :--- | :--- |
| Prime flat: | $\$ 100 * 10$ units | $+\$ 70 * 20$ units $=\$ 2400$ <br> receipts |
| Prime down: | $\$ 80 * 10$ units | $+\$ 40 * 20$ units $=\$ 1600$ <br> receipts |

As MATLAB matrices:

```
Cash = [100 70
    80 40];
Units = [10
    20];
Receipts = Cash * Units
Receipts =
```

    2400
    1600
    Now the investor asks this question: Given these two portfolios and their characteristics, how many units of each should I hold to receive $\$ 7000$ if the prime rate stays flat and $\$ 5000$ if the prime drops one percentage point? Find the answer by solving two linear equations.

|  | M1 | M2 |
| :--- | :--- | :--- |
| Prime flat: | $\$ 100 * x$ units | $+\$ 70 * y$ units $=\$ 7000$ <br> receipts |
| Prime down: | $\$ 80 * x$ units | $+\$ 40 * y$ units $=\$ 5000$ <br> receipts |

In other words, solve for U (units) in the equation R (receipts) $=\mathrm{C}$ (cash) * U (units). Using MATLAB left division

```
Cash = [100 70
    80 40];
```

```
Receipts = [7000
    5000];
Units = Cash \ Receipts
Units =
```

43.7500
37.5000

The investor should hold 43.75 units of portfolio M1 and 37.5 units of portfolio M2 to achieve the annual receipts desired.

## Operating Element by Element

Finally, element-by-element arithmetic operations are called array operations. To indicate a MATLAB array operation, precede the operator with a period (.). Addition and subtraction, and matrix multiplication and division by a scalar, are already array operations so no period is necessary. When using array operations on two matrices, the dimensions of the matrices must be the same. For example, given vectors of stock dividends and closing prices

```
Dividends = [ll.90 0.40 1.56 4.50];
Prices = [l25.625 17.75 26.125 60.50];
Yields = Dividends ./ Prices
Yields =
    0.0741 0.0225 0.0597 0.0744
```


## Function Input and Output Arguments

In this section...<br>"Input Arguments" on page 1-19<br>"Output Arguments" on page 1-21<br>"Interest Rate Arguments" on page 1-22

## Input Arguments

## Matrix Input

MATLAB software was designed to be a large-scale array (vector or matrix) processor. In addition to its linear algebra applications, the general array-based processing facility can perform repeated operations on collections of data. When MATLAB code is written to operate simultaneously on collections of data stored in arrays, the code is said to be vectorized. Vectorized code is not only clean and concise, but is also efficiently processed by the underlying MATLAB engine.

Because MATLAB can process vectors and matrices easily, most Financial Toolbox functions allow vector or matrix input arguments, rather than just single (scalar) values. For example, the irr function computes the internal rate of return of a cash flow stream. It accepts a vector of cash flows and returns a scalar-valued internal rate of return. However, it also accepts a matrix of cash flow streams, a column in the matrix representing a different cash flow stream. In this case, irr returns a vector of internal rates of return, each entry in the vector corresponding to a column of the input matrix. Many other toolbox functions work similarly.

As an example, suppose you make an initial investment of $\$ 100$, from which you then receive by a series of annual cash receipts of $\$ 10, \$ 20, \$ 30, \$ 40$, and $\$ 50$. This cash flow stream may be stored in a vector

$$
\text { CashFlows }=\left[\begin{array}{llllll}
-100 & 10 & 20 & 30 & 40 & 50
\end{array}\right]
$$

which MATLAB displays as

The irr function can compute the internal rate of return of this stream.
Rate = irr(CashFlows)

The internal rate of return of this investment is
0.1201
or $12.01 \%$.
In this case, a single cash flow stream (written as an input vector) produces a scalar output - the internal rate of return of the investment.

Extending this example, if you process a matrix of identical cash flow streams

```
Rate = irr([CashFlows CashFlows CashFlows])
```

you should expect to see identical internal rates of return for each of the three investments.

```
Rate =
    0.1201
        0.1201
        0.1201
```

This simple example illustrates the power of vectorized programming. The example shows how to collect data into a matrix and then use a toolbox function to compute answers for the entire collection. This feature can be useful in portfolio management, for example, where you might want to organize multiple assets into a single collection. Place data for each asset in a different column or row of a matrix, then pass the matrix to a Financial Toolbox function. MATLAB performs the same computation on all of the assets at once.

## Matrices of String Input

Enter MATLAB strings surrounded by single quotes ('string').
Strings are stored as character arrays, one ASCII character per element. Thus, the date string

```
DateString = '9/16/2001'
```

is actually a 1 -by- 9 vector. Strings making up the rows of a matrix or vector all must have the same length. To enter several date strings, therefore, use a column vector and be sure all strings are the same length. Fill in with spaces or zeros. For example, to create a vector of dates corresponding to irregular cash flows

```
DateFields = ['01/12/2001'
    '02/14/2001'
    '03/03/2001'
    '06/14/2001'
    '12/01/2001'];
```

DateFields actually becomes a 5-by-10 character array.
Don't mix numbers and strings in a matrix. If you do, MATLAB treats all entries as characters. For example,

```
Item = [83 90 99 '14-Sep-1999']
```

becomes a 1-by-14 character array, not a 1-by-4 vector, and it contains

```
Item =
```

SZc14-Sep-1999

## Output Arguments

Some functions return no arguments, some return just one, and some return multiple arguments. Functions that return multiple arguments use the syntax

```
[A, B, C] = function(variables...)
```

to return arguments A, B, and C. If you omit all but one, the function returns the first argument. Thus, for this example if you use the syntax

```
X = function(variables...)
```

function returns a value for A , but not for B or C .
Some functions that return vectors accept only scalars as arguments. Why could such functions not accept vectors as arguments and return matrices, where each column in the output matrix corresponds to an entry in the input vector? The answer is that the output vectors can be variable length and thus will not fit in a matrix without some convention to indicate that the shorter columns are missing data.

Functions that require asset life as an input, and return values corresponding to different periods over that life, cannot generally handle vectors or matrices as input arguments. Those functions are:

| amortize | Amortization |
| :--- | :--- |
| depfixdb | Fixed declining-balance depreciation |
| depgendb | General declining-balance depreciation |
| depsoyd | Sum of years' digits depreciation |

For example, suppose you have a collection of assets such as automobiles and you want to compute the depreciation schedules for them. The function depfixdb computes a stream of declining-balance depreciation values for an asset. You might want to set up a vector where each entry is the initial value of each asset. depfixdb also needs the lifetime of an asset. If you were to set up such a collection of automobiles as an input vector, and the lifetimes of those automobiles varied, the resulting depreciation streams would differ in length according to the life of each automobile, and the output column lengths would vary. A matrix must have the same number of rows in each column.

## Interest Rate Arguments

One common argument, both as input and output, is interest rate. All Financial Toolbox functions expect and return interest rates as decimal fractions. Thus an interest rate of $9.5 \%$ is indicated as 0.095 .

# Performing Common Financial Tasks 

- "Introduction" on page 2-2
- "Handling and Converting Dates" on page 2-4
- "Formatting Currency" on page 2-12
- "Charting Financial Data" on page 2-13
- "Analyzing and Computing Cash Flows" on page 2-17
- "Pricing and Computing Yields for Fixed-Income Securities" on page 2-21
- "Term Structure of Interest Rates" on page 2-36
- "Pricing and Analyzing Equity Derivatives" on page 2-39


## Introduction

Financial Toolbox software contains functions that perform many common financial tasks, including:

- "Handling and Converting Dates" on page 2-4

Calendar functions convert dates among different formats (including Excel ${ }^{\circledR}$ formats), determine future or past dates, find dates of holidays and business days, compute time differences between dates, find coupon dates and coupon periods for coupon bonds, and compute time periods based on $360-$ - 365 -, or 366 -day years.

- "Formatting Currency" on page 2-12

The toolbox includes functions for handling decimal values in bank (currency) formats and as fractional prices.

- "Charting Financial Data" on page 2-13

Charting functions produce a variety of financial charts including Bollinger bands, high-low-close charts, candlestick plots, point and figure plots, and moving-average plots.

- "Analyzing and Computing Cash Flows" on page 2-17

Cash-flow evaluation and financial accounting functions compute interest rates, rates of return, payments associated with loans and annuities, future and present values, depreciation, and other standard accounting calculations associated with cash-flow streams.

- "Pricing and Computing Yields for Fixed-Income Securities" on page 2-21

Securities Industry Association (SIA) compliant fixed-income functions compute prices, yields, accrued interest, and sensitivities for securities such as bonds, zero-coupon bonds, and Treasury bills. They handle odd first and last periods in price/yield calculations, compute accrued interest and discount rates, and calculate convexity and duration. Another set of functions analyzes term structure of interest rates, including pricing bonds from yield curves and bootstrapping yield curves from market prices.

- "Pricing and Analyzing Equity Derivatives" on page 2-39

Derivatives analysis functions compute prices, yields, and sensitivities for derivative securities. They deal with both European and American options.

Black-Scholes functions work with European options. They compute delta, gamma, lambda, rho, theta, and vega, as well as values of call and put options.
Binomial functions work with American options, computing put and call prices.

- "Analyzing Portfolios" on page 3-2

Portfolio analysis functions provide basic utilities to compute variances and covariance of portfolios, find combinations to minimize variance, compute Markowitz efficient frontiers, and calculate combined rates of return.

- Modeling volatility in time series.

> Generalized Autoregressive Conditional Heteroskedasticity (GARCH) functions model the volatility of univariate economic time series. (Econometrics Toolbox ${ }^{\mathrm{TM}}$ software provides a more comprehensive and integrated computing environment. For information, see the Econometrics Toolbox User's Guide documentation or the financial products Web page at http://www.mathworks.com/products/finprod.)

## Handling and Converting Dates

In this section...<br>"Date Formats" on page 2-4<br>"Date Conversions" on page 2-5<br>"Current Date and Time" on page 2-8<br>"Determining Dates" on page 2-9

## Date Formats

Since virtually all financial data is dated or derives from a time series, financial functions must have extensive date-handling capabilities. You most often work with date strings (14-Sep-1999) when dealing with dates. Financial Toolbox software works internally with serial date numbers (for example, 730377). A serial date number represents a calendar date as the number of days that has passed since a fixed base date. In MATLAB software, serial date number 1 is January 1, 0000 A.D. MATLAB also uses serial time to represent fractions of days beginning at midnight; for example, 6 p.m. equals 0.75 serial days. So 6:00 p.m. on 14-Sep-1999, in MATLAB, is date number 730377.75.

Note If you specify a two-digit year, MATLAB assumes that the year lies within the 100-year period centered about the current year. See the function datenum for specific information. MATLAB internal date handling and calculations generate no ambiguous values. However, whenever possible, programmers should use serial date numbers or date strings containing four-digit years.

Many toolbox functions that require dates accept either date strings or serial date numbers. If you are dealing with a few dates at the MATLAB command-line level, date strings are more convenient. If you are using toolbox functions on large numbers of dates, as in analyzing large portfolios or cash flows, performance improves if you use date numbers.

The Financial Toolbox software provides functions that convert date strings to serial date numbers, and vice versa.

## Date Conversions

Functions that convert between date formats are

| datedisp | Displays a numeric matrix with date entries formatted <br> as date strings |
| :--- | :--- |
| datenum | Converts a date string to a serial date number |
| datestr | Converts a serial date number to a date string |
| m2xdate | Converts MATLAB serial date number to Excel serial <br> date number |
| x2mdate | Converts Excel serial date number to MATLAB serial <br> date number |

Another function, datevec, converts a date number or date string to a date vector whose elements are [Year Month Day Hour Minute Second]. Date vectors are mostly an internal format for some MATLAB functions; you would not often use them in financial calculations.

## Input Conversions

The datenum function is important for using Financial Toolbox software efficiently. datenum takes an input string in any of several formats, with 'dd-mmm-yyyy', 'mm/dd/yyyy' or 'dd-mmm-yyyy, hh:mm:ss.ss' most common. The input string can have up to six fields formed by letters and numbers separated by any other characters:

- The day field is an integer from 1 through 31.
- The month field is either an integer from 1 through 12 or an alphabetical string with at least three characters.
- The year field is a nonnegative integer: if only two numbers are specified, then the year is assumed to lie within the 100-year period centered about the current year; if the year is omitted, the current year is used as the default.
- The hours, minutes, and seconds fields are optional. They are integers separated by colons or followed by 'am' or 'pm'.

For example, if the current year is 1999, then these are all equivalent

```
'17-May-1999'
'17-May-99'
'17-may'
'May 17, 1999'
'5/17/99'
'5/17'
```

and both of these represent the same time.

```
'17-May-1999, 18:30'
'5/17/99/6:30 pm'
```

Note that the default format for numbers-only input follows the American convention. Thus $3 / 6$ is March 6, not June 3.

With datenum you can convert dates into serial date format, store them in a matrix variable, then later pass the variable to a function. Alternatively, you can use datenum directly in a function input argument list.

For example, consider the function bndprice that computes the price of a bond given the yield-to-maturity. First set up variables for the yield-to-maturity, coupon rate, and the necessary dates.

```
Yield = 0.07;
CouponRate = 0.08;
Settle = datenum('17-May-2000');
Maturity = datenum('01-Oct-2000');
```

Then call the function with the variables

```
bndprice(Yield, CouponRate, Settle, Maturity)
```

Alternatively, convert date strings to serial date numbers directly in the function input argument list.

```
bndprice(0.07, 0.08, datenum('17-May-2000'),...
datenum('01-0ct-2000'))
```

bndprice is an example of a function designed to detect the presence of date strings and make the conversion automatically. For these functions date strings may be passed directly.

```
bndprice(0.07, 0.08, '17-May-2000', '01-Oct-2000')
```

The decision to represent dates as either date strings or serial date numbers is often a matter of convenience. For example, when formatting data for visual display or for debugging date-handling code, it is often much easier to view dates as date strings because serial date numbers are difficult to interpret. Alternatively, serial date numbers are just another type of numeric data, and can be placed in a matrix along with any other numeric data for convenient manipulation.

Remember that if you create a vector of input date strings, use a column vector and be sure all strings are the same length. Fill with spaces or zeros. See "Matrices of String Input" on page 1-21.

## Output Conversions

The function datestr converts a serial date number to one of 19 different date string output formats showing date, time, or both. The default output for dates is a day-month-year string, for example, 24-Aug-2000. This function is quite useful for preparing output reports.

| Format | Description |
| :--- | :--- |
| 01-Mar-2000 15:45:17 | day-month-year hour:minute:second |
| 01-Mar-2000 | day-month-year |
| 03/01/00 | month/day/year |
| Mar | month, three letters |
| M | month, single letter |
| 3 | month |
| 03/01 | month/day |
| 1 | day of month |
| Wed | day of week, three letters |
| W | day of week, single letter |
| 2000 | year, four numbers |
| 99 | year, two numbers |


| Format | Description |
| :--- | :--- |
| Mar01 | month year |
| $15: 45: 17$ | hour:minute:second |
| $03: 45: 17$ PM | hour:minute:second AM or PM |
| $15: 45$ | hour:minute |
| $03: 45$ PM | hour:minute AM or PM |
| Q1-99 | calendar quarter-year |
| Q1 | calendar quarter |

## Current Date and Time

The functions today and now return serial date numbers for the current date, and the current date and time, respectively.

```
today
```

ans =
730693
now
ans =
730693.48

The MATLAB function date returns a string for today's date.

```
date
```

ans $=$
26-Jul-2000

## Determining Dates

The Financial Toolbox software provides many functions for determining specific dates, including functions which account for holidays and other nontrading days. For example, you schedule an accounting procedure for the last Friday of every month. The lweekdate function returns those dates for 2000; the 6 specifies Friday.

Fridates = lweekdate(6, 2000, 1:12);

Fridays = datestr(Fridates)

Fridays =

28-Jan-2000
25-Feb-2000
31-Mar-2000
28-Apr-2000
26-May-2000
30-Jun-2000
28-Jul-2000
25-Aug-2000
29-Sep-2000
27-Oct-2000
24-Nov-2000
29-Dec-2000

Or your company closes on Martin Luther King Jr. Day, which is the third Monday in January. The nweekdate function determines those dates for 2001 through 2004.

```
MLKDates = nweekdate(3, 2, 2001:2004, 1);
MLKDays = datestr(MLKDates)
MLKDays =
15-Jan-2001
21-Jan-2002
20-Jan-2003
19-Jan-2004
```

Accounting for holidays and other nontrading days is important when examining financial dates. The Financial Toolbox software provides the holidays function, which contains holidays and special nontrading days for the New York Stock Exchange between 1950 and 2030, inclusive. In addition, you can use nyseclosures to evaluate all known or anticipated closures of the New York Stock Exchange from January 1, 1885 to December 31, 2050. nyseclosures returns a vector of serial date numbers corresponding to market closures between the dates StartDate and EndDate, inclusive.

In this example, you can use holidays to determine the standard holidays in the last half of 2000 :

```
LHHDates = holidays('1-Jul-2000', '31-Dec-2000');
LHHDays = datestr(LHHDates)
LHHDays =
04-Jul-2000
04-Sep-2000
23-Nov-2000
25-Dec-2000
```

Now use the toolbox busdate function to determine the next business day after these holidays.

```
LHNextDates = busdate(LHHDates);
LHNextDays = datestr(LHNextDates)
LHNextDays =
05-Jul-2000
05-Sep-2000
24-Nov-2000
26-Dec-2000
```

The toolbox also provides the cfdates function to determine cash-flow dates for securities with periodic payments. This function accounts for the coupons per year, the day-count basis, and the end-of-month rule. For example, to
determine the cash-flow dates for a security that pays four coupons per year on the last day of the month, on an actual/365 day-count basis, just enter the settlement date, the maturity date, and the parameters.

```
PayDates = cfdates('14-Mar-2000', '30-Nov-2001', 4, 3, 1);
PayDays = datestr(PayDates)
PayDays =
31-May-2000
31-Aug-2000
30-Nov-2000
28-Feb-2001
31-May-2001
31-Aug-2001
30-Nov-2001
```


## Formatting Currency

Financial Toolbox software provides several functions to format currency and chart financial data. The currency formatting functions are
\(\left.$$
\begin{array}{ll}\text { cur2frac } & \begin{array}{l}\text { Converts decimal currency values to } \\
\text { fractional values }\end{array}
$$ <br>
cur2str \& Converts a value to Financial Toolbox bank <br>

format\end{array}\right\}\)| Converts fractional currency values to |
| :--- |
| decimal values |

These examples show their use.

```
Dec = frac2cur('12.1', 8)
```

returns Dec $=12.125$, which is the decimal equivalent of $12-1 / 8$. The second input variable is the denominator of the fraction.

```
Str = cur2str(-8264, 2)
```

returns the string ( $\$ 8264.00$ ). For this toolbox function, the output format is a numerical format with dollar sign prefix, two decimal places, and negative numbers in parentheses; for example, (\$123.45) and \$6789.01. The standard MATLAB bank format uses two decimal places, no dollar sign, and a minus sign for negative numbers; for example, -123.45 and 6789.01.

## Charting Financial Data

In this section...<br>"Introduction" on page 2-13<br>"High-Low-Close Chart Example" on page 2-14<br>"Bollinger Chart Example" on page 2-15

## Introduction

The following toolbox financial charting functions plot financial data and produce presentation-quality figures quickly and easily.

| bolling | Bollinger band chart |
| :--- | :--- |
| bollinger | Time series Bollinger band |
| candle | Candlestick chart |
| candle | Time series candle plot |
| pointfig | Point and figure chart |
| highlow | High, low, open, close chart |
| highlow | Time series High-Low plot |
| movavg | Leading and lagging moving averages chart |

These functions work with standard MATLAB functions that draw axes, control appearance, and add labels and titles. The toolbox also provides a comprehensive set of charting functions that work with financial time series objects. For lists of these, see "Financial Data Charts" on page 15-6 and "Financial Time Series Indicator" on page 15-27.

Here are two plotting examples: a high-low-close chart of sample IBM $^{\circledR}$ stock price data, and a Bollinger band chart of the same data. These examples load data from an external file (ibm.dat), then call the functions using subsets of the data. The MATLAB variable ibm, which is created by loading ibm.dat, is a six-column matrix where each row is a trading day's data and where columns 2,3 , and 4 contain the high, low, and closing prices, respectively.

Note The data in ibm. dat is fictional and for illustrative use only.

## High-Low-Close Chart Example

First load the data and set up matrix dimensions. load and size are standard MATLAB functions.

```
load ibm.dat;
[ro, co] = size(ibm);
```

Open a figure window for the chart. Use the Financial Toolbox highlow function to plot high, low, and close prices for the last 50 trading days in the data file.

```
figure;
highlow(ibm(ro-50:ro,2),ibm(ro-50:ro,3),ibm(ro-50:ro,4),[],'b');
```

Add labels and title, and set axes with standard MATLAB functions. Use the Financial Toolbox dateaxis function to provide dates for the $x$-axis ticks.

```
xlabel('');
ylabel('Price ($)');
title('International Business Machines, 941231 - 950219');
axis([0 50 -inf inf]);
dateaxis('x',6,'31-Dec-1994')
```

MATLAB produces a figure like this. The plotted data and axes you see may differ. Viewed online, the high-low-close bars are blue.


## Bollinger Chart Example

The bolling function in Financial Toolbox software produces a Bollinger band chart using all the closing prices in the same IBM stock price matrix. A Bollinger band chart plots actual data along with three other bands of data. The upper band is two standard deviations above a moving average; the lower band is two standard deviations below that moving average; and the middle band is the moving average itself. This example uses a 15 -day moving average.

Assuming the previous IBM data is still loaded, execute the function.

```
bolling(ibm(:,4), 15, 0);
```

Specify the axes, labels, and titles. Again, use dateaxis to add the $x$-axis dates.

```
axis([0 ro min(ibm(:,4)) max(ibm(:,4))]);
ylabel('Price ($)');
```

```
    title(['International Business Machines']);
dateaxis('x', 6,'31-Dec-1994')
```



For help using MATLAB plotting functions, see Creating Plots in the MATLAB documentation. See the MATLAB documentation for details on the axis, title, xlabel, and ylabel functions.

## Analyzing and Computing Cash Flows

In this section...<br>"Introduction" on page 2-17<br>"Interest Rates/Rates of Return" on page 2-17<br>"Present or Future Values" on page 2-18<br>"Depreciation" on page 2-19<br>"Annuities" on page 2-19

## Introduction

Financial Toolbox cash-flow functions compute interest rates and rates of return, present or future values, depreciation streams, and annuities.

Some examples in this section use this income stream: an initial investment of $\$ 20,000$ followed by three annual return payments, a second investment of $\$ 5,000$, then four more returns. Investments are negative cash flows, return payments are positive cash flows.

```
Stream = [-20000, 2000, 2500, 3500, -5000, 6500,...
    9500, 9500, 9500];
```


## Interest Rates/Rates of Return

Several functions calculate interest rates involved with cash flows. To compute the internal rate of return of the cash stream, execute the toolbox function irr

$$
\text { ROR }=\operatorname{irr}(\text { Stream })
$$

which gives a rate of return of $11.72 \%$.
Note that the internal rate of return of a cash flow may not have a unique value. Every time the sign changes in a cash flow, the equation defining irr can give up to two additional answers. An irr computation requires solving a polynomial equation, and the number of real roots of such an equation can depend on the number of sign changes in the coefficients. The equation for internal rate of return is

$$
\frac{c f_{1}}{(1+r)}+\frac{c f_{2}}{(1+r)^{2}}+\ldots+\frac{c f_{n}}{(1+r)^{n}}+\text { Investment }=0
$$

where Investment is a (negative) initial cash outlay at time $0, c f_{\mathrm{n}}$ is the cash flow in the $n$th period, and $n$ is the number of periods. irr finds the rate $r$ such that the present value of the cash flow equals the initial investment. If all of the $c f_{\mathrm{n}} \mathrm{s}$ are positive there is only one solution. Every time there is a change of sign between coefficients, up to two additional real roots are possible.

Another toolbox rate function, effrr, calculates the effective rate of return given an annual interest rate (also known as nominal rate or annual percentage rate, APR) and number of compounding periods per year. To find the effective rate of a $9 \%$ APR compounded monthly, enter

$$
\text { Rate }=\operatorname{effrr}(0.09,12)
$$

The answer is $9.38 \%$.
A companion function nomrr computes the nominal rate of return given the effective annual rate and the number of compounding periods.

## Present or Future Values

The toolbox includes functions to compute the present or future value of cash flows at regular or irregular time intervals with equal or unequal payments: fvfix, fvvar, pvfix, and pvvar. The -fix functions assume equal cash flows at regular intervals, while the -var functions allow irregular cash flows at irregular periods.

Now compute the net present value of the sample income stream for which you computed the internal rate of return. This exercise also serves as a check on that calculation because the net present value of a cash stream at its internal rate of return should be zero. Enter

```
NPV = pvvar(Stream, ROR)
```

which returns an answer very close to zero. The answer usually is not exactly zero due to rounding errors and the computational precision of the computer.

Note Other toolbox functions behave similarly. The functions that compute a bond's yield, for example, often must solve a nonlinear equation. If you then use that yield to compute the net present value of the bond's income stream, it usually does not exactly equal the purchase price, but the difference is negligible for practical applications.

## Depreciation

The toolbox includes functions to compute standard depreciation schedules: straight line, general declining-balance, fixed declining-balance, and sum of years' digits. Functions also compute a complete amortization schedule for an asset, and return the remaining depreciable value after a depreciation schedule has been applied.

This example depreciates an automobile worth $\$ 15,000$ over five years with a salvage value of $\$ 1,500$. It computes the general declining balance using two different depreciation rates: $50 \%$ (or 1.5), and $100 \%$ (or 2.0, also known as double declining balance). Enter

```
Decline1 = depgendb(15000, 1500, 5, 1.5)
Decline2 = depgendb(15000, 1500, 5, 2.0)
```

which returns

| Decline $1=$ |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
| 4500.00 | 3150.00 | 2205.00 | 1543.50 | 2101.50 |
| Decline $=$ |  |  |  |  |
| 6000.00 | 3600.00 | 2160.00 | 1296.00 | 444.00 |

These functions return the actual depreciation amount for the first four years and the remaining depreciable value as the entry for the fifth year.

## Annuities

Several toolbox functions deal with annuities. This first example shows how to compute the interest rate associated with a series of loan payments when only the payment amounts and principal are known. For a loan whose original value was $\$ 5000.00$ and which was paid back monthly over four years at $\$ 130.00 /$ month

```
Rate = annurate(4*12, 130, 5000, 0, 0)
```

The function returns a rate of 0.0094 monthly, or about $11.28 \%$ annually.
The next example uses a present-value function to show how to compute the initial principal when the payment and rate are known. For a loan paid at $\$ 300.00 /$ month over four years at $11 \%$ annual interest

```
Principal = pvfix(0.11/12, 4*12, 300, 0, 0)
```

The function returns the original principal value of $\$ 11,607.43$.
The final example computes an amortization schedule for a loan or annuity. The original value was $\$ 5000.00$ and was paid back over 12 months at an annual rate of $9 \%$.

```
[Prpmt, Intpmt, Balance, Payment] = ...
    amortize(0.09/12, 12, 5000, 0, 0);
```

This function returns vectors containing the amount of principal paid,

```
Prpmt = [lllla9.76 402.76 405.78 408.82 411.89 414.97
    418.09 421.22 424.38 427.56 430.77 434.00]
```

the amount of interest paid,

```
Intpmt = [lllllllll}37.50 34.50 31.48 28.44 25.37 22.28
    19.17 16.03 12.88 9.69 6.49 3.26]
```

the remaining balance for each period of the loan,

```
Balance = [lllllo0.24 4197.49 3791.71 3382.89 2971.01
    2556.03 2137.94 1716.72 1292.34 864.77
    434.00 0.00]
```

and a scalar for the monthly payment.

```
Payment = 437.26
```


## Pricing and Computing Yields for Fixed-Income Securities

In this section...<br>"Introduction" on page 2-21<br>"Terminology" on page 2-21<br>"Framework" on page 2-26<br>"Default Parameter Values" on page 2-27<br>"Coupon Date Calculations" on page 2-30<br>"Yield Conventions" on page 2-31<br>"Pricing Functions" on page 2-31<br>"Yield Functions" on page 2-32<br>"Fixed-Income Sensitivities" on page 2-33

## Introduction

The Financial Toolbox product provides functions for computing accrued interest, price, yield, convexity, and duration of fixed-income securities. Various conventions exist for determining the details of these computations. The Financial Toolbox software supports conventions specified by the Securities Industry and Financial Markets Association (SIFMA), used in the US markets, the International Capital Market Association (ICMA), used mainly in the European markets, and the International Swaps and Derivatives Association (ISDA). Note that for historical reasons, SIFMA is referred to in Financial Toolbox documentation as SIA and ICMA is referred to as International Securities Market Association (ISMA).

## Terminology

Since terminology varies among texts on this subject, here are some basic definitions that apply to these Financial Toolbox functions. The "Glossary" on page Glossary-1 contains additional definitions.

The settlement date of a bond is the date when money first changes hands; that is, when a buyer pays for a bond. It need not coincide with the issue date, which is the date a bond is first offered for sale.

The first coupon date and last coupon date are the dates when the first and last coupons are paid, respectively. Although bonds typically pay periodic annual or semiannual coupons, the length of the first and last coupon periods may differ from the standard coupon period. The toolbox includes price and yield functions that handle these odd first and/or last periods.

Successive quasi-coupon dates determine the length of the standard coupon period for the fixed income security of interest, and do not necessarily coincide with actual coupon payment dates. The toolbox includes functions that calculate both actual and quasi-coupon dates for bonds with odd first and/or last periods.

Fixed-income securities can be purchased on dates that do not coincide with coupon payment dates. In this case, the bond owner is not entitled to the full value of the coupon for that period. When a bond is purchased between coupon dates, the buyer must compensate the seller for the pro-rata share of the coupon interest earned from the previous coupon payment date. This pro-rata share of the coupon payment is called accrued interest. The purchase price, the price actually paid for a bond, is the quoted market price plus accrued interest.

The maturity date of a bond is the date when the issuer returns the final face value, also known as the redemption value or par value, to the buyer. The yield-to-maturity of a bond is the nominal compound rate of return that equates the present value of all future cash flows (coupons and principal) to the current market price of the bond.

The period of a bond refers to the frequency with which the issuer of a bond makes coupon payments to the holder.

## Period of a Bond

| Period Value | Payment Schedule |
| :--- | :--- |
| 0 | No coupons (Zero coupon bond) |
| 1 | Annual |
| 2 | Semiannual |
| 3 | Tri-annual |
| 4 | Quarterly |

## Period of a Bond (Continued)

| Period Value | Payment Schedule |
| :--- | :--- |
| 6 | Bi-monthly |
| 12 | Monthly |

The basis of a bond refers to the basis or day-count convention for a bond. Basis is normally expressed as a fraction in which the numerator determines the number of days between two dates, and the denominator determines the number of days in the year. For example, the numerator of actual/actual means that when determining the number of days between two dates, count the actual number of days; the denominator means that you use the actual number of days in the given year in any calculations (either 365 or 366 days depending on whether the given year is a leap year).

The day count convention determines how accrued interest is calculated and determines how cash flows for the bond are discounted, thereby effecting price and yield calculations. Furthermore, the SIA convention is to use the actual/actual day count convention for discounting cash flows in all cases.

## Basis of a Bond

| Basis Value | Meaning | Description |
| :--- | :--- | :--- |
| 0 (default) | actual/actual | Actual days held over <br> actual days in coupon <br> period. Denominator is <br> 365 in most years and <br>  |
|  |  | 366 in a leap year. |
| 1 | $30 / 360$ (SIA) | Each month contains <br> 30 days; a year contains <br> 360 days. Payments <br> are adjusted for bonds <br> that pay coupons on the <br> last day of February. |

Basis of a Bond (Continued)

| Basis Value | Meaning | Description |
| :--- | :--- | :--- |
| 2 | actual/360 | Actual days held over <br> 360. |
| 3 | actual/365 | Actual days held over <br> 365, even in leap years. |
| 4 | $30 / 360$ BMA (Bond <br> Market Association) | Each month contains <br> 30 days; a year contains <br> 360 days. If the last <br> date of the period is the <br> last day of February, <br> the month is extended <br> to 30 days. |
| 5 | $30 / 360$ ISDA <br> (International Swap <br> Dealers Association) | Variant of 30/360 with <br> slight differences for <br> calculating number of <br> days in a month. |
| 6 | $30 / 360$ European | Variant of 30/360 used <br> primarily in Europe. |
| 7 | actual/365 Japanese | All years contain 365 <br> days. Leap days are <br> ignored. |
| 8 | actual/actual (ICMA) | Actual days held over <br> actual days in coupon <br> period. Denominator <br> is 365 in most years <br> and 366 in a leap year. <br> This basis assumes an <br> annual compounding <br> period. |
| 9 | actual/360 (ICMA) | Actual days held <br> over 360. This basis <br> assumes an annual <br> compounding period. |
|  |  |  |

## Basis of a Bond (Continued)

| Basis Value | Meaning | Description |
| :--- | :--- | :--- |
| 10 | actual/365 (ICMA) | Actual days held over <br> 365, even in leap years. <br> This basis assumes an <br> annual compounding <br> period. |
| 11 | $30 / 360 \mathrm{E}$ (ICMA) | The number of days in <br> every month is set to 30. <br> If the start date or the <br> end date of the period <br> is the 31st of a month, <br> that date is set to the <br> 30th. The number of <br> days in a year is 360. |
| 12 | actual/365 (ISDA) | This day count fraction <br> is equal to the sum <br> of number of interest <br> accrual days falling <br> with a leap year divided <br> by 366 and the number <br> of interest accrual days <br> not falling within a leap <br> year divided by 365. |
| 13 |  | BUS/252 |

Note Although the concept of day count sounds deceptively simple, the actual calculation of day counts can be quite complex. You can find a good discussion of day counts and the formulas for calculating them in Chapter 5 of Stigum and Robinson, Money Market and Bond Calculations in Appendix A, "Bibliography".

The end-of-month rule affects a bond's coupon payment structure. When the rule is in effect, a security that pays a coupon on the last actual day of a month will always pay coupons on the last day of the month. This means, for example, that a semiannual bond that pays a coupon on February 28 in nonleap years will pay coupons on August 31 in all years and on February 29 in leap years.

## End-of-Month Rule

| End-of-Month Rule Value | Meaning |
| :--- | :--- |
| 1 (default) | Rule in effect. |
| 0 | Rule not in effect. |

## Framework

Although not all Financial Toolbox functions require the same input arguments, they all accept the following common set of input arguments.

## Common Input Arguments

| Input | Meaning |
| :--- | :--- |
| Settle | Settlement date |
| Maturity | Maturity date |
| Period | Coupon payment period |
| Basis | Day-count basis |
| EndMonthRule | End-of-month payment rule |

## Common Input Arguments (Continued)

| Input | Meaning |
| :--- | :--- |
| IssueDate | Bond issue date |
| FirstCouponDate | First coupon payment date |
| LastCouponDate | Last coupon payment date |

Of the common input arguments, only Settle and Maturity are required. All others are optional. They will be set to the default values if you do not explicitly set them. Note that, by default, the FirstCouponDate and LastCouponDate are nonapplicable. In other words, if you do not specify FirstCouponDate and LastCouponDate, the bond is assumed to have no odd first or last coupon periods. In this case, the bond is a standard bond with a coupon payment structure based solely on the maturity date.

## Default Parameter Values

To illustrate the use of default values in Financial Toolbox functions, consider the cfdates function, which computes actual cash flow payment dates for a portfolio of fixed income securities regardless of whether the first and/or last coupon periods are normal, long, or short.

The complete calling syntax with the full input argument list is

```
CFlowDates = cfdates(Settle, Maturity, Period, Basis, ...
EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)
```

while the minimal calling syntax requires only settlement and maturity dates

```
CFlowDates = cfdates(Settle, Maturity)
```


## Single Bond Example

As an example, suppose you have a bond with these characteristics

| Settle | $=' 20-$ Sep-1999' |
| :--- | :--- |
| Maturity | $=' 15-$ Oct-2007' |
| Period | $=2$ |

```
Basis = 0
EndMonthRule = 1
IssueDate = NaN
FirstCouponDate = NaN
LastCouponDate = NaN
```

Note that Period, Basis, and EndMonthRule are set to their default values, and IssueDate, FirstCouponDate, and LastCouponDate are set to NaN.

Formally, a NaN is an IEEE ${ }^{\circledR}$ arithmetic standard for Not-a-Number and is used to indicate the result of an undefined operation (for example, zero divided by zero). However, NaN is also a very convenient placeholder. In the SIA functions of Financial Toolbox software, NaN indicates the presence of a nonapplicable value. It tells the Financial Toolbox functions to ignore the input value and apply the default. Setting IssueDate, FirstCouponDate, and LastCouponDate to NaN in this example tells cfdates to assume that the bond has been issued before settlement and that no odd first or last coupon periods exist.

Having set these values, all these calls to cfdates produce the same result.

```
cfdates(Settle, Maturity)
cfdates(Settle, Maturity, Period)
cfdates(Settle, Maturity, Period, [])
cfdates(Settle, Maturity, [], Basis)
cfdates(Settle, Maturity, [], [])
cfdates(Settle, Maturity, Period, [], EndMonthRule)
cfdates(Settle, Maturity, Period, [], NaN)
cfdates(Settle, Maturity, Period, [], [], IssueDate)
cfdates(Settle, Maturity, Period, [], [], IssueDate, [], [])
cfdates(Settle, Maturity, Period, [], [], [], [],LastCouponDate)
cfdates(Settle, Maturity, Period, Basis, EndMonthRule, ...
IssueDate, FirstCouponDate, LastCouponDate)
```

Thus, leaving a particular input unspecified has the same effect as passing an empty matrix ([ ]) or passing a NaN - all three tell cfdates (and other Financial Toolbox functions) to use the default value for a particular input parameter.

## Bond Portfolio Example

Since the previous example included only a single bond, there was no difference between passing an empty matrix or passing a NaN for an optional input argument. For a portfolio of bonds, however, using NaN as a placeholder is the only way to specify default acceptance for some bonds while explicitly setting nondefault values for the remaining bonds in the portfolio.

Now suppose you have a portfolio of two bonds.

```
Settle = '20-Sep-1999'
Maturity = ['15-Oct-2007'; '15-Oct-2010']
```

These calls to cfdates all set the coupon period to its default value (Period $=2$ ) for both bonds.

```
cfdates(Settle, Maturity, 2)
cfdates(Settle, Maturity, [2 2])
cfdates(Settle, Maturity, [])
cfdates(Settle, Maturity, NaN)
cfdates(Settle, Maturity, [NaN NaN])
cfdates(Settle, Maturity)
```

The first two calls explicitly set Period = 2. Since Maturity is a 2-by-1 vector of maturity dates, cfdates knows you have a two-bond portfolio.

The first call specifies a single (that is, scalar) 2 for Period. Passing a scalar tells cfdates to apply the scalar-valued input to all bonds in the portfolio. This is an example of implicit scalar-expansion. Note that the settlement date has been implicit scalar-expanded as well.

The second call also applies the default coupon period by explicitly passing a two-element vector of 2's. The third call passes an empty matrix, which cfdates interprets as an invalid period, for which the default value will be used. The fourth call is similar, except that a NaN has been passed. The fifth call passes two NaN's, and has the same effect as the third. The last call passes the minimal input set.

Finally, consider the following calls to cfdates for the same two-bond portfolio.

```
cfdates(Settle, Maturity, [4 NaN])
cfdates(Settle, Maturity, [4 2])
```

The first call explicitly sets Period $=4$ for the first bond and implicitly sets the default Period $=2$ for the second bond. The second call has the same effect as the first but explicitly sets the periodicity for both bonds.

The optional input Period has been used for illustrative purpose only. The default-handling process illustrated in the examples applies to any of the optional input arguments.

## Coupon Date Calculations

Calculating coupon dates, either actual or quasi dates, is notoriously complicated. Financial Toolbox software follows the SIA conventions in coupon date calculations.

The first step in finding the coupon dates associated with a bond is to determine the reference, or synchronization date (the sync date). Within the SIA framework, the order of precedence for determining the sync date is:

1 The first coupon date
2 The last coupon date
3 The maturity date

In other words, a Financial Toolbox function first examines the FirstCouponDate input. If FirstCouponDate is specified, coupon payment dates and quasi-coupon dates are computed with respect to FirstCouponDate; if FirstCouponDate is unspecified, empty ([ ]), or NaN, then the LastCouponDate is examined. If LastCouponDate is specified, coupon payment dates and quasi-coupon dates are computed with respect to LastCouponDate. If both FirstCouponDate and LastCouponDate are unspecified, empty ([ ]), or NaN, the Maturity (a required input argument) serves as the sync date.

## Yield Conventions

There are two yield and time factor conventions that are used in the Financial Toolbox software - these are determined by the input basis. Specifically, bases 0 to 7 are assumed to have semiannual compounding, while bases 8 to 12 are assumed to have annual compounding regardless of the period of the bond's coupon payments (including zero-coupon bonds). In addition, any yield-related sensitivity (that is, duration and convexity), when quoted on a periodic basis, follows this same convention. (See bndconvp, bndconvy, bnddurp, bnddury, and bndkrdur.)

## Pricing Functions

This example shows how easily you can compute the price of a bond with an odd first period using the function bndprice. Assume you have a bond with these characteristics:

```
Settle = '11-Nov-1992';
Maturity = '01-Mar-2005';
IssueDate = '15-0ct-1992';
FirstCouponDate = '01-Mar-1993';
CouponRate = 0.0785;
Yield = 0.0625;
```

Allow coupon payment period (Period = 2), day-count basis (Basis = 0), and end-of-month rule (EndMonthRule $=1$ ) to assume the default values. Also, assume there is no odd last coupon date and that the face value of the bond is $\$ 100$. Calling the function

```
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle, ...
Maturity, [], [], [], IssueDate, FirstCouponDate)
```

returns a price of $\$ 113.60$ and accrued interest of $\$ 0.59$.
Similar functions compute prices with regular payments, odd first and last periods, and prices of Treasury bills and discounted securities such as zero-coupon bonds.

Note bndprice and other functions use nonlinear formulas to compute the price of a security. For this reason, Financial Toolbox software uses Newton's method when solving for an independent variable within a formula. See any elementary numerical methods textbook for the mathematics underlying Newton's method.

## Yield Functions

To illustrate toolbox yield functions, compute the yield of a bond that has odd first and last periods and settlement in the first period. First set up variables for settlement, maturity date, issue, first coupon, and a last coupon date.

```
Settle = '12-Jan-2000';
Maturity = '01-Oct-2001';
IssueDate = '01-Jan-2000';
FirstCouponDate = '15-Jan-2000';
LastCouponDate = '15-Apr-2000';
```

Assume a face value of $\$ 100$. Specify a purchase price of $\$ 95.70$, a coupon rate of $4 \%$, quarterly coupon payments, and a 30/360 day-count convention (Basis = 1).

```
Price = 95.7;
CouponRate = 0.04;
Period = 4;
Basis = 1;
EndMonthRule = 1;
```

Calling the function

```
Yield = bndyield(Price, CouponRate, Settle, Maturity, Period,...
Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)
```

returns
Yield = 0.0659 (6.60\%).

## Fixed-Income Sensitivities

Financial Toolbox software supports the following options for managing interest-rate risk for one or more bonds:

- bnddurp and bnddury support duration and convexity analysis based on market quotes and assume parallel shifts in the bond yield curve.
- bndkrdur supports key rate duration based on a market yield curve and can model nonparallel shifts in the bond yield curve.


## Calculating Duration and Convexity for Bonds

The toolbox includes functions to perform sensitivity analysis such as convexity and the Macaulay and modified durations for fixed-income securities. The Macaulay duration of an income stream, such as a coupon bond, measures how long, on average, the owner waits before receiving a payment. It is the weighted average of the times payments are made, with the weights at time T equal to the present value of the money received at time T . The modified duration is the Macaulay duration discounted by the per-period interest rate; that is, divided by (1+rate/frequency).

To illustrate, the following example computes the annualized Macaulay and modified durations, and the periodic Macaulay duration for a bond with settlement (12-Jan-2000) and maturity (01-Oct-2001) dates as above, a $5 \%$ coupon rate, and a $4.5 \%$ yield to maturity. For simplicity, any optional input arguments assume default values (that is, semiannual coupons, and day-count basis = 0 (actual/actual), coupon payment structure synchronized to the maturity date, and end-of-month payment rule in effect).

```
CouponRate = 0.05;
Yield = 0.045;
[ModDuration, YearDuration, PerDuration] = bnddury(Yield,...
CouponRate, Settle, Maturity)
```

The durations are

```
ModDuration = 1.6107 (years)
YearDuration = 1.6470 (years)
PerDuration = 3.2940 (semiannual periods)
```

Note that the semiannual periodic Macaulay duration (PerDuration) is twice the annualized Macaulay duration (YearDuration).

## Calculating Key Rate Durations for Bonds

Key rate duration enables you to evaluate the sensitivity and price of a bond to nonparallel changes in the spot or zero curve by decomposing the interest rate risk along the spot or zero curve. Key rate duration refers to the process of choosing a set of key rates and computing a duration for each rate. Specifically, for each key rate, while the other rates are held constant, the key rate is shifted up and down (and intermediate cash flow dates are interpolated), and then the present value of the security given the shifted curves is computed.

The calculation of bndkrdur supports:

$$
k r d u r_{i}=\frac{\left(P V_{\text {down }}-P V_{u p}\right)}{(P V \times \text { ShiftValue } \times 2)}
$$

Where $P V$ is the current value of the instrument, $P V \_u p$ and $P V \_d o w n$ are the new values after the discount curve has been shocked, and ShiftValue is the change in interest rate. For example, if key rates of 3 months, 1, 2, 3, $5,7,10,15,20,25,30$ years were chosen, then a 30 -year bond might have corresponding key rate durations of:

| 3 M | 1 Y | 2 Y | 3 Y | 5 Y | 7 Y | 10 Y | 15 Y | 20 Y | 25 Y | 30 Y |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| .01 | .04 | .09 | .21 | .4 | .65 | 1.27 | 1.71 | 1.68 | 1.83 | 7.03 |

The key rate durations add up to approximately equal the duration of the bond.

For example, compute the key rate duration of the U.S. Treasury Bond with maturity date of August 15, 2028 and coupon rate of $5.5 \%$. (For further information on this bond, refer to .)

```
Settle = datenum('18-Nov-2008');
CouponRate = 5.500/100;
Maturity = datenum('15-Aug-2028');
Price = 114.83;
```

For the ZeroData information on the current spot curve for this bond, refer to :

```
ZeroDates = daysadd(Settle ,[30 90 180 360 360*2 360*3 360*5 ...
360*7 360*10 360*20 360*30]);
ZeroRates = ([0.06 0.12 0.81 1.08 1.22 1.53 2.32 2.92 3.68 4.42 4.20]/100)';
```

Compute the key rate duration for a specific set of rates (choose this based on the maturities of the available hedging instruments):

```
krd = bndkrdur([ZeroDates ZeroRates],CouponRate,Settle,Maturity,'keyrates',[2 5 10 20])
krd =
    0.2865 0.8729 2.6451 8.5778
```

Note, the sum of the key rate durations approximately equals the duration of the bond:

```
[sum(krd) bnddurp(Price,CouponRate,Settle,Maturity)]
ans =
    12.3823 12.3919
```


## Term Structure of Interest Rates

In this section...<br>"Introduction" on page 2-36<br>"Deriving an Implied Zero Curve" on page 2-37

## Introduction

The Financial Toolbox product contains several functions to derive and analyze interest rate curves, including data conversion and extrapolation, bootstrapping, and interest-rate curve conversion functions.

One of the first problems in analyzing the term structure of interest rates is dealing with market data reported in different formats. Treasury bills, for example, are quoted with bid and asked bank-discount rates. Treasury notes and bonds, on the other hand, are quoted with bid and asked prices based on $\$ 100$ face value. To examine the full spectrum of Treasury securities, analysts must convert data to a single format. Financial Toolbox functions ease this conversion. This brief example uses only one security each; analysts often use 30,100 , or more of each.

First, capture Treasury bill quotes in their reported format

| $\%$ | Maturity | Days | Bid | Ask | AskYield |
| :--- | :--- | :--- | :--- | :--- | :--- |
| TBill $=$ [datenum('12/26/2000') | 53 | 0.0503 | 0.0499 | $0.0510] ;$ |  |

then capture Treasury bond quotes in their reported format

| $\%$ | Coupon | Maturity | Bid | Ask | AskYield |
| :--- | :---: | :--- | :--- | :--- | :--- |
| TBond $=[0.08875$ | datenum $(2001,11,5)$ | $103+4 / 32$ | $103+6 / 32$ | $0.0564] ;$ |  |

and note that these quotes are based on a November 3, 2000 settlement date.

```
Settle = datenum('3-Nov-2000');
```

Next use the toolbox tbl2bond function to convert the Treasury bill data to Treasury bond format.

```
TBTBond = tbl2bond(TBill)
```


(The second element of TBTBond is the serial date number for December 26, 2000.)

Now combine short-term (Treasury bill) with long-term (Treasury bond) data to set up the overall term structure.

```
TBondsAll = [TBTBond; TBond]
TBondsAll =
\begin{tabular}{rrrrr}
0 & 730846 & 99.26 & 99.27 & 0.05 \\
0.09 & 731160 & 103.13 & 103.19 & 0.06
\end{tabular}
```

The Financial Toolbox software provides a second data-preparation function, tr2bonds, to convert the bond data into a form ready for the bootstrapping functions. tr2bonds generates a matrix of bond information sorted by maturity date, plus vectors of prices and yields.

```
[Bonds, Prices, Yields] = tr2bonds(TBondsAll);
```


## Deriving an Implied Zero Curve

Using this market data, you can use one of the Financial Toolbox bootstrapping functions to derive an implied zero curve. Bootstrapping is a process whereby you begin with known data points and solve for unknown data points using an underlying arbitrage theory. Every coupon bond can be valued as a package of zero-coupon bonds which mimic its cash flow and risk characteristics. By mapping yields-to-maturity for each theoretical zero-coupon bond, to the dates spanning the investment horizon, you can create a theoretical zero-rate curve. The Financial Toolbox software provides two bootstrapping functions: zbtprice derives a zero curve from bond data and prices, and zbtyield derives a zero curve from bond data and yields. Using zbtprice

```
[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle)
ZeroRates =
```

```
                        0.05
        0.06
        CurveDates =
        730846
        7 3 1 1 6 0
CurveDates gives the investment horizon.
datestr(CurveDates)
ans =
26-Dec-2000
05-Nov-2001
```

Additional Financial Toolbox functions construct discount, forward, and par yield curves from the zero curve, and vice versa.

```
[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates,...
Settle);
[FwdRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, Settle);
[PYldRates, CurveDates] = zero2pyld(ZeroRates, CurveDates,...
Settle);
```


## Pricing and Analyzing Equity Derivatives

In this section...<br>"Introduction" on page 2-39<br>"Sensitivity Measures" on page 2-39<br>"Analysis Models" on page 2-40

## Introduction

These toolbox functions compute prices, sensitivities, and profits for portfolios of options or other equity derivatives. They use the Black-Scholes model for European options and the binomial model for American options. Such measures are useful for managing portfolios and for executing collars, hedges, and straddles.

## Sensitivity Measures

There are six basic sensitivity measures associated with option pricing: delta, gamma, lambda, rho, theta, and vega - the "greeks." The toolbox provides functions for calculating each sensitivity and for implied volatility.

## Delta

Delta of a derivative security is the rate of change of its price relative to the price of the underlying asset. It is the first derivative of the curve that relates the price of the derivative to the price of the underlying security. When delta is large, the price of the derivative is sensitive to small changes in the price of the underlying security.

## Gamma

Gamma of a derivative security is the rate of change of delta relative to the price of the underlying asset; that is, the second derivative of the option price relative to the security price. When gamma is small, the change in delta is small. This sensitivity measure is important for deciding how much to adjust a hedge position.

## Lambda

Lambda, also known as the elasticity of an option, represents the percentage change in the price of an option relative to a $1 \%$ change in the price of the underlying security.

## Rho

Rho is the rate of change in option price relative to the risk-free interest rate.

## Theta

Theta is the rate of change in the price of a derivative security relative to time. Theta is usually very small or negative since the value of an option tends to drop as it approaches maturity.

## Vega

Vega is the rate of change in the price of a derivative security relative to the volatility of the underlying security. When vega is large the security is sensitive to small changes in volatility. For example, options traders often must decide whether to buy an option to hedge against vega or gamma. The hedge selected usually depends upon how frequently one rebalances a hedge position and also upon the standard deviation of the price of the underlying asset (the volatility). If the standard deviation is changing rapidly, balancing against vega is usually preferable.

## Implied Volatility

The implied volatility of an option is the standard deviation that makes an option price equal to the market price. It helps determine a market estimate for the future volatility of a stock and provides the input volatility (when needed) to the other Black-Scholes functions.

## Analysis Models

Toolbox functions for analyzing equity derivatives use the Black-Scholes model for European options and the binomial model for American options. The Black-Scholes model makes several assumptions about the underlying securities and their behavior. The binomial model, on the other hand, makes far fewer assumptions about the processes underlying an option. For further
explanation, see Options, Futures, and Other Derivatives by John Hull in Appendix A, "Bibliography".

## Black-Scholes Model

Using the Black-Scholes model entails several assumptions:

- The prices of the underlying asset follow an Ito process. (See Hull, page 222.)
- The option can be exercised only on its expiration date (European option).
- Short selling is permitted.
- There are no transaction costs.
- All securities are divisible.
- There is no riskless arbitrage.
- Trading is a continuous process.
- The risk-free interest rate is constant and remains the same for all maturities.

If any of these assumptions is untrue, Black-Scholes may not be an appropriate model.

To illustrate toolbox Black-Scholes functions, this example computes the call and put prices of a European option and its delta, gamma, lambda, and implied volatility. The asset price is $\$ 100.00$, the exercise price is $\$ 95.00$, the risk-free interest rate is $10 \%$, the time to maturity is 0.25 years, the volatility is 0.50 , and the dividend rate is 0 . Simply executing the toolbox functions

```
[OptCall, OptPut] = blsprice(100, 95, 0.10, 0.25, 0.50, 0);
[CallVal, PutVal] = blsdelta(100, 95, 0.10, 0.25, 0.50, 0);
GammaVal = blsgamma(100, 95, 0.10, 0.25, 0.50, 0);
VegaVal = blsvega(100, 95, 0.10, 0.25, 0.50, 0);
[LamCall, LamPut] = blslambda(100, 95, 0.10, 0.25, 0.50, 0);
```

yields:

- The option call price OptCall $=\$ 13.70$
- The option put price OptPut $=\$ 6.35$
- delta for a call CallVal $=0.6665$ and delta for a put PutVal $=-0.3335$
- gamma GammaVal $=0.0145$
- vega VegaVal $=18.1843$
- lambda for a call LamCall $=4.8664$ and lambda for a put LamPut $=-5.2528$

Now as a computation check, find the implied volatility of the option using the call option price from blsprice.

```
Volatility = blsimpv(100, 95, 0.10, 0.25, OptCall);
```

The function returns an implied volatility of 0.500 , the original blsprice input.

## Binomial Model

The binomial model for pricing options or other equity derivatives assumes that the probability over time of each possible price follows a binomial distribution. The basic assumption is that prices can move to only two values, one up and one down, over any short time period. Plotting the two values, and then the subsequent two values each, and then the subsequent two values each, and so on over time, is known as "building a binomial tree." This model applies to American options, which can be exercised any time up to and including their expiration date.

This example prices an American call option using a binomial model. Again, the asset price is $\$ 100.00$, the exercise price is $\$ 95.00$, the risk-free interest rate is $10 \%$, and the time to maturity is 0.25 years. It computes the tree in increments of 0.05 years, so there are $0.25 / 0.05=5$ periods in the example. The volatility is 0.50 , this is a call ( $\mathrm{flag}=1$ ), the dividend rate is 0 , and it pays a dividend of $\$ 5.00$ after three periods (an ex-dividend date). Executing the toolbox function

```
[StockPrice, OptionPrice] = binprice(100, 95, 0.10, 0.25,...
0.05, 0.50, 1, 0, 5.0, 3);
```

returns the tree of prices of the underlying asset

```
StockPrice =
```

| 100.00 | 111.27 | 123.87 | 137.96 | 148.69 | 166.28 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 89.97 | 100.05 | 111.32 | 118.90 | 132.96 |
| 0 | 0 | 81.00 | 90.02 | 95.07 | 106.32 |
| 0 | 0 | 0 | 72.98 | 76.02 | 85.02 |
| 0 | 0 | 0 | 0 | 60.79 | 67.98 |
| 0 | 0 | 0 | 0 | 0 | 54.36 |

and the tree of option values.
OptionPrice =

| 12.10 | 19.17 | 29.35 | 42.96 | 54.17 | 71.28 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 5.31 | 9.41 | 16.32 | 24.37 | 37.96 |
| 0 | 0 | 1.35 | 2.74 | 5.57 | 11.32 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |

The output from the binomial function is a binary tree. Read the StockPrice matrix this way: column 1 shows the price for period 0 , column 2 shows the up and down prices for period 1 , column 3 shows the up-up, up-down, and down-down prices for period 2, and so on. Ignore the zeros. The OptionPrice matrix gives the associated option value for each node in the price tree. Ignore the zeros that correspond to a zero in the price tree.

## Portfolio Analysis

- "Analyzing Portfolios" on page 3-2
- "Portfolio Optimization Functions" on page 3-3
- "Portfolio Construction Examples" on page 3-5
- "Portfolio Selection and Risk Aversion" on page 3-8
- "Constraint Specification" on page 3-12
- "Active Returns and Tracking Error Efficient Frontier" on page 3-20


## Analyzing Portfolios

Portfolio managers concentrate their efforts on achieving the best possible trade-off between risk and return. For portfolios constructed from a fixed set of assets, the risk/return profile varies with the portfolio composition. Portfolios that maximize the return, given the risk, or, conversely, minimize the risk for the given return, are called optimal. Optimal portfolios define a line in the risk/return plane called the efficient frontier.

A portfolio may also have to meet additional requirements to be considered. Different investors have different levels of risk tolerance. Selecting the adequate portfolio for a particular investor is a difficult process. The portfolio manager can hedge the risk related to a particular portfolio along the efficient frontier with partial investment in risk-free assets. The definition of the capital allocation line, and finding where the final portfolio falls on this line, if at all, is a function of:

- The risk/return profile of each asset
- The risk-free rate
- The borrowing rate
- The degree of risk aversion characterizing an investor

Financial Toolbox software includes a set of portfolio optimization functions designed to find the portfolio that best meets investor requirements.

## Portfolio Optimization Functions

The portfolio optimization functions assist portfolio managers in constructing portfolios that optimize risk and return.

| Capital <br> Allocation | Description |
| :--- | :--- |
| portalloc | Computes the optimal risky portfolio on the efficient <br> frontier, based on the risk-free rate, the borrowing rate, <br> and the investor's degree of risk aversion. Also generates <br> the capital allocation line, which provides the optimal <br> allocation of funds between the risky portfolio and the <br> risk-free asset. |


| Efficient <br> Frontier <br> Computation | Description |
| :--- | :--- | frontcon | Computes portfolios along the efficient frontier for a |
| :--- |
| given group of assets. The computation is based on sets |
| of constraints representing the maximum and minimum |
| weights for each asset, and the maximum and minimum |
| total weight for specified groups of assets. |$|$


| Constraint <br> Specification | Description |
| :--- | :--- |
| portcons | Generates the portfolio constraints matrix for a portfolio <br> of asset investments using linear inequalities. The <br> inequalities are of the type A *Wts ' <= b, where Wts is a <br> row vector of weights. |
| portvrisk | Portfolio value at risk (VaR) returns the maximum <br> potential loss in the value of a portfolio over one period of <br> time, given the loss probability level RiskThreshold. |
| pcalims | Asset minimum and maximum allocation. Generates a <br> constraint set to fix the minimum and maximum weight <br> for each individual asset. |
| pcgcomp | Group-to-group ratio constraint. Generates a constraint <br> set specifying the maximum and minimum ratios <br> between pairs of groups. |
| pcglims | Asset group minimum and maximum allocation. <br> Generates a constraint set to fix the minimum and <br> maximum total weight for each defined group of assets. |
| pcpval | Total portfolio value. Generates a constraint set to fix the <br> total value of the portfolio. |


| Constraint <br> Conversion | Description |
| :--- | :--- |
| abs2active | Transforms a constraint matrix expressed in absolute <br> weight format to an equivalent matrix expressed in active <br> weight format. |
| active2abs | Transforms a constraint matrix expressed in active <br> weight format to an equivalent matrix expressed in <br> absolute weight format. |

## Portfolio Construction Examples

In this section...<br>"Introduction" on page 3-5<br>"Efficient Frontier Example" on page 3-5

## Introduction

The efficient frontier computation functions require information about each asset in the portfolio. This data is entered into the function via two matrices: an expected return vector and a covariance matrix. The expected return vector contains the average expected return for each asset in the portfolio. The covariance matrix is a square matrix representing the interrelationships between pairs of assets. This information can be directly specified or can be estimated from an asset return time series with the function ewstats.

## Efficient Frontier Example

This example computes the efficient frontier of portfolios consisting of three different assets using the function frontcon. To visualize the efficient frontier curve clearly, consider 10 different evenly spaced portfolios.

Assume that the expected return of the first asset is $10 \%$, the second is $20 \%$, and the third is $15 \%$. The covariance is defined in the matrix ExpCovariance.
ExpReturn $=\left[\begin{array}{llrr}0.1 & 0.20 .15\end{array}\right] ;$
ExpCovariance $=\left[\begin{array}{rrrr} & 0.005 & -0.010 & 0.004 ; \\ & -0.010 & 0.040 & -0.002 ; \\ & 0.004 & -0.002 & 0.023\end{array}\right] ;$
NumPorts $=10 ;$

Since there are no constraints, you can call frontcon directly with the data you already have. If you call frontcon without specifying any output arguments, you get a graph representing the efficient frontier curve.
frontcon (ExpReturn, ExpCovariance, NumPorts);


Calling frontcon while specifying the output arguments returns the corresponding vectors and arrays representing the risk, return, and weights for each of the 10 points computed along the efficient frontier.
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn,... ExpCovariance, NumPorts)

PortRisk =
0.0392
0.0445
0.0559
0.0701
0.0858
0.1023
0.1192
0.1383
0.1661
0.2000

PortReturn =

$$
\begin{array}{rll}
0.1231 & & \\
0.1316 \\
0.1402 & & \\
0.1487 & & \\
0.1573 & & \\
0.1658 & & \\
0.1744 & & \\
0.1829 & & \\
0.1915 & & \\
0.2000 & & 0.0000 \\
& & 0.0342 \\
\text { PortWts }= & & 0.1079 \\
& & 0.1816 \\
0.7692 & 0.2308 & 0.2553 \\
0.6667 & 0.2991 & \\
0.5443 & 0.3478 & \\
0.4220 & 0.3964 & \\
0.2997 & 0.4450 & \\
0.1774 & 0.4936 & 0.3290 \\
0.0550 & 0.5422 & 0.4027 \\
0 & 0.6581 & 0.3419 \\
0 & 0.8291 & 0.1709 \\
0 & 1.0000 & 0.0000
\end{array}
$$

The output data is represented row-wise. Each portfolio's risk, rate of return, and associated weights are identified as corresponding rows in the vectors and matrix.

For example, you can see from these results that the second portfolio has a risk of 0.0445 , an expected return of $13.16 \%$, and allocations of about $67 \%$ in the first asset, $30 \%$ in the second, and $3 \%$ in the third.

## Portfolio Selection and Risk Aversion

In this section...<br>"Introduction" on page 3-8<br>"Optimal Risky Portfolio Example" on page 3-9

## Introduction

One of the factors to consider when selecting the optimal portfolio for a particular investor is degree of risk aversion. This level of aversion to risk can be characterized by defining the investor's indifference curve. This curve consists of the family of risk/return pairs defining the trade-off between the expected return and the risk. It establishes the increment in return that a particular investor will require in order to make an increment in risk worthwhile. Typical risk aversion coefficients range between 2.0 and 4.0 , with the higher number representing lesser tolerance to risk. The equation used to represent risk aversion in Financial Toolbox software is

$$
U=E(r)-0.005^{*} A * \operatorname{sig}^{\wedge} 2
$$

where:

U is the utility value.
$E(r)$ is the expected return.
A is the index of investor's aversion.
sig is the standard deviation.


## Optimal Risky Portfolio Example

This example computes the optimal risky portfolio on the efficient frontier based upon the risk-free rate, the borrowing rate, and the investor's degree of risk aversion. You do this with the function portalloc.

First generate the efficient frontier data using either portopt or frontcon. This example uses portopt and the same asset data from the previous example.

```
ExpReturn = [0.1 0.2 0.15];
```



This time consider 20 different points along the efficient frontier.
NumPorts = 20;

```
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,...
ExpCovariance, NumPorts);
```

As with frontcon, calling portopt while specifying output arguments returns the corresponding vectors and arrays representing the risk, return, and weights for each of the portfolios along the efficient frontier. Use them as the first three input arguments to the function portalloc.

Now find the optimal risky portfolio and the optimal allocation of funds between the risky portfolio and the risk-free asset, using these values for the risk-free rate, borrowing rate and investor's degree of risk aversion.
RisklessRate $=0.08$
BorrowRate $=0.12$
RiskAversion $=3$

Calling portalloc without specifying any output arguments gives a graph displaying the critical points.

```
portalloc (PortRisk, PortReturn, PortWts, RisklessRate,...
BorrowRate, RiskAversion);
```



Calling portalloc while specifying the output arguments returns the variance (RiskyRisk), the expected return (RiskyReturn), and the weights (RiskyWts) allocated to the optimal risky portfolio. It also returns the fraction (RiskyFraction) of the complete portfolio allocated to the risky portfolio, and the variance (OverallRisk) and expected return (OverallReturn) of the optimal overall portfolio. The overall portfolio combines investments in the risk-free asset and in the risky portfolio. The actual proportion assigned to each of these two investments is determined by the degree of risk aversion characterizing the investor.

```
[RiskyRisk, RiskyReturn, RiskyWts,RiskyFraction, OverallRisk,...
OverallReturn] = portalloc (PortRisk, PortReturn, PortWts,...
RisklessRate, BorrowRate, RiskAversion)
RiskyRisk = 0.1288
RiskyReturn = 0.1791
RiskyWts = 0.0057 0.5879 0.4064
RiskyFraction = 1.1869
OverallRisk = 0.1529
OverallReturn = 0.1902
```

The value of RiskyFraction exceeds 1 (100\%), implying that the risk tolerance specified allows borrowing money to invest in the risky portfolio, and that no money will be invested in the risk-free asset. This borrowed capital is added to the original capital available for investment. In this example the customer will tolerate borrowing $18.69 \%$ of the original capital amount.

## Constraint Specification

```
In this section...
"Example" on page 3-12
"Linear Constraint Equations" on page 3-14
"Specifying Additional Constraints" on page 3-17
```


## Example

This example computes the efficient frontier of portfolios consisting of three different assets, INTC, XON, and RD, given a list of constraints. The expected returns for INTC, XON, and RD are respectively as follows:

```
ExpReturn = [0.1 0.2 0.15];
```

The covariance matrix is

- Constraint 1
- Allow short selling up to $10 \%$ of the portfolio value in any asset, but limit the investment in any one asset to $110 \%$ of the portfolio value.


## - Constraint 2

- Consider two different sectors, technology and energy, with the following table indicating the sector each asset belongs to.

| Asset | INTC | XON | RD |
| :--- | :--- | :--- | :--- |
| Sector | Technology | Energy | Energy |

Constrain the investment in the Energy sector to $80 \%$ of the portfolio value, and the investment in the Technology sector to $70 \%$.

To solve this problem, use frontcon, passing in a list of asset constraints. Consider eight different portfolios along the efficient frontier:

```
NumPorts = 8;
```

To introduce the asset bounds constraints specified in Constraint 1, create the matrix AssetBounds, where each column represents an asset. The upper row represents the lower bounds, and the lower row represents the upper bounds.

```
AssetBounds = [-0.10, -0.10, -0.10;
    1.10, 1.10, 1.10];
```

Constraint 2 needs to be entered in two parts, the first part defining the groups, and the second part defining the constraints for each group. Given the information above, you can build a matrix of 1 s and 0 s indicating whether a specific asset belongs to a group. Each column represents an asset, and each row represents a group. This example has two groups: the technology group, and the energy group. Create the matrix Groups as follows.

$$
\text { Groups } \left.=\begin{array}{rrr}
{[0} & 1 & 1 ; \\
1 & 0 & 0
\end{array}\right]
$$

The GroupBounds matrix allows you to specify an upper and lower bound for each group. Each row in this matrix represents a group. The first column represents the minimum allocation, and the second column represents the maximum allocation to each group. Since the investment in the Energy sector is capped at $80 \%$ of the portfolio value, and the investment in the Technology sector is capped at $70 \%$, create the GroupBounds matrix using this information.

$$
\text { GroupBounds }=\left[\begin{array}{cc}
0 & 0.80 ; \\
0 & 0.70
\end{array}\right] ;
$$

Now use frontcon to obtain the vectors and arrays representing the risk, return, and weights for each of the eight portfolios computed along the efficient frontier.
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn,... ExpCovariance, NumPorts, [], AssetBounds, Groups, GroupBounds)

PortRisk =

```
    0.0416
        0.0499
        0.0624
        0.0767
        0.0920
        0.1100
        0.1378
        0.1716
PortReturn =
    0.1279
    0.1361
    0.1442
    0.1524
    0.1605
    0.1687
    0.1768
    0.1850
PortWts =
\begin{tabular}{lrr}
0.7000 & 0.2582 & 0.0418 \\
0.6031 & 0.3244 & 0.0725 \\
0.4864 & 0.3708 & 0.1428 \\
0.3696 & 0.4172 & 0.2132 \\
0.2529 & 0.4636 & 0.2835 \\
0.2000 & 0.5738 & 0.2262 \\
0.2000 & 0.7369 & 0.0631 \\
0.2000 & 0.9000 & -0.1000
\end{tabular}
```

The output data is represented row-wise, where each portfolio's risk, rate of return, and associated weight is identified as corresponding rows in the vectors and matrix.

## Linear Constraint Equations

While frontcon allows you to enter a fixed set of constraints related to minimum and maximum values for groups and individual assets, you often need to specify a larger and more general set of constraints when finding
the optimal risky portfolio. The function portopt addresses this need, by accepting an arbitrary set of constraints as an input matrix.

The auxiliary function portcons can be used to create the matrix of constraints, with each row representing an inequality. These inequalities are of the type $A * W t s{ }^{\prime}<=b$, where $A$ is a matrix, $b$ is a vector, and Wts is a row vector of asset allocations. The number of columns of the matrix A, and the length of the vector Wts correspond to the number of assets. The number of rows of the matrix $A$, and the length of vector $b$ correspond to the number of constraints. This method allows you to specify any number of linear inequalities to the function portopt.

In actuality, portcons is an entry point to a set of functions that generate matrices for specific types of constraints. portcons allows you to specify all the constraints data at once, while the specific portfolio constraint functions allow you to build the constraints incrementally. These constraint functions are pcpval, pcalims, pcglims, and pcgcomp.

Consider an example to help understand how to specify constraints to portopt while bypassing the use of portcons. This example requires specifying the minimum and maximum investment in various groups.

Maximum and Minimum Group Exposure

| Group | Minimum Exposure | Maximum Exposure |
| :--- | :--- | :--- |
| North America | 0.30 | 0.75 |
| Europe | 0.10 | 0.55 |
| Latin America | 0.20 | 0.50 |
| Asia | 0.50 | 0.50 |

Note that the minimum and maximum exposure in Asia is the same. This means that you require a fixed exposure for this group.

Also assume that the portfolio consists of three different funds. The correspondence between funds and groups is shown in the table below.

## Group Membership

| Group | Fund 1 | Fund 2 | Fund 3 |
| :--- | :--- | :--- | :--- |
| North America | X | X |  |
| Europe |  |  | X |
| Latin America | X |  |  |
| Asia |  | X | X |

Using the information in these two tables, build a mathematical representation of the constraints represented. Assume that the vector of weights representing the exposure of each asset in a portfolio is called Wts = [W1 W2 W3].

Specifically

| 1. | $W 1+W 2$ | $\geq$ | 0.30 |
| :--- | :--- | :--- | :--- |
| 2. | $W 1+W 2$ | $\leq$ | 0.75 |
| 3. | $W 3$ | $\geq$ | 0.10 |
| 4. | $W 3$ | $\leq$ | 0.55 |
| 5. | $W 1$ | $\leq$ | 0.20 |
| 6. | $W 2+W 3$ | $=$ | 0.50 |
| 7. |  |  | 0.50 |

Since you need to represent the information in the form A*Wts <= b, multiply equations 1,3 and 5 by -1 . Also turn equation 7 into a set of two inequalities: $W 2+W 3 \geq 0.50$ and $W 2+W 3 \leq 0.50$. (The intersection of these two inequalities is the equality itself.) Thus
1.
-W1-W2
$\leq$
-0.30
2.
$W 1+W 2$
$\leq$
0.75
3.
-W3
$\leq$
-0.10

| 4. | $W 3$ | $\leq$ | 0.55 |
| :--- | :--- | :--- | :--- |
| 5. | $-W 1$ | $\leq$ | -0.20 |
| 6. | $W 1$ | $\leq$ | 0.50 |
| 7. | $-W 2-W 3$ | $\leq$ | -0.50 |
| 8. | $W 2+W 3$ | $\leq$ | 0.50 |

Bringing these equations into matrix notation gives
$A=\left[\begin{array}{rrr}-1 & -1 & 0 ; \\ 1 & 1 & 0 ; \\ 0 & 0 & -1 ; \\ 0 & 0 & 1 ; \\ -1 & 0 & 0 ; \\ 1 & 0 & 0 ; \\ 0 & -1 & -1 ; \\ 0 & 1 & 1] \\ b=[-0.30 ; & \\ 0.75 ; & \\ -0.10 ; & \\ 0.55 ; & \\ -0.20 ; & \\ 0.50 ; & \\ -0.50 ; & \\ 0.50]\end{array}\right.$

Build the constraint matrix ConSet by concatenating the matrix $A$ to the vector b.

```
ConSet = [A, b]
```


## Specifying Additional Constraints

The example above defined a constraints matrix that specified a set of typical scenarios. It defined groups of assets, specified upper and lower bounds for total allocation in each of these groups, and it set the total allocation of one of the groups to a fixed value. Constraints like these are common occurrences.

The function portcons was created to simplify the creation of the constraint matrix for these and other common portfolio requirements. portcons takes as input arguments a list of constraint-specifier strings, followed by the data necessary to build the constraint specified by the strings.

Assume that you need to add more constraints to the previous example. Specifically, add a constraint indicating that the sum of weights in any portfolio should be equal to 1 , and another set of constraints (one per asset) indicating that the weight for each asset must greater than 0 . This translates into five more constraint rows: two for the new equality, and three indicating that each weight must be greater or equal to 0 . The total number of inequalities in the example is now 13. Clearly, creating the constraint matrix can turn into a tedious task.

To create the new constraint matrix using portcons, use two separate constraint-specifier strings:

- 'Default', which indicates that each weight is greater than 0 and that the total sum of the weights adds to 1
- 'GroupLims ', which defines the minimum and maximum allocation on each group

The only data requirement for the constraint-specifier string 'Default' is NumAssets (the total number of assets). The constraint-specifier string 'GroupLims ' requires three different arguments: a Groups matrix indicating the assets that belong to each group, the GroupMin vector indicating the minimum bounds for each group, and the GroupMax vector indicating the maximum bounds for each group. Based on the table Group Membership on page 3-16, build the Group matrix, with each row representing a group, and each column representing an asset.

Group $=$| $[1$ | 1 | $0 ;$ |
| ---: | ---: | ---: |
| 0 | 0 | $1 ;$ |
| 1 | 0 | $0 ;$ |
| 0 | 1 | $1]$ |

The table Maximum and Minimum Group Exposure on page 3-15 has the information to build GroupMin and GroupMax.

$$
\text { GroupMin }=\left[\begin{array}{llll}
0.30 & 0.10 & 0.20 & 0.50
\end{array}\right] ;
$$

```
GroupMax = [lll.75 0.55 0.50 0.50];
```

Given that the number of assets is three, build the constraint matrix by calling portcons.

```
ConSet = portcons('Default', 3, 'GroupLims', Group, GroupMin,...
GroupMax);
```

In most cases, portcons('Default') returns the minimal set of constraints required for calling portopt. If ConSet is not specified in the call to portopt, the function calls portcons passing 'Default' as its only specifier.

Now use portopt to obtain the vectors and arrays representing the risk, return, and weights for the portfolios computed along the efficient frontier.

```
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,...
ExpCovariance, [], [], ConSet)
PortRisk = 0.0586
Port Return = 0.1375
PortWts = 0.5 0.25 0.25
```

In this case, the constraints allow only one optimum portfolio.

## Active Returns and Tracking Error Efficient Frontier

Suppose you want to identify an efficient set of portfolios that minimize the variance of the difference in returns with respect to a given target portfolio, subject to a given expected excess return. The mean and standard deviation of this excess return are often called the active return and active risk, respectively. Active risk is sometimes referred to as the tracking error. Since the objective is to track a given target portfolio as closely as possible, the resulting set of portfolios is sometimes referred to as the tracking error efficient frontier.

Specifically, assume that the target portfolio is expressed as an index weight vector, such that the index return series may be expressed as a linear combination of the available assets. This example illustrates how to construct a frontier that minimizes the active risk (tracking error) subject to attaining a given level of return. That is, it computes the tracking error efficient frontier.

One way to construct the tracking error efficient frontier is to explicitly form the target return series and subtract it from the return series of the individual assets. In this manner, you specify the expected mean and covariance of the active returns, and compute the efficient frontier subject to the usual portfolio constraints.

This example works directly with the mean and covariance of the absolute (unadjusted) returns but converts the constraints from the usual absolute weight format to active weight format.

Consider a portfolio of five assets with the following expected returns, standard deviations, and correlation matrix based on absolute weekly asset returns.

| NumAssets | $=5 ;$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| ExpReturn | $=\left[\begin{array}{llllll}0.2074 & 0.1971 & 0.2669 & 0.1323 & 0.2535\end{array}\right] / 100 ;$ |  |  |  |  |
| Sigmas | $=\left[\begin{array}{lllll}2.6570 & 3.6297 & 3.9916 & 2.7145 & 2.6133\end{array}\right] / 100 ;$ |  |  |  |  |
|  |  |  |  |  |  |
| Correlations $=\left[\begin{array}{lllll}1.0000 & 0.6092 & 0.6321 & 0.5833 & 0.7304 \\ & 0.6092 & 1.0000 & 0.8504 & 0.8038 \\ & 0.6321 & 0.8504 & 1.0000 & 0.7723 \\ & & & & 0.7236\end{array}\right.$ |  |  |  |  |  |


| 0.5833 | 0.8038 | 0.7723 | 1.0000 | 0.7225 |
| :--- | :--- | :--- | :--- | :--- |
| 0.7304 | 0.7176 | 0.7236 | 0.7225 | $1.0000] ;$ |

Convert the correlations and standard deviations to a covariance matrix using corr2cov.

```
ExpCovariance = corr2cov(Sigmas, Correlations);
```

Next, assume that the target index portfolio is an equally-weighted portfolio formed from the five assets. Note that the sum of index weights equals 1 , satisfying the standard full investment budget equality constraint.

```
Index = ones(NumAssets, 1)/NumAssets;
```

Generate an asset constraint matrix using portcons. The constraint matrix AbsConset is expressed in absolute format (unadjusted for the index), and is formatted as [A b], corresponding to constraints of the form $A^{*} w<=b$. Each row of AbsConSet corresponds to a constraint, and each column corresponds to an asset. Allow no short-selling and full investment in each asset (lower and upper bounds of each asset are 0 and 1, respectively). In particular, note that the first two rows correspond to the budget equality constraint; the remaining rows correspond to the upper/lower investment bounds.

```
AbsConSet = portcons('PortValue', 1, NumAssets, ...
'AssetLims', zeros(NumAssets,1), ones(NumAssets,1));
```

Now transform the absolute constraints to active constraints with abs2active.

```
ActiveConSet = abs2active(AbsConSet, Index);
```

An examination of the absolute and active constraint matrices reveals that they are differ only in the last column (the columns corresponding to the $b$ in $A^{*} w<=b$ ).

```
[AbsConSet(:,end) ActiveConSet(:,end)]
ans =
    1.0000 0
```

| 1.0000 | 0.8000 |
| ---: | ---: |
| 1.0000 | 0.8000 |
| 1.0000 | 0.8000 |
| 1.0000 | 0.8000 |
| 1.0000 | 0.8000 |
| 0 | 0.2000 |
| 0 | 0.2000 |
| 0 | 0.2000 |
| 0 | 0.2000 |
| 0 | 0.2000 |

In particular, note that the sum-to-one absolute budget constraint becomes a sum-to-zero active budget constraint. The general transformation is as follows:

$$
b_{\text {active }}=b_{\text {absolute }}-A \times \text { Index } .
$$

Now construct and plot the tracking error efficient frontier with 21 portfolios.

```
[ActiveRisk, ActiveReturn, ActiveWeights] = ...
portopt(ExpReturn,ExpCovariance, 21, [], ActiveConSet);
ActiveRisk = real(ActiveRisk);
plot(ActiveRisk*100, ActiveReturn*100, 'blue')
grid('on')
xlabel('Active Risk (Standard Deviation in Percent)')
ylabel('Active Return (Percent)')
title('Tracking Error Efficient Frontier')
```



Of particular interest is the lower-left portfolio along the frontier. This zero-risk/zero-return portfolio has a practical economic significance. It represents a full investment in the index portfolio itself. Note that each tracking error efficient portfolio (each row in the array ActiveWeights) satisfies the active budget constraint, and thus represents portfolio investment allocations with respect to the index portfolio. To convert these allocations to absolute investment allocations, add the index to each efficient portfolio.

```
AbsoluteWeights = ActiveWeights + repmat(Index', 21, 1);
```


## Portfolio Optimization Tools

- "Portfolio Optimization Theory" on page 4-2
- "Portfolio Object" on page 4-12
- "Constructing the Portfolio Object" on page 4-22
- "Common Operations on the Portfolio Object" on page 4-29
- "Working with Asset Returns and Moments of Asset Returns" on page 4-36
- "Working with Portfolio Constraints" on page 4-52
- "Validating the Portfolio Problem" on page 4-73
- "Estimate Efficient Portfolios" on page 4-77
- "Estimate Efficient Frontiers" on page 4-88
- "Post-Processing" on page 4-95
- "Asset Allocation Example" on page 4-100


## Portfolio Optimization Theory

In this section...<br>"Portfolio Optimization Problems" on page 4-2<br>"Portfolio Problem Specification" on page 4-2<br>"Return Proxy" on page 4-3<br>"Risk Proxy" on page 4-5<br>"Portfolio Set for Mean-Variance Portfolio Optimization" on page 4-5<br>"Default Portfolio Problem" on page 4-11

## Portfolio Optimization Problems

Portfolio optimization problems involve identifying portfolios that satisfy three criteria: minimize a proxy for risk, match or exceed a proxy for return, and satisfy basic feasibility requirements.

Portfolios are points from a feasible set of assets that constitute an asset universe. A portfolio specifies either holdings or weights in each individual asset in the asset universe. The convention is to specify portfolios in terms of weights, although the portfolio object tools work with holdings as well.

The set of feasible portfolios is necessarily a nonempty, closed, and bounded set. The proxy for risk is a function that characterizes either the variability or losses associated with portfolio choices. The proxy for return is a function that characterizes either the gross or net benefits associated with portfolio choices. The terms "risk" and "risk proxy" and "return" and "return proxy" are interchangeable. The fundamental insight of Markowitz (see "Portfolio Optimization" on page A-12) is that the goal of the portfolio choice problem is to seek minimum risk for a given level of return and to seek maximum return for a given level of risk. Portfolios satisfying these criteria are efficient portfolios and the graph of the risks and returns of these portfolios forms a curve called the efficient frontier.

## Portfolio Problem Specification

To specify a portfolio optimization problem, you need:

- Proxy for portfolio return ( $\mu$ )
- Proxy for portfolio risk ( $\Sigma$ )
- Set of feasible portfolios ( $X$ ), called a portfolio set.

Financial Toolbox software supports a portfolio object for mean-variance portfolio optimization. The portfolio object has either gross or net portfolio returns as the return proxy, the variance of portfolio returns as the risk proxy, and a portfolio set that is any combination of the specified constraints to form a portfolio set.

## Return Proxy

The proxy for portfolio return is a function $\mu: X \rightarrow R$ on a portfolio set
$X \subset R^{n}$ that characterizes the rewards associated with portfolio choices. In most cases, the proxy for portfolio return has two general forms, gross and net portfolio returns. Both portfolio return models separate the risk-free rate $r_{0}$ so that the portfolio $x \in X$ contains only risky assets.

Regardless of the underlying distribution of asset returns, a collection of $S$ asset returns $y_{1}, \ldots, y_{S}$ has a mean of asset returns

$$
m=\frac{1}{S} \sum_{s=1}^{S} y_{s}
$$

and (sample) covariance of asset returns

$$
C=\frac{1}{S-1} \sum_{s=1}^{S}\left(y_{s}-m\right)\left(y_{s}-m\right)^{T}
$$

These moments (or alternative estimators that characterize these moments) are used directly in mean-variance portfolio optimization to form proxies for portfolio risk and return.

## Gross Portfolio Returns

The gross portfolio return for a portfolio $x \in X$ is

$$
\mu(x)=r_{0}+\left(m-r_{0} 1\right)^{T} x
$$

where:
$r_{0}$ is the risk-free rate (scalar).
$m$ is the mean of asset returns ( $n$ vector).
If the portfolio weights sum to 1 , the risk-free rate is irrelevant. The properties in the portfolio object to specify gross portfolio returns are:

- RiskFreeRate for $r_{0}$
- AssetMean for $m$


## Net Portfolio Returns

The net portfolio return for a portfolio $x \in X$ is

$$
\mu(x)=r_{0}+\left(m-r_{0} 1\right)^{T} x-b^{T} \max \left\{0, x-x_{0}\right\}-s^{T} \max \left\{0, x_{0}-x\right\}
$$

where:
$r_{0}$ is the risk-free rate (scalar).
$m$ is the mean of asset returns ( $n$ vector).
$b$ is the proportional cost to purchase assets ( $n$ vector).
$s$ is the proportional cost to sell assets ( $n$ vector).
You can incorporate fixed transaction costs in this model also, although in this case, it is necessary to incorporate prices into such costs. The properties in the portfolio object to specify net portfolio returns are:

- RiskFreeRate for $r_{0}$
- AssetMean for $m$
- InitPort for $x_{0}$
- BuyCost for $b$
- SellCost for $s$.


## Risk Proxy

The proxy for portfolio risk is a function $\sum: X \rightarrow R$ on a portfolio set $X \subset R^{n}$ that characterizes the risks associated with portfolio choices.

## Variance of Portfolio Returns

The variance of portfolio returns for a portfolio $x \in X$ is:

$$
\Sigma(x)=x^{T} C x
$$

where $C$ is covariance of asset returns ( $n$-by- $n$ positive-semidefinite matrix).
The property in the portfolio object to specify the variance of portfolio returns is AssetCovar for C.

Although the risk proxy in mean-variance portfolio optimization is the variance of portfolio returns, the square root, which is the standard deviation of portfolio returns, is often reported and displayed. Moreover, this quantity is often called the "risk" of the portfolio. For details, see Markowitz ("Portfolio Optimization" on page A-12).

## Portfolio Set for Mean-Variance Portfolio Optimization

The final element for a complete specification of a portfolio optimization problem is the set of feasible portfolios, which is called a portfolio set. A portfolio set $X \subset R^{n}$ is specified by construction as the intersection of sets formed by a collection of constraints on portfolio weights. A portfolio set necessarily and sufficiently must be a nonempty, closed, and bounded set.

When you set up your portfolio set, you need to ensure that the portfolio set satisfies these conditions. The most basic or "default" portfolio set requires portfolio weights to be nonnegative (using the lower-bound constraint) and to
sum to 1 ( using the budget constraint). The most general portfolio set handled by the portfolio optimization tools can have any of the following constraints:

- Linear inequality constraints
- Linear equality constraints
- Bound constraints
- Budget constraints
- Group constraints
- Group ratio constraints
- Turnover constraints


## Linear Inequality Constraints

Linear inequality constraints are general linear constraints that model relationships among portfolio weights that satisfy a system of inequalities. Linear inequality constraints take the form

$$
A_{I} x \leq b_{I}
$$

where:
$x$ is the portfolio ( $n$ vector).
$A_{I}$ is the linear inequality constraint matrix ( $n_{I}$-by- $n$ matrix).
$b_{I}$ is the linear inequality constraint vector ( $n_{I}$ vector).
$n$ is the number of assets in the universe and $n_{I}$ is the number of constraints.
Portfolio object properties to specify linear inequality constraints are:

- AInequality for $A_{I}$
- bInequality for $b_{I}$

The default is to ignore these constraints.

## Linear Equality Constraints

Linear equality constraints are general linear constraints that model relationships among portfolio weights that satisfy a system of equalities. Linear equality constraints take the form

$$
A_{E} x=b_{E}
$$

where:
$x$ is the portfolio ( $n$ vector).
$A_{E}$ is the linear equality constraint matrix ( $n_{E}$-by- $n$ matrix).
$b_{I}$ is the linear equality constraint vector ( $n_{E}$ vector).
$n$ is the number of assets in the universe and $n_{E}$ is the number of constraints.
Portfolio object properties to specify linear equality constraints are:

- AEquality for $A_{E}$
- bEquality for $b_{E}$

The default is to ignore these constraints.

## Bound Constraints

Bound constraints are specialized linear constraints that confine portfolio weights to fall either above or below specific bounds. Since every portfolio set must be bounded, it is often a good practice, albeit not necessary, to set explicit bounds for the portfolio problem. To obtain explicit bounds for a given portfolio set, use the method estimateBounds. Bound constraints take the form

$$
l_{B} \leq x \leq u_{B}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{B}$ is the lower-bound constraint ( $n$ vector).
$u_{B}$ is the upper-bound constraint ( $n$ vector).
$n$ is the number of assets in the universe.
Portfolio object properties to specify bound constraints are:

- LowerBound for $l_{B}$
- UpperBound for $u_{B}$

The default is to ignore these constraints.
Note, the default portfolio optimization problem (see "Default Portfolio Problem" on page $4-11$ ) has $l_{B}=0$ with $u_{B}$ set implicitly through a budget constraint.

## Budget Constraints

Budget constraints are specialized linear constraints that confine the sum of portfolio weights to fall either above or below specific bounds. The constraints take the form

$$
l_{S} \leq 1^{T} x \leq u_{S}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{S}$ is the lower-bound budget constraint (scalar).
$u_{S}$ is the upper-bound budget constraint (scalar).
$n$ is the number of assets in the universe.
Portfolio object properties to specify budget constraints are:

- LowerBudget for $l_{S}$
- UpperBudget for $u_{S}$

The default is to ignore this constraint.

The default portfolio optimization problem (see "Default Portfolio Problem" on page $4-11$ ) has $l_{S}=u_{S}=1$, which means that the portfolio weights sum to 1. If the portfolio optimization problem includes possible movements in and out of cash, the budget constraint is used to specify how far portfolios can go into cash. For example, if $l_{S}=0$ and $u_{S}=1$, then the portfolio can have $0 \%$ to $100 \%$ invested in cash. If cash is to be a portfolio choice, set RiskFreeRate $\left(r_{0}\right)$ to a suitable value (see "Return Proxy" on page 4-3 and "Working with a Riskless Asset" on page 4-48).

## Group Constraints

Group constraints are specialized linear constraints that provide a useful way to enforce "membership" among groups of assets. The constraints take the form

$$
l_{G} \leq G x \leq u_{G}
$$

where:
$x$ is the portfolio ( $n$ vector).
$l_{G}$ is the lower-bound group constraint ( $n_{G}$ vector).
$u_{G}$ is the upper-bound group constraint ( $n_{G}$ vector).
$G$ is the matrix of group membership indexes ( $n_{G}$-by- $n$ matrix).
Each row of $G$ identifies which assets belong to a group associated with that row. Each row contains either 0 s or 1 s with 1 indicating that an asset is part of the group or 0 indicating that the asset is not part of the group.

Portfolio object properties to specify group constraints are:

- GroupMatrix for $G$
- LowerGroup for $l_{G}$
- UpperGroup for $u_{G}$

The default is to ignore these constraints.

## Group Ratio Constraints

Group ratio constraints are specialized linear constraints that provide a useful way to enforce relationships among groups of assets. The constraints take the form

$$
l_{R i}\left(G_{B} x\right)_{i} \leq\left(G_{A} x\right)_{i} \leq u_{R i}\left(G_{B} x\right)_{i}
$$

for $i=1, \ldots, n_{R}$ where:
$x$ is the portfolio ( $n$ vector).
$l_{R}$ is the vector of lower-bound group ratio constraints ( $n_{R}$ vector).
$u_{R}$ is the vector matrix of upper-bound group ratio constraints ( $n_{R}$ vector).
$G_{A}$ is the matrix of base group membership indexes ( $n_{R}$-by- $n$ matrix).
$G_{B}$ is the matrix of comparison group membership indexes ( $n_{R}$-by- $n$ matrix).
$n$ is the number of assets in the universe and $n_{R}$ is the number of constraints. Each row of $G_{A}$ and $G_{B}$ identify which assets belong to a base and comparison group associated with that row. Each row contains either 0s or 1s with 1 indicating that an asset is part of the group or 0 indicating that the asset is not part of the group.

Portfolio object properties to specify group ratio constraints are:

- GroupA for $G_{A}$
- GroupB for $G_{B}$
- LowerRatio for $l_{R}$
- UpperRatio for $u_{R}$

The default is to ignore these constraints.

## Turnover Constraints

Turnover constraint is a linear absolute value constraint that ensures estimated optimal portfolios differ from an initial portfolio by no more than a
specified amount. Although portfolio turnover is defined in many ways, the turnover constraints implemented in Financial Toolbox software computes portfolio turnover as the average of purchases and sales. Turnover constraints takes the form

$$
\frac{1}{2} 1^{T}\left|x-x_{0}\right| \leq \tau
$$

where:
$x$ is the portfolio ( $n$ vector).
$x_{0}$ is the initial portfolio ( $n$ vector).
$\tau$ is the upper-bound for turnover (scalar).
$n$ is the number of assets in the universe.

Portfolio object properties to specify the turnover constraint are:

- Turnover for $\tau$
- InitPort for $x_{0}$

The default is to ignore this constraint.

## Default Portfolio Problem

The default portfolio optimization problem has a risk and return proxy associated with a given problem, and a portfolio set that specifies portfolio weights to be nonnegative and to sum to 1 . The lower bound combined with the budget constraint is sufficient to ensure that the portfolio set is nonempty, closed, and bounded. The default portfolio optimization problem characterizes a long-only investor who is fully invested in a collection of assets.

## Portfolio Object

In this section...
"Portfolio Object Properties and Methods" on page 4-12
"Working with Portfolio Objects" on page 4-17
"Setting and Getting Properties" on page 4-18
"Displaying Portfolio Objects" on page 4-18
"Saving and Loading Portfolio Objects" on page 4-19
"Estimating Efficient Portfolios and Frontiers" on page 4-19
"Arrays of Portfolio Objects" on page 4-19
"Subclassing Portfolio Objects" on page 4-20
"Conventions for Representation of Data" on page 4-20
Portfolio Object Properties and MethodsThe portfolio object implements mean-variance portfolio optimization and isderived from the abstract class AbstractPortfolio. Every property andmethod of the portfolio object is public, although some properties and methodsare hidden. See Portfolio Object Properties on page 4-12 and Portfolio ObjectMethods on page 4-14 for the properties and methods of a portfolio object. Theportfolio object is a value object where every instance of the object is a distinctversion of the object. Since the portfolio object is also a MATLAB object, itinherits the default methods associated with MATLAB objects.

## Portfolio Object Properties

| Property | Description | Characteristics |
| :--- | :--- | :--- |
| AEquality | Matrix for equality <br> constraints | Matrix |
| AInequality | Matrix for inequality <br> constraints | Matrix |

## Portfolio Object Properties (Continued)

| Property | Description | Characteristics |
| :---: | :---: | :---: |
| AssetCovar | Covariance of asset returns | Symmetric positive-semidefinite matrix |
| AssetList | List of asset names or identifiers | Vector cell array of strings |
| AssetMean | Mean of asset returns | Vector |
| bEquality | Vector for equality constraints | Vector |
| bInequality | Vector for inequality constraints | Vector |
| BuyCost | Cost to purchase assets | Vector |
| GroupA | Base group ratio constraint matrix | Boolean matrix or matrix |
| GroupB | Comparison group ratio constraint matrix | Boolean matrix or matrix |
| GroupMatrix | Group membership matrix | Boolean matrix or matrix |
| InitPort | Initial or current portfolio | Vector |
| LowerBound | Lower bound constraint | Vector |
| LowerBudget | Lower budget constraint | Scalar |
| LowerGroup | Lower group constraint | Vector |
| LowerRatio | Lower group ratio constraint ratio | Vector |
| Name | Name for instance of portfolio object | String |
| NumAssets | Number of assets in universe | Scalar positive integer |
| RiskFreeRate | Period return of riskless asset | Scalar |
| SellCost | Cost to sell assets | Vector |

## Portfolio Object Properties (Continued)

| Property | Description | Characteristics |
| :--- | :--- | :--- |
| Turnover | Upper bound portfolio <br> turnover | Scalar |
| UpperBound | Upper bound constraint | Vector |
| UpperBudget | Upper budget constraint | Scalar |
| UpperGroup | Upper group constraint | Vector |
| UpperRatio | Upper group ratio constraint <br> ratio | Vector |

## Portfolio Object Methods

| Method | Description |
| :--- | :--- |
| addEquality | Add equality constraints for portfolio <br> weights to existing constraints. |
| addGroupRatio | Add group ratio constraints for <br> portfolio weights to existing <br> constraints. |
| addGroups | Add group constraints for portfolio <br> weights to existing constraints. |
| addInequality | Add inequality constraints for portfolio <br> weights to existing constraints. |
| checkFeasibility | Determine if portfolios are members of <br> the set of feasible portfolios. |
| estimateAssetMoments | Estimate mean and covariance of asset <br> returns from price or return data. |
| estimateBounds | Determine if set of feasible portfolios <br> is nonempty and bounded. |
| estimateFrontier | Estimate portfolios on the entire <br> efficient frontier. |

## Portfolio Object Methods (Continued)

| Method | Description |
| :--- | :--- |
| estimateFrontierByReturn | Estimate portfolios on the efficient <br> frontier with targeted returns or <br> return proxies. |
| estimateFrontierByRisk | Estimate portfolios on the efficient <br> frontier with targeted risks or risk <br> proxies. |
| estimateFrontierLimits | Estimate portfolios at the extreme <br> ends of the efficient frontier (minimum <br> risk and maximum return). |
| estimatePortMoments | Estimate mean and standard <br> deviation of portfolio returns for <br> specified portfolios. |
| estimatePortReturn | Estimate return or return proxy for <br> specified portfolios. |
| estimatePortRisk | Estimate risk or risk proxy for <br> specified portfolios. |
| getAssetMoments | Get mean and covariance of asset <br> returns from object. |
| getBounds | Get lower and upper bounds from <br> object. |
| getBudget | Get lower and upper budget <br> constraints from object. |
| getCosts | Get purchase and sales proportional <br> transaction costs from object. |
| getEquality | Get equality constraint matrix and <br> vector from object. |
| getGroupRatio | Get base matrix, comparison matrix, <br> and lower and upper bounds for group <br> ratio constraints from object. |

## Portfolio Object Methods (Continued)

| Method | Description |
| :--- | :--- |
| getGroups | Get group matrix and lower and upper <br> bounds for group constraints from <br> object. |
| getInequality | Get inequality constraint matrix and <br> vector from object. |
| plotFrontier | Plot efficient frontier and optionally <br> obtain risks and returns for portfolios <br> on the efficient frontier. |
| setAssetList | Set up a list of asset names and <br> symbols to be associated with assets <br> in universe. |
| setAssetMoments | Set up mean and covariance of asset <br> returns. |
| setBounds | Set up lower and upper bounds for <br> portfolio weights. |
| setBudget | Set up lower and upper budget <br> constraints for portfolio weights. |
| setCosts | Set up purchase and sale proportional <br> transaction costs for assets in universe. |
| setDefaultConstraints | Set up default constraints for portfolio <br> weights (nonnegative weights that <br> must sum to 1). |
| setEquality | Set up equality constraints for portfolio <br> weights. |
| setGroupRatio | Set up group ratio constraints for <br> portfolio weights. |
| setGroups | Set up group constraints for portfolio <br> weights. |
| setInequality | Set up inequality constraints for <br> portfolio weights. |
|  |  |

## Portfolio Object Methods (Continued)

| Method | Description |
| :--- | :--- |
| setInitPort | Set up initial portfolio weights. |
| setOptions | Set up hidden control properties in <br> object (not implemented). |
| setSolver | Set up solver to estimate efficient <br> portfolios. |
| setTurnover | Set up turnover constraints for <br> portfolio weights. |

## Working with Portfolio Objects

The portfolio object and its methods are an interface for mean-variance portfolio optimization. Consequently, almost everything you do with the portfolio object can be done using the methods. The basic workflow is:

1 Design your portfolio problem.
2 Use the portfolio constructor (Portfolio.) to create the portfolio object or use the various set methods to set up your portfolio problem.

3 Use estimate methods to solve your portfolio problem.
In addition, methods are available to help you view intermediate results and to diagnose your computations. Since MATLAB features are part of a portfolio object, you can save and load objects from your workspace and create and manipulate arrays of objects. After settling upon a problem, which, in the case of mean-variance portfolio optimization, means that you have either data or moments for asset returns and a collection of constraints on your portfolios, use the portfolio constructor to set the properties for the portfolio object. The portfolio constructor lets you create an object from scratch or update an existing object. Since the portfolio object is a value object, it is easy to create a basic object, then use methods to build upon the basic object to create new versions of the basic object. This is useful to compare a basic problem with alternatives derived from the basic problem. For details, see "Constructing the Portfolio Object" on page 4-22.

## Setting and Getting Properties

You can set properties of a portfolio object with either the constructor (Portfolio.) or various set methods.

Note Although you can also set properties directly, it is not recommended since error-checking is not performed when you set a property directly.

The portfolio constructor supports setting properties with parameter name and value pairs such that each parameter name is a property and each value is the value to assign to that property. For example, to set the AssetMean and AssetCovar properties in an existing portfolio object $p$ with the values $m$ and $C$, use the syntax:

```
p = Portfolio(p, 'AssetMean', m, 'AssetCovar', C);
```

In addition to the portfolio constructor, which lets you set individual properties one at a time, groups of properties are set in a portfolio object with various "set" and "add" methods. For example, to set up a turnover constraint, use the setTurnover method to specify the bound on portfolio turnover and the initial portfolio. To get individual properties from a portfolio object, obtain properties directly or use an assortment of "get" methods that obtain groups of properties from a portfolio object. The portfolio object constructor and set methods have several useful features:

- The constructor and set methods try to determine the dimensions of your problem with either explicit or implicit inputs.
- The constructor and set methods try to resolve ambiguities with default choices.
- The constructor and set methods perform scalar expansion on arrays when possible.
- The methods try to diagnose and warn about problems.


## Displaying Portfolio Objects

The portfolio object uses the default display method provided by MATLAB, where display and disp display a portfolio object and its properties with or without the object variable name.

## Saving and Loading Portfolio Objects

Save and load portfolio objects with the MATLAB save and load commands.

## Estimating Efficient Portfolios and Frontiers

Estimating efficient portfolios and efficient frontiers is the primary purpose of the portfolio optimization tools. A collection of "estimate" and "plot" methods provides ways to explore the efficient frontier. The "estimate" methods obtain either efficient portfolios or risk and return proxies to form efficient frontiers. At the portfolio level, a collection of methods estimates efficient portfolios on the efficient frontier with methods to obtain efficient portfolios:

- At the endpoints of the efficient frontier
- That attain targeted values for return proxies
- That attain targeted values for risk proxies
- Along the entire efficient frontier

These methods also provide purchases and sales needed to shift from an initial or current portfolio to each efficient portfolio. At the efficient frontier level, a collection of methods plot the efficient frontier and estimate either risk or return proxies for efficient portfolios on the efficient frontier. The resultant efficient portfolios or risk and return proxies can be used in subsequent analyses.

## Arrays of Portfolio Objects

Although all methods associated with a portfolio object are designed to work on a scalar portfolio object, the array capabilities of MATLAB enables you to set up and work with arrays of portfolio objects. The easiest way to do this is with the repmat function. For example, to create a 3-by-2 array of portfolio objects:

```
p = repmat(Portfolio, 3, 2);
disp(p)
```

Once you have set up an array of portfolio objects, you can work on individual portfolio objects in the array by indexing. For example:

```
p(i,j) = Portfolio(p(i,j), ... );
```

This example calls the portfolio object constructor for the (i, j) element of a matrix of portfolio objects in the variable $p$.

If you set up an array of portfolio objects, you can access properties of a particular portfolio object in the array by indexing so that you can set the lower and upper bounds lb and ub for the (i,j,k) element of a 3-D array of portfolio objects with

$$
p(i, j, k)=p(i, j, k) . s e t B o u n d s(l b, u b) ;
$$

and, once set, you can access these bounds with

$$
[l b, u b]=p(i, j, k) . \text { getBounds; }
$$

Portfolio object methods work on only one portfolio object at a time.

## Subclassing Portfolio Objects

You can subclass the portfolio object to override existing methods or to add new properties or methods. To do so, create a derived class from the Portfolio class. This gives you all the properties and methods of the Portfolio class along with any new features that you choose to add to your subclassed object. Since the Portfolio class is derived from an abstract class called AbstractPortfolio, you can also create a derived class from AbstractPortfolio that implements an entirely different form of portfolio optimization using the properties and methods of the AbstractPortfolio class.

## Conventions for Representation of Data

The portfolio optimization tools follow these conventions regarding the representation of different quantities associated with portfolio optimization:

- Asset returns or prices are in matrix form with samples for a given asset going down the rows and assets going across the columns.
- The mean and covariance of asset returns are stored in a vector and a matrix and the tools have no requirement that the mean must be either a column or row vector.
- Portfolios are in vector or matrix form with weights for a given portfolio going down the rows and distinct portfolios going across the columns.
- Constraints on portfolios are formed in such a way that a portfolio is a column vector.
- Portfolio risks and returns are either scalars or column vectors (for multiple portfolio risks and returns).


## Constructing the Portfolio Object

In this section...<br>"Syntax" on page 4-22<br>"Portfolio Problem Sufficiency" on page 4-23<br>"Constructor Examples" on page 4-23

To create a fully specified mean-variance portfolio optimization problem, instantiate the portfolio object using the portfolio constructor.

## Syntax

Use the portfolio constructor Portfolio. to create an instance of an object of the Portfolio. class. The portfolio constructor can be used in several ways. To set up a portfolio optimization problem in a portfolio object, the simplest syntax is:
p = Portfolio;

This syntax creates a portfolio object $p$ such that all object properties are empty.

The constructor also accepts collections of parameter name-value pairs for properties and their values. The constructor accepts inputs for public properties (see Portfolio Object Properties on page 4-12) with the general syntax:

```
p = Portfolio('property1', value1, 'property2', value2, ... );
```

If a portfolio object already exists, the syntax permits the first (and only the first argument) of the portfolio constructor to be an existing object with subsequent parameter name-value pairs for properties to be added or modified. For example, given an existing portfolio object in $p$, the general syntax is:

```
p = Portfolio(p, 'property1', value1, 'property2', value2, ... );
```

Input parameter names are not case sensitive, but must be completely specified. In addition, several properties can be specified with alternative
parameter names (see on page 4-26). The constructor Portfolio tries to detect problem dimensions from the inputs and, once set, subsequent inputs can undergo various scalar or matrix expansion operations that simplify the overall process to formulate a problem. In addition, a portfolio object is a value object so that, given portfolio $p$, the following code creates two objects,p and q , that are distinct:

$$
q=\text { Portfolio }(p, \ldots)
$$

## Portfolio Problem Sufficiency

A mean-variance portfolio optimization is completely specified with the portfolio object if these two conditions are met:

- The moments of asset returns must be specified such that the property AssetMean contains a valid finite mean vector of asset returns and the property AssetCovar contains a valid symmetric positive-semidefinite matrix for the covariance of asset returns.

The first condition is satisfied by setting the properties associated with the moments of asset returns.

- The set of feasible portfolios must be a nonempty compact set, where a compact set is closed and bounded.

The second condition is satisfied by an extensive collection of properties that define different types of constraints to form a set of feasible portfolios. Since such sets must be bounded, either explicit or implicit constraints can be imposed, and several methods, such as estimateBounds, provide ways to ensure that your problem is properly formulated.

Although the general sufficiency conditions for mean-variance portfolio optimization go beyond these two conditions, the portfolio object implemented in Financial Toolbox software implicitly handles all these additional conditions. For more information on the Markowitz model for mean-variance portfolio optimization, see "Portfolio Optimization" on page A-12.

## Constructor Examples

If you create a portfolio object $p$ with no input arguments, you can display it using disp:

```
p = Portfolio;
```

```
disp(p); Portfolio
Properties:
            BuyCost: []
            SellCost: []
    RiskFreeRate: []
            AssetMean: []
            AssetCovar: []
            Turnover: []
            Name: []
            NumAssets: []
            AssetList: []
            InitPort: []
    AInequality: []
    bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: []
            UpperBound: []
            LowerBudget: []
            UpperBudget: []
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
                    GroupA: []
                    GroupB: []
            LowerRatio: []
            UpperRatio: []
                    Methods, Superclasses
```

The approaches listed provide a way to set up a portfolio optimization problem with the portfolio constructor. The custom set methods offer additional ways to set and modify collections of properties in the portfolio object.

## Using the Constructor for a Single Step Setup

You can use the constructor to directly set up a "standard" portfolio optimization problem, given a mean and covariance of asset returns in the variables $m$ and $C$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('assetmean', m, 'assetcovar', C, ...
'lowerbudget', 1, 'upperbudget', 1, 'lowerbound', 0);
```

Note, the LowerBound property value undergoes scalar expansion since AssetMean and AssetCovar provide the dimensions of the problem.

You can use dot notation with the method plotFrontier:

```
p.plotFrontier;
```



## Using the Constructor with a Sequence of Steps

An alternative way to accomplish the same task of setting up a "standard" portfolio optimization problem, given a mean and covariance of asset returns in the variables $m$ and $C$ (which also illustrates that parameter names are not case sensitive):

```
p = Portfolio;
p = Portfolio(p, 'assetmean', m, 'assetcovar', C);
p = Portfolio(p, 'lowerbudget', 1, 'upperbudget', 1);
p = Portfolio(p, 'lowerbound', 0);
p.plotFrontier;
```

This alternative works because the calls to the constructor are in this particular order. In this case, the call to initialize AssetMean and AssetCovar provides the dimensions for the problem. If you were to do this step last, you would have to explicitly dimension the LowerBound property as follows:

```
p = Portfolio;
p = Portfolio(p, 'LowerBound', zeros(size(m)));
p = Portfolio(p, 'LowerBudget', 1, 'UpperBudget', 1);
p = Portfolio(p, 'AssetMean', m, 'AssetCovar', C);
p.plotFrontier;
```

If you did not specify the size of LowerBound but, instead, input a scalar argument, the constructor assumes that you are defining a single-asset problem and produces an error at the call to set asset moments with four assets.

## Shortcuts for Property Names

The portfolio constructor has shorter parameter names that replace longer parameter names associated with specific properties of the portfolio object. For example, rather than enter 'assetcovar', the constructor accepts the case-insensitive name 'covar' to set the AssetCovar property in a portfolio object. Although every parameter name corresponds with a single property in the portfolio constructor, one exception exists with the alternative parameter name 'budget', which signifies that both the LowerBudget and UpperBudget properties are set to the same value to form an equality budget constraint.

## (Continued)

| Shortcut Parameter <br> Name | Equivalent Parameter / Property Name |
| :--- | :--- |
| ae | AEquality |
| ai | AInequality |
| covar | AssetCovar |
| assetnames or assets | AssetList |
| mean | AssetMean |
| be | bEquality |
| bi | bInequality |
| group | GroupMatrix |
| lb | LowerBound |
| budget | LowerBudget |
| n or num | NumAssets |
| rfr | RiskFreeRate |
| ub | UpperBound |
| budget | UpperBudget and LowerBudget |

For example, this call to the constructor uses these shortcuts for properties and is equivalent to the previous examples:

```
p = Portfolio('mean', m, 'covar', C, 'budget', 1, 'lb', 0);
p.plotFrontier;
```


## Direct Setting of Portfolio Object Properties

Although not recommended, you can set properties directly, however no error-checking is done on your inputs:

```
p = Portfolio;
p.NumAssets = numel(m);
p.AssetMean = m;
```

```
p.AssetCovar = C;
p.LowerBudget = 1;
p.UpperBudget = 1;
p.LowerBound = zeros(size(m));
p.plotFrontier;
```


## Common Operations on the Portfolio Object

In this section...<br>"Naming a Portfolio Object" on page 4-29<br>"Setting Up the Number of Assets in the Asset Universe" on page 4-29<br>"Setting Up a List of Asset Identifiers" on page 4-30<br>"Truncating and Padding Asset Lists" on page 4-31<br>"Setting Up an Initial or Current Portfolio" on page 4-32

## Naming a Portfolio Object

To name a portfolio object, use the Name property. Name is informational and has no effect on any portfolio calculations. If the Name property is nonempty, Name is the title for the efficient frontier plot generated by plotFrontier. For example, if you set up an asset allocation fund, you could name the portfolio object Asset Allocation Fund:

```
p = Portfolio('Name','Asset Allocation Fund');
disp(p.Name);
Asset Allocation Fund
```


## Setting Up the Number of Assets in the Asset Universe

The fundamental quantity in the portfolio object is the number of assets in the asset universe. This quantity is maintained in the NumAssets property . Although you can set this property directly, it is usually derived from other properties such as the mean of asset returns and the initial portfolio. In some instances, the number of assets may need to be set directly. This example shows how to set up a portfolio object that has four assets:

```
p = Portfolio('NumAssets', 4);
disp(p.NumAssets);
4
```

Once the NumAssets property is set, you cannot modify it (unless no other properties are set that depend upon NumAssets). The only way to change the number of assets in an existing portfolio object with a known number of assets is to create a new portfolio object.

## Setting Up a List of Asset Identifiers

When working with portfolios, you must specify a universe of assets. Although you can perform a complete analysis without naming the assets in your universe, it is helpful to have an identifier associated with each asset as you create and work with portfolios. You can create a list of asset identifiers as a cell vector of strings in the property AssetList. You can set up the list using two methods.

## Setting Up Asset Lists Using the Constructor

Suppose you have a portfolio object $p$ with assets with symbols 'AA', 'BA', 'CAT', 'DD', and 'ETR'. You can create a list of these asset symbols in the object using the constructor:

```
p = Portfolio('assetlist', { 'AA', 'BA', 'CAT', 'DD', 'ETR' });
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```

Notice that the property AssetList is maintained as a cell array that contains strings, and that it is necessary to pass a cell array into the constructor to set AssetList. In addition, notice that the property NumAssets is set to 5 based on the number of symbols used to create the asset list:

```
disp(p.NumAssets);
5
```


## Setting Up Asset Lists Using the setAssetList Method

You can also specify a list of assets using the method setAssetList. Given the list of asset symbols 'AA', 'BA', 'CAT', 'DD', and'ETR', you can use setAssetList with:

```
p = Portfolio;
p = p.setAssetList({ 'AA', 'BA', 'CAT', 'DD', 'ETR' });
disp(p.AssetList);
    'AA' 'BA' 'CAT' 'DD' 'ETR'
```

setAssetList also enables you enter symbols directly as a comma-separated list without creating a cell array of strings. For example, given the list of assets symbols 'AA', 'BA', 'CAT', 'DD', and 'ETR', use setAssetList:

```
p = Portfolio;
p = p.setAssetList('AA', 'BA', 'CAT', 'DD', 'ETR');
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```

setAssetList has many additional features to create lists of asset identifiers. If you use setAssetList with just a portfolio object, it creates a default asset list according to the name specified in the hidden public property defaultforAssetList (which is 'Asset' by default). The number of asset names created depends on the number of assets in the property NumAssets. If NumAssets is not set, then NumAssets is assumed to be 1 .

For example, if a portfolio object $p$ is created with NumAssets $=5$, then this code fragment demonstrates the default naming behavior:

```
p = Portfolio('numassets',5);
p = p.setAssetList;
disp(p.AssetList);
'Asset1' 'Asset2' 'Asset3' 'Asset4' 'Asset5'
```

Suppose that your assets are, for example, ETFs and you change the hidden property defaultforAssetList to 'ETF', you can then create a default list for ETFs:

```
p = Portfolio('numassets',5);
p.defaultforAssetList = 'ETF';
p = p.setAssetList;
disp(p.AssetList);
'ETF1' 'ETF2' 'ETF3' 'ETF4' 'ETF5'
```


## Truncating and Padding Asset Lists

If the NumAssets property is already set and you pass in too many or too few identifiers, the portfolio constructor, and the setAssetList method truncate or pad the list with numbered default asset names that use the name specified in the hidden public property defaultforAssetList. If the list is truncated or padded, a warning message indicates the discrepancy. For example, assume you have a portfolio object with five ETFs and you only know the first three CUSIPs '921937835', '922908769', and '922042775'. Use this
syntax to create an asset list that pads the remaining asset identifiers with numbered "UnknownCUSIP" placeholders:

```
p = Portfolio('numassets',5);
p.defaultforAssetList = 'UnknownCUSIP';
p = p.setAssetList('921937835', '922908769', '922042775');
disp(p.AssetList);
Warning: Input list of assets has 2 too few identifiers. Padding with numbered assets.
> In Portfolio.setAssetList at 130
'921937835' '922908769' '922042775' 'UnknownCUSIP4' 'UnknownCUSIP5'
```

Alternatively, suppose you have too many identifiers and need only the first four assets. This example illustrates truncation of the asset list using the portfolio constructor:

```
p = Portfolio('numassets',4);
p = Portfolio(p, 'assetlist', { 'AGG', 'EEM', 'MDY', 'SPY', 'VEU' });
disp(p.AssetList);
Warning: AssetList has 1 too many identifiers. Using first 4 assets.
> In Portfolio.checkarguments at 477
    In Portfolio.Portfolio>Portfolio.Portfolio at 180
'AGG' 'EEM' 'MDY' 'SPY'
```

The hidden public property uppercaseAssetList is a Boolean flag to specify whether to convert asset names to uppercase letters. The default value for uppercaseAssetList is false. This example shows how to use the uppercaseAssetList flag to force identifiers to be uppercase letters:

```
p = Portfolio;
p.uppercaseAssetList = true;
p = p.setAssetList({ 'aa', 'ba', 'cat', 'dd', 'etr' });
disp(p.AssetList);
'AA' 'BA' 'CAT' 'DD' 'ETR'
```


## Setting Up an Initial or Current Portfolio

In many applications, creating a new optimal portfolio requires comparing the new portfolio with an initial or current portfolio to form lists of purchases and sales. The portfolio object property InitPort lets you identify an initial or current portfolio. The initial portfolio also plays an essential role if you have either transaction costs or a turnover constraint. The initial portfolio need not be feasible within the constraints of the problem. This can
happen if the weights in a portfolio have shifted such that some constraints become violated. To check if your initial portfolio is feasible, use the method checkFeasibility described in "Validating Portfolios" on page 4-75. Suppose you have an initial portfolio in $\times 0$, then use the portfolio object constructor to set up an initial portfolio:

```
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = Portfolio('InitPort', x0);
disp(p.InitPort);
    0.3000
    0.2000
    0.2000
        0
```

As with all array properties, InitPort can be set with scalar expansion. This is helpful to set up an equally weighted initial portfolio of, for example, 10 assets:

```
p = Portfolio('NumAssets', 10, 'InitPort', 1/10);
disp(p.InitPort);
    0.1000
    0.1000
    0.1000
    0.1000
    0.1000
    0.1000
    0.1000
    0.1000
    0.1000
    0.1000
```

To clear an initial portfolio from your portfolio object, use either the constructor or the setInitPort method with an empty input for the InitPort property. If transaction costs or turnover constraints are set, it is not possible to clear the InitPort property in this way. In this case, to clear InitPort, first clear the dependent properties and then clear theInitPort property.

The InitPort property can also be set with setInitPort which lets you specify the number of assets if you want to use scalar expansion. For example, given an initial portfolio in $\times 0$, use setInitPort to set the InitPort property:

```
p = Portfolio;
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = p.setInitPort(x0);
disp(p.InitPort);
    0.3000
    0.2000
    0.2000
        0
```

To create, an equally weighted portfolio of four assets, use setInitPort:

```
p = Portfolio;
p = p.setInitPort(1/4, 4);
disp(p.InitPort);
    0.2500
    0.2500
    0.2500
    0.2500
```

Portfolio object methods that work with either transaction costs or turnover constraints also depend on the InitPort property. Consequently, the set methods for transaction costs or turnover constraints permit the assignment of a value for the InitPort property as part of their implementation. For details, see "Working with Turnover Constraints" on page 4-70 and "Working with Transaction Costs" on page 4-49 for details. If either transaction costs or turnover constraints are used, then the InitPort property must have a nonempty value. Absent a specific value assigned through the constructor or various set methods, the portfolio object sets InitPort to 0 and warns if BuyCost, SellCost, or Turnover properties are set. The following example illustrates what happens if a turnover constraint is specified with an initial portfolio:

```
p = Portfolio('Turnover', 0.3, 'InitPort', [ 0.3; 0.2; 0.2; 0.0 ]);
disp(p.InitPort);
    0.3000
    0.2000
    0.2000
        0
```

In contrast, this example shows what happens if a turnover constraint is specified without an initial portfolio:

```
p = Portfolio('Turnover', 0.3);
disp(p.InitPort);
Warning: InitPort and NumAssets are empty and either transaction costs or turnover constraints
specified. Will set NumAssets = 1 and InitPort = 0.
> In Portfolio.checkarguments at 403
    In Portfolio.Portfolio>Portfolio.Portfolio at 180
```

    0
    
## Working with Asset Returns and Moments of Asset Returns

In this section...<br>"Assignment Using the Portfolio Constructor" on page 4-36<br>"Assignment Using the setAssetMoments Method" on page 4-38<br>"Scalar Expansion of Arguments" on page 4-39<br>"Estimating Asset Moments from Asset Prices or Returns" on page 4-40<br>"Estimating Asset Moments from Asset Returns or Prices with Missing Data" on page 4-44<br>"Estimating Asset Moments from Financial Time Series Data" on page 4-46<br>"Working with a Riskless Asset" on page 4-48<br>"Working with Transaction Costs" on page 4-49

Since mean-variance portfolio optimization problems require estimates for the mean and covariance of asset returns, the portfolio object has several ways to set and get the properties AssetMean (for the mean) and AssetCovar (for the covariance). In addition, the return for a riskless asset is kept in the property RiskFreeRate so that all assets in AssetMean and AssetCovar are risky assets.

## Assignment Using the Portfolio Constructor

Suppose you have a mean and covariance of asset returns in variables $m$ and $C$. The properties for the moments of asset returns are set using the constructor:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio('AssetMean', m, 'AssetCovar', C);
disp(p.NumAssets);
disp(p.AssetMean);
disp(p.AssetCovar);
```

4

| 0.0042 |  |  |  |
| ---: | ---: | ---: | ---: |
| 0.0083 |  |  |  |
| 0.0100 |  |  | 0 |
| 0.0150 |  | 0.0002 | 0.0010 |
|  |  | 0.0017 | 0.0028 |
| 0.0005 | 0.0003 | 0.0048 | 0.0102 |

Notice that the portfolio object determines the number of assets in NumAssets from the moments. The portfolio constructor enables separate initialization of the moments, for example:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio;
p = Portfolio(p, 'AssetMean', m);
p = Portfolio(p, 'AssetCovar', C);
[assetmean, assetcovar] = p.getAssetMoments
assetmean =
    0.0042
    0.0083
    0.0100
    0.0150
assetcovar =
```

| 0.0005 | 0.0003 | 0.0002 | 0 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0048 | 0.0028 |
| 0 | 0.0010 | 0.0028 | 0.0102 |

The getAssetMoments method lets you get the values for AssetMean and AssetCovar properties at the same time.

## Assignment Using the setAssetMoments Method

You can also set asset moment properties using the setAssetMoments method. For example, given the mean and covariance of asset returns in the variables $m$ and $C$, the asset moment properties can be set:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio;
p = p.setAssetMoments(m, C);
[assetmean, assetcovar] = p.getAssetMoments
assetmean =
    0.0042
    0.0083
    0.0100
    0.0150
assetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```


## Scalar Expansion of Arguments

Both the constructor Portfolio. and the setAssetMoments method perform scalar expansion on arguments for the moments of asset returns. When using the constructor, the number of assets must be already specified in the variable NumAssets. If NumAssets has not already been set, a scalar argument is interpreted as a scalar with NumAssets set to 1. setAssetMoments provides an additional optional argument to specify the number of assets so that scalar expansion works with the correct number of assets. In addition, if either a scalar or vector is input for the covariance of asset returns, a diagonal matrix is formed such that a scalar expands along the diagonal and a vector becomes the diagonal. This example demonstrates scalar expansion for four jointly independent assets with a common mean 0.1 and common variance 0.03 :

```
p = Portfolio;
p = p.setAssetMoments(0.1, 0.03, 4);
[assetmean, assetcovar] = p.getAssetMoments
assetmean =
    0.1000
    0.1000
    0.1000
    0.1000
assetcovar =
\begin{tabular}{rrrr}
0.0300 & 0 & 0 & 0 \\
0 & 0.0300 & 0 & 0 \\
0 & 0 & 0.0300 & 0 \\
0 & 0 & 0 & 0.0300
\end{tabular}
```

If at least one argument is properly dimensioned, you don't need to include the additional NumAssets argument. This example illustrates a constant-diagonal covariance matrix and a mean of asset returns for four assets:

```
p = Portfolio;
p = p.setAssetMoments([ 0.05; 0.06; 0.04; 0.03 ], 0.03);
[assetmean, assetcovar] = p.getAssetMoments
assetmean =
```

$$
\begin{array}{rrrr}
0.0500 \\
0.0600 & & \\
0.0400 & & \\
0.0300 & & \\
\text { assetcovar }= & & & \\
& & & \\
0.0300 & 0 & 0 & 0 \\
0 & 0.0300 & 0 & 0 \\
0 & 0 & 0.0300 & 0 \\
0 & 0 & 0 & 0.0300
\end{array}
$$

In addition, scalar expansion works with the portfolio constructor if NumAssets is known, or is deduced from the inputs.

## Estimating Asset Moments from Asset Prices or Returns

Another way to set the moments of asset returns is to use the method estimateAssetMoments which accepts either prices or returns and estimates the mean and covariance of asset returns. Either prices or returns are stored as matrices with samples going down the rows and assets going across the columns. In addition, prices or returns can be stored in a financial time series (fints) object (see "Estimating Asset Moments from Financial Time Series Data" on page 4-46). To illustrate using estimateAssetMoments, generate random samples of 120 observations of asset returns for four assets from the mean and covariance of asset returns in the variables $m$ and $C$ with portsim. The default behavior of portsim creates simulated data with estimated mean and covariance identical to the input moments $m$ and $C$. In addition to a return series created by portsim in the variable $X$, a price series is created in the variable $Y$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
```

```
Y = ret2tick(X);
```

Note Portfolio optimization requires that you use total returns and not just price returns. Consequently, "returns" should be total returns and "prices" should be total return prices.

Given asset returns and prices in variables $X$ and $Y$ from above, this sequence of examples demonstrates equivalent ways to estimate asset moments for the portfolio object. A portfolio object is created in p with the moments of asset returns set directly in the constructor, and a second portfolio object is created in $q$ to obtain the mean and covariance of asset returns from asset return data in X using estimateAssetMoments:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
p = Portfolio('mean', m, 'covar', C);
q = Portfolio;
q = q.estimateAssetMoments(X);
[passetmean, passetcovar] = p.getAssetMoments
[qassetmean, qassetcovar] = q.getAssetMoments
passetmean =
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar =
```

```
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
qassetmean =
    0.0042
    0.0083
    0.0100
    0.0150
qassetcovar =
\begin{tabular}{llll}
0.0005 & 0.0003 & 0.0002 & 0.0000 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0.0000 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```

Notice how either approach has the same moments. The default behavior of estimateAssetMoments is to work with asset returns. If, instead, you have asset prices in the variable Y , estimateAssetMoments accepts a parameter name 'DataFormat' with a corresponding value set to 'prices' to indicate that the input to the method is in the form of asset prices and not returns (the default parameter value for 'DataFormat' is 'returns'). This example compares direct assignment of moments in the portfolio object $p$ with estimated moments from asset price data in $Y$ in the portfolio object q:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
Y = ret2tick(X);
```

```
p = Portfolio('mean',m,'covar',C);
q = Portfolio;
q = q.estimateAssetMoments(Y, 'dataformat', 'prices');
[passetmean, passetcovar] = p.getAssetMoments
[qassetmean, qassetcovar] = q.getAssetMoments
passetmean =
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
qassetmean =
    0.0042
    0.0083
    0.0100
    0.0150
qassetcovar =
\begin{tabular}{llll}
0.0005 & 0.0003 & 0.0002 & 0.0000 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0.0000 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```


## Estimating Asset Moments from Asset Returns or Prices with Missing Data

Often when working with multiple assets, you have missing data indicated by NaN values in your return or price data. Although Chapter 7, "Regression with Missing Data" goes into detail about regression with missing data, the method estimateAssetMoments has a parameter name 'MissingData' that indicates with a Boolean value whether to use the missing data capabilities of Financial Toolbox software. The default value for 'MissingData' is false which removes all samples with NaN values. If, however, 'MissingData' is set to true, estimateAssetMoments uses the ECM algorithm to estimate asset moments. This example illustrates how this works on price data with missing values:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
Y = ret2tick(X);
Y(1:20,1) = NaN;
Y(1:12,4) = NaN;
p = Portfolio('mean',m,'covar',C);
q = Portfolio;
q = q.estimateAssetMoments(Y, 'dataformat', 'prices');
r = Portfolio;
r = r.estimateAssetMoments(Y, 'dataformat', 'prices', 'missingdata', true);
[passetmean, passetcovar] = p.getAssetMoments
[qassetmean, qassetcovar] = q.getAssetMoments
[rassetmean, rassetcovar] = r.getAssetMoments
passetmean =
```

```
        0.0042
        0.0083
        0.0100
        0.0150
passetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
qassetmean =
    0.0046
    0.0104
    0.0157
    0.0159
qassetcovar =
\begin{tabular}{llll}
0.0005 & 0.0004 & 0.0003 & 0.0001 \\
0.0004 & 0.0023 & 0.0015 & 0.0009 \\
0.0003 & 0.0015 & 0.0044 & 0.0027 \\
0.0001 & 0.0009 & 0.0027 & 0.0106
\end{tabular}
rassetmean =
    0.0043
    0.0083
    0.0100
    0.0125
rassetcovar =
\begin{tabular}{llll}
0.0007 & 0.0005 & 0.0004 & 0.0001 \\
0.0005 & 0.0032 & 0.0022 & 0.0012 \\
0.0004 & 0.0022 & 0.0063 & 0.0037 \\
0.0001 & 0.0012 & 0.0037 & 0.0135
\end{tabular}
```

The portfolio object $p$ contains raw moments, the object $q$ contains estimated moments in which NaN values are discarded, and the object $r$ contains raw moments that accommodate missing values. Each time you run this example, you will get different estimates for the moments in $q$ and $r$ and these will also differ from the moments in $p$.

## Estimating Asset Moments from Financial Time Series Data

The estimateAssetMoments method also accepts asset returns or prices stored in financial time series (fints) objects. estimateAssetMoments implicitly works with matrices of data or data in a fints object using the same rules for whether the data are returns or prices.

To illustrate, use fints to create a fints objects Xfts that contains asset returns generated with portsim (see "Estimating Asset Moments from Asset Prices or Returns" on page 4-40) and add series labels:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
c=[ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    00.0119 0.0336 0.1225 ];
m = m/12;
c= c/12;
x = portsim(m', c, 120);
d = (datenum('31-jan-2001'):datenum('31-dec-2010'))';
xfts = fints(d, zeros(numel(d),4), {'Bonds', 'LargeCap', 'SmallCap', 'Emerging'});
xfts = tomonthly(Xfts);
xfts.Bonds = x(:,1);
xfts.LargeCap = x(:,2);
xfts.SmallCap = x(:,3);
xfts.Emerging = x(:,4);
p = Portfolio('mean',m,'covar',c);
q = Portfolio;
```

```
q = q.estimateAssetMoments(Xfts);
[passetmean, passetcovar] = p.getAssetMoments
[qassetmean, qassetcovar] = q.getAssetMoments
passetmean =
    0.0042
    0.0083
    0.0100
    0.0150
passetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
qassetmean =
    0.0042
    0.0083
    0.0100
    0.0150
qassetcovar =
\begin{tabular}{llll}
0.0005 & 0.0003 & 0.0002 & 0.0000 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0.0000 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```

As you can see, the moments match. The parameter name-value inputs 'DataFormat' to handle return or price data and 'MissingData' to ignore or use samples with missing values also work for fints data. In addition, estimateAssetMoments also extracts asset names or identifiers from a fints object with the parameter name 'GetAssetList' set to true (its default value is false). If the 'GetAssetList' value is true, the identifiers are used to set the AssetList property of the object. Thus, repeating the formation of the
portfolio object q from the previous example with the 'GetAssetList' flag set to true extracts the series labels from the fints object:

```
q = q.estimateAssetMoments(Xfts, 'getassetlist', true);
disp(q.AssetList)
'Bonds' 'LargeCap' 'SmallCap' 'Emerging'
```

Note if you set the 'GetAssetList' flag set to true and your input data is in a matrix, estimateAssetMoments uses the default labeling scheme from setAssetList described in "Setting Up a List of Asset Identifiers" on page 4-30.

## Working with a Riskless Asset

You can specify a riskless asset with the mean and covariance of asset returns in the AssetMean and AssetCovar properties such that the riskless asset has variance of 0 and is completely uncorrelated with all other assets. In this case, the portfolio object uses a separate RiskFreeRate property that stores the rate of return of a riskless asset. Thus, you can separate your universe into a riskless asset and a collection of risky assets. For example, assume that your riskless asset has a return in the scalar variable ro, then the property for the RiskFreeRate is set using the constructor:

```
r0 = 0.01/12;
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio('RiskFreeRate', ro, 'AssetMean', m, 'AssetCovar', C);
disp(p.RiskFreeRate);
    8.3333e-004
```

Note If your problem has a budget constraint such that your portfolio weights must sum to 1 , then the riskless asset is irrelevant.

## Working with Transaction Costs

The difference between net and gross portfolio returns is transaction costs. The net portfolio return proxy has distinct proportional costs to purchase and to sell assets which are maintained in the portfolio object properties BuyCost and SellCost. Transaction costs are in units of total return and, as such, are proportional to the price of an asset so that they enter the model for net portfolio returns in return form. For example, suppose you have a stock currently priced $\$ 40$ and your usual transaction costs are 5 cents per share. Then the transaction cost for the stock is $0.05 / 40=0.00125$ (as defined in "Net Portfolio Returns" on page 4-4). Costs are entered as positive values and credits are entered as negative values.

## Setting Transaction Costs Using the Constructor

To set up transaction costs, you must specify an initial or current portfolio in the InitPort property. If the initial portfolio is not set at the time that you set up the transaction cost properties, InitPort is 0 . The properties for transaction costs can be set through the constructor Portfolio. For example, assume that purchase and sale transaction costs are in the variables bc and sc and an initial portfolio is in the variable $\times 0$, then transaction costs are set:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('BuyCost', bc, 'SellCost', sc, 'InitPort', x0);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```

5
0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070

$$
\begin{aligned}
& 0.0013 \\
& 0.0013 \\
& 0.0024 \\
& \\
& 0.4000 \\
& 0.2000 \\
& 0.2000 \\
& 0.1000 \\
& 0.1000
\end{aligned}
$$

## Setting Transaction Costs Using setCosts Method

You can also set the properties for transaction costs using the setCosts method. Assume that you have the same costs and initial portfolio as in the previous example. Given a portfolio object $p$ with an initial portfolio already set, use setCosts to set up transaction costs:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('InitPort', x0);
p = p.setCosts(bc, sc);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```

You can also set up the initial portfolio's InitPort value as an optional argument to setCosts so that the following is an equivalent way to set up transaction costs:

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio;
p = p.setCosts(bc, sc, x0);
disp(p.NumAssets);
```

```
disp(p.BuyCost);
disp(p.SellCost);
disp(p.InitPort);
```


## Using the Constructor or Method to Set Bounds

Both the constructor Portfolio and setCosts method implement scalar expansion on the arguments for transaction costs and the initial portfolio. If the NumAssets property is already set in the portfolio object, scalar arguments for these properties are expanded to have the same value across all dimensions. In addition, setCosts lets you specify NumAssets as an optional final argument. For example, assume that you have an initial portfolio x0 and you want to set common transaction costs on all assets in your universe. You can set these costs in any of these equivalent ways:

```
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('InitPort', x0, 'BuyCost', 0.002, 'SellCost', 0.002);
```

or

```
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('InitPort', x0);
p = p.setCosts(0.002, 0.002);
```

or

```
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio;
p = p.setCosts(0.002, 0.002, x0);
```

To clear costs from your portfolio object, use either the constructor or setCosts with empty inputs for the properties to be cleared. For example, you can clear sales costs from the portfolio object $p$ in the previous example:

```
p = Portfolio(p, 'SellCost', []);
```


## Working with Portfolio Constraints

In this section...<br>"Setting Default Constraints for Portfolio Weights" on page 4-52<br>"Working with Bound Constraints" on page 4-55<br>"Working with Budget Constraints" on page 4-57<br>"Working with Group Constraints" on page 4-59<br>"Working with Group Ratio Constraints" on page 4-62<br>"Working with Linear Equality Constraints" on page 4-66<br>"Working with Linear Inequality Constraints" on page 4-68<br>"Working with Turnover Constraints" on page 4-70

## Setting Default Constraints for Portfolio Weights

The "default" portfolio problem has two constraints on portfolio weights:

- Portfolio weights must be nonnegative.
- Portfolio weights must sum to 1 .

Implicitly, these constraints imply that portfolio weights are no greater than 1 , although this is a superfluous constraint to impose on the problem.

## Setting Default Constraints Using Constructor

Given a portfolio optimization problem with NumAssets $=20$ assets, use the constructor Portfolio to set up a default problem and explicitly set bounds and budget constraints:

```
p = Portfolio('NumAssets', 20, 'LowerBound', 0, 'Budget', 1);
disp(p);
Portfolio
Properties:
    BuyCost: []
    SellCost: []
```

```
RiskFreeRate: []
    AssetMean: []
    AssetCovar: []
        Turnover: []
            Name: []
        NumAssets: 20
        AssetList: []
        InitPort: []
    AInequality: []
    bInequality: []
        AEquality: []
        bEquality: []
    LowerBound: [20x1 double]
    UpperBound: []
    LowerBudget: 1
    UpperBudget: 1
    GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
Methods, Superclasses
```


## Setting Default Constraints Using setDefaultConstraints Method

An alternative approach is to use the setDefaultConstraints method. If the number of assets is already known in a portfolio object, use setDefaultConstraints with no arguments to set up the necessary bound and budget constraints. Suppose you have 20 assets to set up the portfolio set for a default problem:

```
p = Portfolio('NumAssets', 20);
p = p.setDefaultConstraints;
disp(p);
```

    Portfolio
    ```
Properties:
            BuyCost: []
            SellCost: []
    RiskFreeRate: []
            AssetMean: []
            AssetCovar: []
            Turnover: []
            Name: []
            NumAssets: 20
            AssetList: []
            InitPort: []
            AInequality: []
            bInequality: []
                AEquality: []
                bEquality: []
            LowerBound: [20x1 double]
            UpperBound: []
            LowerBudget: 1
            UpperBudget: 1
            GroupMatrix: []
            LowerGroup: []
            UpperGroup: []
            GroupA: []
            GroupB: []
            LowerRatio: []
            UpperRatio: []
                    Methods, Superclasses
```

If the number of assets is unknown, setDefaultConstraints accepts NumAssets as an optional argument to form a portfolio set for a default problem. Suppose you have 20 assets:

```
p = Portfolio;
p = p.setDefaultConstraints(20);
disp(p);
    Portfolio
    Properties:
```

```
            BuyCost: []
            SellCost: []
RiskFreeRate: []
    AssetMean: []
    AssetCovar: []
        Turnover: []
            Name: []
        NumAssets: 20
        AssetList: []
        InitPort: []
    AInequality: []
    bInequality: []
        AEquality: []
        bEquality: []
        LowerBound: [20x1 double]
        UpperBound: []
        LowerBudget: 1
        UpperBudget: 1
        GroupMatrix: []
    LowerGroup: []
    UpperGroup: []
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
Methods, Superclasses
```


## Working with Bound Constraints

Bound constraints are optional linear constraints that maintain upper and lower bounds on portfolio weights (see "Bound Constraints" on page 4-7). Although every portfolio set must be bounded, it is not necessary to specify a portfolio set with explicit bound constraints. For example, you can create a portfolio set with an implicit upper bound constraint or a portfolio set with just turnover constraints. The bound constraints have properties LowerBound for the lower-bound constraint and UpperBound for the upper-bound constraint. Set default values for these constraints using the setDefaultConstraints method (see "Setting Default Constraints for Portfolio Weights" on page 4-52).

## Setting Bounds Using the Constructor

The properties for bound constraints are set through the constructor Portfolio.. Suppose you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. The bound constraints for a balanced fund are set with:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = Portfolio('LowerBound', lb, 'UpperBound', ub);
disp(p.NumAssets);
disp(p.LowerBound);
disp(p.UpperBound);
    2
    0.5000
    0.2500
    0.7500
    0.5000
```

To continue with this example, you must set up a budget constraint. For details, see "Working with Budget Constraints" on page 4-57.

## Setting Bounds Using the setBounds Method

You can also set the properties for bound constraints using the setBounds method. Suppose you have a balanced fund with stocks that can range from $50 \%$ to $75 \%$ of your portfolio and bonds that can range from $25 \%$ to $50 \%$ of your portfolio. Given a portfolio object $p$, use setBounds to set the bound constraints:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = Portfolio;
p = p.setBounds(lb, ub);
disp(p.NumAssets);
disp(p.LowerBound);
disp(p.UpperBound);
```

2
0.5000
0.2500
0.7500
0.5000

## Setting Bounds Using the Constructor or setBounds Method

Both the constructor Portfolio. and setBounds method implement scalar expansion on either the LowerBound or UpperBound properties. If the NumAssets property is already set in the portfolio object, scalar arguments for either property are expanded to have the same value across all dimensions. In addition, setBounds lets you specify NumAssets as an optional argument. Suppose you have a universe of 500 assets and you want to set common bound constraints on all assets in your universe. Specifically, you are a long-only investor and want to hold no more than $5 \%$ of your portfolio in any single asset. You can set these bound constraints in any of these equivalent ways:

```
p = Portfolio('NumAssets', 500, 'LowerBound', 0, 'UpperBound', 0.05);
```

or

```
p = Portfolio('NumAssets', 500);
p = p.setBounds(0, 0.05);
```

or

```
p = Portfolio;
p = p.setBounds(0, 0.05, 500);
```

To clear bound constraints from your portfolio object, use either the constructor Portfolio. or setBounds with empty inputs for the properties to be cleared. For example, to clear the upper bound constraint from the portfolio object $p$ in the previous example:

```
p = Portfolio(p, 'UpperBound', []);
```


## Working with Budget Constraints

The budget constraint is an optional linear constraint that maintains upper and lower bounds on the sum of portfolio weights (see "Budget Constraints" on page 4-8). Budget constraints have properties LowerBudget for the lower
budget constraint and UpperBudget for the upper budget constraint. If you set up a portfolio optimization problem that requires portfolios to be fully invested in your universe of assets, you can set LowerBudget to be equal to UpperBudget. These budget constraints can be set with default values equal to 1 using setDefaultConstraints (see "Setting Default Constraints for Portfolio Weights" on page 4-52).

## Setting Budget Constraints Using the Constructor

The properties for the budget constraint can also be set using the constructor Portfolio. Suppose you have an asset universe with many risky assets and a riskless asset and you want to ensure that your portfolio never holds more than $1 \%$ cash, that is, you want to ensure that you are $99 \%$ to $100 \%$ invested in risky assets. The budget constraint for this portfolio can be set with:

```
p = Portfolio('LowerBudget', 0.99, 'UpperBudget', 1);
disp(p.LowerBudget);
disp(p.UpperBudget);
```

0.9900

1

## Setting Budget Constraints Using setBudget Method

You can also set the properties for a budget constraint using the setBudget method. Suppose you have a fund that permits up to $10 \%$ leverage which means that your portfolio can be between $100 \%$ and $110 \%$ invested in risky assets. Given a portfolio object p , use setBudget to set the budget constraints:

```
p = Portfolio;
p = p.setBudget(1, 1.1);
disp(p.LowerBudget);
disp(p.UpperBudget);
```

1
1.1000

If you were to continue with this example, then set the RiskFreeRate property to the borrowing rate to finance possible leveraged positions. For details on
the RiskFreeRate property, see "Working with a Riskless Asset" on page 4-48. To clear either bound for the budget constraint from your portfolio object, use either the constructor Portfolio. or setBudget with empty inputs for the properties to be cleared. For example, clear the upper budget constraint from the portfolio object $p$ in the previous example with:

```
p = Portfolio(p, 'UpperBudget', []);
```


## Working with Group Constraints

Group constraints are optional linear constraints that group assets together and enforce bounds on the group weights (see "Group Constraints" on page 4-9). Although the constraints are implemented as general constraints, the usual convention is to form a group matrix that identifies membership of each asset within a specific group with Boolean indicators (either true or false or with 1 or 0) for each element in the group matrix. Group constraints have properties GroupMatrix for the group membership matrix, LowerGroup for the lower-bound constraint on groups, and UpperGroup for the upper-bound constraint on groups.

## Setting Group Constraints Using the Constructor

The properties for group constraints are set through the constructor Portfolio.. Suppose you have a portfolio of five assets and want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio, then you can set group constraints:

```
G = [ 1 1 1 1 0 0 ];
p = Portfolio('GroupMatrix', G, 'UpperGroup', 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
0.3000

The group matrix $G$ can also be a logical matrix so that the following code achieves the same result:

```
G = [ true true true false false ];
p = Portfolio('GroupMatrix', G, 'UpperGroup', 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
0.3000

## Setting Group Constraints Using setGroups and addGroups Methods

You can also set the properties for group constraints using the setGroups method. Suppose you have a portfolio of five assets and want to ensure that the first three assets constitute no more than $30 \%$ of your portfolio. Given a portfolio object $p$, use setGroups to set the group constraints:

```
G = [ true true true false false ];
p = Portfolio;
p = p.setGroups(G, [], 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);
```

5

| 1 | 1 | 1 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |

0.3000

In this example, you would set the LowerGroup property to be empty ([ ]).
Suppose you want to add another group constraint to make odd-numbered assets constitute at least $20 \%$ of your portfolio. Set up an augmented group matrix and introduce infinite bounds for unconstrained group bounds or use the addGroups method to build up group constraints. For this example, create another group matrix for the second group constraint:

```
p = Portfolio;
G = [ true true true false false ]; % group matrix for first group constraint
p = p.setGroups(G, [], 0.3);
G = [ true false true false true ]; % group matrix for second group constraint
p = p.addGroups(G, 0.2);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.LowerGroup);
disp(p.UpperGroup);
5
1
1 0
    -Inf
0.2000
0.3000
    Inf
```

addGroups determines which bounds are unbounded so you only need to focus on the constraints that you want to set.

Both the constructor Portfolio. and setGroups and addGroups implement scalar expansion on either the LowerGroup or UpperGroup properties based on the dimension of the group matrix in the property GroupMatrix. Suppose you have a universe of 30 assets with 6 asset classes such that assets $1-5$, assets $6-12$, assets $13-18$, assets $19-22$, assets $23-27$, and assets $28-30$ constitute each of your asset classes and you want each asset class to fall between $0 \%$ and $25 \%$ of your portfolio. Let the following group matrix define your groups and scalar expansion define the common bounds on each group:

```
p = Portfolio;
G = blkdiag(true(1,5), true(1,7), true(1,6), true(1,4), true(1,5), true(1,3));
p = p.setGroups(G, 0, 0.25);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.LowerGroup);
disp(p.UpperGroup);
```

Columns 1 through 16

| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Columns 17 through 30

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

0.2500
0.2500
0.2500
0.2500
0.2500
0.2500

## Working with Group Ratio Constraints

Group ratio constraints are optional linear constraints that maintain bounds on proportional relationships among groups of assets (see "Group Ratio Constraints" on page 4-10). Although the constraints are implemented as general constraints, the usual convention is to specify a pair of group matrices
that identify membership of each asset within specific groups with Boolean indicators (either true or false or with 1 or 0 ) for each element in each of the group matrices. The goal is to ensure that the ratio of a base group to a comparison group fall within specified bounds. Group ratio constraints have properties:

- GroupA for the base membership matrix.
- GroupB for the comparison membership matrix.
- LowerRatio for the lower-bound constraint on the ratio of groups.
- UpperRatio, for the upper-bound constraint on the ratio of groups.


## Setting Group Ratio Constraints Using the Constructor

The properties for group ratio constraints are set using constructor Portfolio.. For example, assume you want the ratio of financial to nonfinancial companies in your portfolios to never go above $50 \%$. Suppose you have 12 assets with 6 financial companies (assets 1-6) and 6 nonfinanical companies (assets 7-12). To set group ratio constraints:

```
GA = [ 1 1 1 1 1 0 0 0 ]; % financial companies
GB = [ llllllll
p = Portfolio('GroupA', GA, 'GroupB', GB, 'UpperRatio', 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
1
0
0.5000
```

Group matrices GA and GB in this example can be logical matrices with true and false elements that yield the same result:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % non-financial companies
p = Portfolio('GroupA', GA, 'GroupB', GB, 'UpperRatio', 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
```

6

| 1 | 1 | 1 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 | 1 | 1 |

0.5000

## Setting Group Ratio Constraints Using the setGroupRatio and addGroupRatio Methods

You can also set the properties for group ratio constraints using the setGroupRatio method. For example, assume that you want the ratio of financial to nonfinancial companies in your portfolios to never go above $50 \%$. Suppose you have 12 assets with 6 financial companies (assets 1-6) and 6 nonfinanical companies (assets $7-12$ ). Given a portfolio object p, use setGroupRatio to set the group constraints:

```
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % non-financial companies
p = Portfolio;
p = p.setGroupRatio(GA, GB, [], 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
1
0
```


### 0.5000

In this example, you would set the LowerRatio property to be empty ([ ]).
Suppose you want to add another group ratio constraint to ensure that the weights in odd-numbered assets constitute at least $20 \%$ of the weights in nonfinancial assets your portfolio. You can set up augmented group ratio matrices and introduce infinite bounds for unconstrained group ratio bounds, or you can use the addGroupRatio method to build up group ratio constraints. For this example, create another group matrix for the second group constraint:

```
p = Portfolio;
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % non-financial companies
p = p.setGroupRatio(GA, GB, [], 0.5);
GA = [ true false true false true false ]; % odd-numbered companies
GB = [ false false false true true true ]; % non-financial companies
p = p.addGroupRatio(GA, GB, 0.2);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.LowerRatio);
disp(p.UpperRatio);
```

6

| 1 | 1 | 1 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 |

- Inf
0.2000
0.5000

Notice that addGroupRatio determines which bounds are unbounded so you only need to focus on the constraints you want to set.

Both the constructor Portfolio., setGroupRatio, and addGroupRatio implement scalar expansion on either the LowerRatio or UpperRatio properties based on the dimension of the group matrices in GroupA and GroupB properties.

## Working with Linear Equality Constraints

Linear equality constraints are optional linear constraints that impose systems of equalities on portfolio weights (see "Linear Equality Constraints" on page 4-7). Linear equality constraints have properties AEquality, for the equality constraint matrix, and bEquality, for the equality constraint vector.

## Setting Linear Equality Constraints Using the Constructor

The properties for linear equality constraints are set using the constructor Portfolio.. Suppose you have a portfolio of five assets and want to ensure that the first three assets are exactly $50 \%$ of your portfolio. To set this constraint:

```
A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio('AEquality', A, 'bEquality', b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
0.5000

## Setting Linear Equality Constraints Using the setEquality and addEquality Methods

You can also set the properties for linear equality constraints using the setEquality method. Suppose you have a portfolio of five assets and want to ensure that the first three assets are exactly $50 \%$ of your portfolio. Given a portfolio object $p$, use setEquality to set the linear equality constraints:

```
A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = p.setEquality(A, b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
```

```
5
1 1 1 1 1 0
0.5000
```

Suppose you want to add another linear equality constraint to ensure that the last three assets also constitute $50 \%$ of your portfolio. You can set up an augmented system of linear equalities or use the addEquality method to build up linear equality constraints. For this example, create another system of equalities:

```
p = Portfolio;
A = [ 1 1 1 0 0 ]; % first equality constraint
b = 0.5;
p = p.setEquality(A, b);
A = [ 0 0 1 1 1 ]; % second equality constraint
b = 0.5;
p = p.addEquality(A, b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
```

5

| 1 | 1 | 1 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 1 | 1 | 1 |

0.5000
0.5000

Both the constructor Portfolio. and setEquality and addEquality implement scalar expansion on the bEquality property based on the dimension of the matrix in the AEquality property.

## Working with Linear Inequality Constraints

Linear inequality constraints are optional linear constraints that impose systems of inequalities on portfolio weights (see "Linear Inequality Constraints" on page 4-6). Linear inequality constraints have properties AInequality for the inequality constraint matrix, and bInequality for the inequality constraint vector.

## Setting Linear Inequality Constraints Using the Constructor

The properties for linear inequality constraints are set using the constructor Portfolio.. Suppose you have a portfolio of five assets and you want to ensure that the first three assets are no more than $50 \%$ of your portfolio. To set up these constraints:

```
A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio('AInequality', A, 'bInequality', b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
```

5
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
0.5000

## Setting Linear Inequality Constraints Using setInequality and addInequality Methods

You can also set the properties for linear inequality constraints using the setInequality method. Suppose you have a portfolio of five assets and you want to ensure that the first three assets constitute no more than $50 \%$ of your portfolio. Given a portfolio object $p$, use setInequality to set the linear inequality constraints:

```
A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = p.setInequality(A, b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
5
1
0.5000
```

Suppose you want to add another linear inequality constraint to ensure that the last three assets constitute at least $50 \%$ of your portfolio. You can set up an augmented system of linear inequalities or use the addInequality method to build up linear inequality constraints. For this example, create another system of inequalities:

```
p = Portfolio;
A = [ 1 1 1 1 0 0 ]; % first inequality constraint
b = 0.5;
p = p.setInequality(A, b);
A = [ 0 0 -1 -1 -1 ]; % second inequality constraint
b = -0.5;
p = p.addInequality(A, b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
```

| 1 | 1 | 1 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | -1 | -1 | -1 |

0.5000
-0.5000

Both the constructor Portfolio. and setInequality and addInequality implement scalar expansion on the bInequality property based on the dimension of the matrix in the AInequality property.

## Working with Turnover Constraints

The turnover constraint is an optional linear absolute value constraint (see "Turnover Constraints" on page 4-10) that enforces an upper bound on the average of purchases and sales. The turnover constraint can be set through either the portfolio constructor Portfolio. or the setTurnover method. The turnover constraint depends upon an initial or current portfolio, which is assumed to be zero if not set when the turnover constraint is set. The turnover constraint has properties Turnover, for the upper bound on average turnover, and InitPort, for the portfolio against which turnover is computed.

## Setting Turnover Constraints Using the Constructor

The properties for the turnover constraint are set using the constructor Portfolio.. Suppose you have an initial portfolio 10 assets in a variable $\mathrm{x0}$ and you want to ensure that average turnover is no more than $30 \%$. To set this turnover constraint:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('Turnover', 0.3, 'InitPort', x0);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
```

0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000

Note if the NumAssets or InitPort properties are not set before or when the turnover constraint is set, various rules are applied to assign default values to these properties (see "Setting Up an Initial or Current Portfolio" on page 4-32 for details).

## Setting Turnover Constraints Using setTurnover Method

You can also set properties for portfolio turnover using the setTurnover method to specify both the upper bound for average turnover and an initial portfolio. Suppose you have an initial portfolio of 10 assets in a variable x0 and want to ensure that average turnover is no more than $30 \%$. Given a portfolio object $p$, use setTurnover to set the turnover constraint with and without the initial portfolio being set previously:

```
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('InitPort', x0);
p = p.setTurnover(0.3);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
0.1200
0.0900
0.0800
0.0700
```

```
    0.1000
    0.1000
    0.1500
    0.1100
    0.0800
    0.1000
or
x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio;
p = p.setTurnover(0.3, x0);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000
```

setTurnover implements scalar expansion on the argument for the initial portfolio. If the NumAssets property is already set in the portfolio object, a scalar argument for InitPort is expanded to have the same value across all dimensions. In addition, setTurnover lets you specify NumAssets as an optional argument. To clear turnover from your portfolio object, use the constructor Portfolio. or setTurnover with empty inputs for the properties to be cleared.

## Validating the Portfolio Problem

In this section...<br>"Validating a Portfolio Set" on page 4-73<br>"Validating Portfolios" on page 4-75

In some cases, you may want to validate either your inputs to, or outputs from, a portfolio optimization problem. Although most of the error-checking that occurs during the problem setup phase catches most difficulties with a portfolio optimization problem, the processes to validate portfolio sets and portfolios are time consuming and are best done offline. Consequently, the portfolio optimization tools have specialized methods to validate portfolio sets and portfolios.

## Validating a Portfolio Set

Since it is necessary and sufficient that your portfolio set must be a nonempty, closed, and bounded set to have a valid portfolio optimization problem, the method estimateBounds lets you examine your portfolio set to determine if it is nonempty and, if nonempty, whether it is bounded. Suppose you have the following portfolio set which is an empty set because the initial portfolio at 0 is too far from a portfolio that satisfies the budget and turnover constraint:

```
p = Portfolio('NumAssets', 3, 'Budget', 1);
p = p.setTurnover(0.3, 0);
```

If a portfolio set is empty, estimateBounds returns NaN bounds and sets the isbounded flag to []:

```
[lb, ub, isbounded] = p.estimateBounds
lb =
    NaN
    NaN
    NaN
ub =
```

```
    NaN
        NaN
        NaN
isbounded =
```

    []
    Suppose you create an unbounded portfolio set as follows:

```
p = Portfolio('AInequality', [1 -1; 1 1 ], 'bInequality', 0);
[lb, ub, isbounded] = p.estimateBounds
lb =
    -Inf
    - Inf
ub =
    1.0e-008 *
        -0.3712
        Inf
    isbounded =
```

        0
    In this case, estimateBounds returns (possibly infinite) bounds and sets the isbounded flag to false. The result shows which assets are unbounded so that you can apply bound constraints as necessary.

Finally, suppose you created a portfolio set that is both nonempty and bounded. estimateBounds not only validates the set, but also obtains tighter bounds which is useful if you are concerned with the actual range of portfolio choices for individual assets in your portfolio set:

```
p = Portfolio;
p = p.setBudget(1,1);
p = p.setBounds([ -0.1; 0.2; 0.3; 0.2 ], [ 0.5; 0.3; 0.9; 0.8 ]);
```

```
[lb, ub, isbounded] = p.estimateBounds
lb =
    -0.1000
    0.2000
    0.3000
    0.2000
ub =
    0.3000
    0.3000
    0.7000
    0.6000
isbounded =
1
```

In this example, all but the second asset have tighter upper bounds than the input upper bound implies.

## Validating Portfolios

Given a portfolio set specified in a portfolio object, often you want to check if specific portfolios are feasible with respect to the portfolio set. This can occur with, for example, initial portfolios and with portfolios obtained from other procedures. The checkFeasibility method determines whether a collection of portfolios is feasible. Suppose you perform the following portfolio optimization and want to determine if the resultant efficient portfolios are feasible relative to a modified problem.

First, set up a problem in the portfolio objectp, estimate efficient portfolios in pwgt, and then confirm that these portfolios are feasible relative to the initial problem:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
```

```
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontier;
p.checkFeasibility(pwgt)
ans =
```

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Next, set up a different portfolio problem that starts with the initial problem with an additional a turnover constraint and an equally weighted initial portfolio:

```
q = p.setTurnover(0.3, 0.25);
q.checkFeasibility(pwgt)
ans =
```

| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

In this case, only two of the ten efficient portfolios from the initial problem are feasible relative to the new problem in portfolio object q. Solving the second problem using checkFeasibility demonstrates that the efficient portfolio for portfolio object $q$ is feasible relative to the initial problem:

```
qwgt = q.estimateFrontier;
p.checkFeasibility(qwgt)
ans =
    1
```


## Estimate Efficient Portfolios

In this section...<br>"Obtaining Portfolios Along the Entire Efficient Frontier" on page 4-77<br>"Obtaining Endpoints of the Efficient Frontier" on page 4-79<br>"Obtaining Efficient Portfolios for Target Returns" on page 4-81<br>"Obtaining Efficient Portfolios for Target Risks" on page 4-83<br>"Choosing and Controlling the Solver" on page 4-86

There are two ways to look at a portfolio optimization problem that depends upon what you are trying to do. One goal is to estimate efficient portfolios and the other is to estimate efficient frontiers. This section focuses on the former goal and the subsequent section ("Estimate Efficient Frontiers" on page 4-88) focuses on the latter goal.

## Obtaining Portfolios Along the Entire Efficient Frontier

The most basic way to obtain optimal portfolios is to obtain points over the entire range of the efficient frontier. Given a portfolio optimization problem in a portfolio object, the estimateFrontier method computes efficient portfolios spaced evenly according to the return proxy from the minimum to maximum return efficient portfolios. The number of portfolios estimated is controlled by the hidden property defaultNumPorts which is set to 10 . A different value for the number of portfolios estimated is specified as input to estimateFrontier. This example shows the default number of efficient portfolios over the entire range of the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m,C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontier;
```

| disp(pwgt); |  |  |  |  | 0 | 0 | 0 |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.8891 | 0.7215 | 0.5540 | 0.3865 | 0.2190 | 0.0515 | 0 | 0 | 0 |  |
| 0.0369 | 0.1289 | 0.2209 | 0.3129 | 0.4049 | 0.4969 | 0.4049 | 0.2314 | 0.0579 | 0 |
| 0.0404 | 0.0567 | 0.0730 | 0.0893 | 0.1056 | 0.1219 | 0.1320 | 0.1394 | 0.1468 | 0 |
| 0.0336 | 0.0929 | 0.1521 | 0.2113 | 0.2705 | 0.3297 | 0.4630 | 0.6292 | 0.7953 | 1.0000 |

If you want only four portfolios in the previous example:

```
pwgt = p.estimateFrontier(4);
```

| disp(pwgt); |  |  |  |
| ---: | ---: | ---: | ---: |
| 0.8891 | 0.3865 | 0 | 0 |
| 0.0369 | 0.3129 | 0.4049 | 0 |
| 0.0404 | 0.0893 | 0.1320 | 0 |
| 0.0336 | 0.2113 | 0.4630 | 1.0000 |

Starting from the initial portfolio, estimateFrontier also returns purchases and sales to get from your initial portfolio to each efficient portfolio on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales:

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = p.setInitPort(pwgt0);
[pwgt, pbuy, psell] = p.estimateFrontier;
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{rrrrrrrrrr}
0 \\
0.8891 & 0.7215 & 0.5540 & 0.3865 & 0.2190 & 0.0515 & 0 & 0 & 0 & 0 \\
0.0369 & 0.1289 & 0.2209 & 0.3129 & 0.4049 & 0.4969 & 0.4049 & 0.2314 & 0.0579 & 0 \\
0.0404 & 0.0567 & 0.0730 & 0.0893 & 0.1056 & 0.1219 & 0.1320 & 0.1394 & 0.1468 & 0 \\
0.0336 & 0.0929 & 0.1521 & 0.2113 & 0.2705 & 0.3297 & 0.4630 & 0.6292 & 0.7953 & 1.0000
\end{tabular}
pbuy =
0.5891 0.4215 0.2540 0.0865 
```

| 0 | 0 | 0 | 0.0129 |  | 0.1049 |  | 0.1969 |  | 0.1049 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 |  | 0 |  | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 0 | 0 | 0.0521 | 0.1113 |  | 0.1705 |  | 0.2297 |  | 0.3630 | 0.5292 | 0.6953 | 0.9000 |
| psell = |  |  |  |  |  |  |  |  |  |  |  |  |
| 0 | 0 | 0 | $0 \quad 0.0810$ |  | 0.2485 |  | 50.3000 |  | 0.3000 | $0.3000 \quad 0.3000$ |  |  |
| 0.2631 | 0.1711 | 0.0791 | 0 | 0 |  | 0 | 0 |  | 0 | 0.0686 | 0.2421 | 0.3000 |
| 0.1596 | 0.1433 | 0.1270 | 0.1107 |  | 0.0944 |  | 0.0781 |  | 0.0680 | 0.0606 | 0.0532 | 0.2000 |
| 0.0664 | 0.0071 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## Obtaining Endpoints of the Efficient Frontier

In many cases, you might be interested in the endpoint portfolios for the efficient frontier. Suppose you want to determine the range of returns from minimum to maximum to refine a search for a portfolio with a specific target return. Use the estimateFrontierLimits method to obtain the endpoint portfolios:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontierLimits;
disp(pwgt);
disp(pwgt);
    0.8891 0
    0.0369 0
    0.0404 0
```

$$
0.0336 \quad 1.0000
$$

The estimatePortMoments method shows the range of risks and returns for efficient portfolios:

```
[prsk, pret] = p.estimatePortMoments(pwgt);
disp([prsk, pret]);
    0.0769 0.0590
    0.3500 0.1800
```

Starting from an initial portfolio, estimateFrontierLimits also returns purchases and sales to get from the initial portfolio to the endpoint portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = p.setInitPort(pwgt0);
[pwgt, pbuy, psell] = p.estimateFrontierLimits;
display(pwgt);
display(pbuy);
display(psell);
pwgt =
    0.8891 0
    0.0369 0
    0.0404 0
    0.0336 1.0000
pbuy =
```

```
        0.5891 0
        0 0
        0 0
        0 0.9000
psell =
\(0 \quad 0.3000\)
    0.2631 0.3000
    0.1596 0.2000
    0.0664 0
```

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## Obtaining Efficient Portfolios for Target Returns

To obtain efficient portfolios that have targeted portfolio returns, the estimateFrontierByReturn method accepts one or more target portfolio returns and obtains efficient portfolios with the specified returns. For example, assume that you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of $6 \%, 9 \%$, and $12 \%$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontierByReturn([0.06, 0.09, 0.12]);
display(pwgt);
pwgt =
    0.8772 0.5032 0.1293
    0.0434 0.2488 0.4541
```

$$
\begin{array}{lll}
0.0416 & 0.0780 & 0.1143 \\
0.0378 & 0.1700 & 0.3022
\end{array}
$$

In some cases, you can request a return for which no efficient portfolio exists. Based on the previous example, suppose you want a portfolio with a $5 \%$ return (which is the return of the first asset). A portfolio that is fully invested in the first asset, however, is inefficient. estimateFrontierByReturn warns if your target returns are outside the range of efficient portfolio returns and replaces it with the endpoint portfolio of the efficient frontier closest to your target return:

```
Warning: One or more target return values are outside the feasible range [ 0.0590468, 0.18 ].
Will return portfolios associated with endpoints of the range for these values.
> In Portfolio.estimateFrontierByReturn at 74
pwgt =
    0.8891
    0.0369
    0.0404
    0.0336
```

The best way to avoid this situation is to bracket your target portfolio returns with estimateFrontierLimits and estimatePortReturns (see "Obtaining Endpoints of the Efficient Frontier" on page 4-79 and "Obtaining Portfolio Risks and Returns" on page 4-88).

```
pret = p.estimatePortReturn(p.estimateFrontierLimits);
display(pret);
pret =
    0.0590
    0.1800
```

This result indicates that efficient portfolios have returns that range between $5.9 \%$ and $18 \%$.

If you have an initial portfolio, estimateFrontierByReturn also returns purchases and sales to get from your initial portfolio to the target portfolios
on the efficient frontier. For example, given an initial portfolio in pwgt0, to obtain purchases and sales with target returns of $6 \%, 9 \%$, and $12 \%$ :

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = p.setInitPort(pwgt0);
[pwgt, pbuy, psell] = p.estimateFrontierByReturn([0.06, 0.09, 0.12]);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{lll}
0.8772 & 0.5032 & 0.1293 \\
0.0434 & 0.2488 & 0.4541 \\
0.0416 & 0.0780 & 0.1143 \\
0.0378 & 0.1700 & 0.3022
\end{tabular}
pbuy =
\begin{tabular}{rrr}
0.5772 & 0.2032 & 0 \\
0 & 0 & 0.1541 \\
0 & 0 & 0 \\
0 & 0.0700 & 0.2022
\end{tabular}
psell =
\begin{tabular}{rrr}
0 & 0 & 0.1707 \\
0.2566 & 0.0512 & 0 \\
0.1584 & 0.1220 & 0.0857 \\
0.0622 & 0 & 0
\end{tabular}
```

If you do not have an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## Obtaining Efficient Portfolios for Target Risks

To obtain efficient portfolios that have targeted portfolio risks, the estimateFrontierByRisk method accepts one or more target portfolio risks
and obtains efficient portfolios with the specified risks. Suppose you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of $12 \%, 14 \%$, and $16 \%$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontierByRisk([0.12, 0.14, 0.16]);
display(pwgt);
pwgt =
    0.3984 0.2659 0.1416
    0.3064 0.3791 0.4474
    0.0882 0.1010 0.1131
    0.2071 0.2540 0.2979
```

In some cases, you can request a risk for which no efficient portfolio exists. Based on the previous example, suppose you want a portfolio with $7 \%$ risk (individual assets in this universe have risks ranging from $8 \%$ to $35 \%$ ). It turns out that a portfolio with $7 \%$ risk cannot be formed with these four assets. estimateFrontierByRisk warns if your target risks are outside the range of efficient portfolio risks and replaces it with the endpoint of the efficient frontier closest to your target risk:

```
pwgt = p.estimateFrontierByRisk(0.07)
Warning: One or more target risk values are outside the feasible range [ 0.0769288, 0.35 ].
Will return portfolios associated with endpoints of the range for these values.
> In Portfolio.estimateFrontierByRisk at 87
pwgt =
    0.8891
    0.0369
```

> 0.0404
> 0.0336

The best way to avoid this situation is to bracket your target portfolio risks with estimateFrontierLimits and estimatePortRisk (see "Obtaining Endpoints of the Efficient Frontier" on page 4-79 and "Obtaining Portfolio Risks and Returns" on page 4-88).

```
prsk = p.estimatePortRisk(p.estimateFrontierLimits);
display(prsk);
prsk =
    0.0769
    0.3500
```

This result indicates that efficient portfolios have risks that range between $7.7 \%$ and $35 \%$.

Starting with an initial portfolio, estimateFrontierByRisk also returns purchases and sales to get from your initial portfolio to the target portfolios on the efficient frontier. For example, given an initial portfolio in pwgt0, you can obtain purchases and sales from the example with target risks of $12 \%, 14 \%$, and $16 \%$ :

```
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = p.setInitPort(pwgt0);
[pwgt, pbuy, psell] = p.estimateFrontierByRisk([0.12, 0.14, 0.16]);
display(pwgt);
display(pbuy);
display(psell);
pwgt =
\begin{tabular}{lll}
0.3984 & 0.2659 & 0.1416 \\
0.3064 & 0.3791 & 0.4474 \\
0.0882 & 0.1010 & 0.1131 \\
0.2071 & 0.2540 & 0.2979
\end{tabular}
```

```
pbuy =
    0.0984 0 0
        0.0064 0.0791 0.1474
        0 0 0
        0.1071 0.1540 0.1979
psell =
        0 0.0341 0.1584
        0 0
        0.1118 0.0990 0.0869
        0 0 0
```

If you do not specify an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

## Choosing and Controlling the Solver

The default solver for mean-variance portfolio optimization is lcprog, which implements a linear complementarity programming (LCP) algorithm. Although lcprog works for most problems, you can adjust parameters to control the algorithm. Alternatively, the mean-variance portfolio optimization tools let you use any of the variations of quadprog from Optimization Toolbox ${ }^{\text {TM }}$ software. Unlike Optimization Toolbox which uses the trust-region-reflective algorithm as the default algorithm for quadprog, the portfolio optimization tools use the interior-point-convex algorithm. For details about quadprog and quadratic programming algorithms and options, see "Quadratic Programming Algorithms".

To modify either lcprog or to specify quadprog as your solver, use the setSolver method to set the hidden properties solverType and solverOptions that specify and control the solver. Since the solver properties are hidden, you cannot set these using the portfolio constructor Portfolio.. The default solver is lcprog so you do not need to use setSolver to specify this solver. To use quadprog, you must set up the interior-point-convex version of quadprog using:
p = p.setSolver('quadprog');

```
display(p.solverType);
quadprog
```

and you can switch back tolcprog with:

```
p = p.setSolver('lcprog');
display(p.solverType);
lcprog
```

In both cases, setSolver sets up default options associated with either solver. If you want to specify additional options associated with a given solver, setSolver accepts these options with parameter name and value pairs in the function call. For example, if you intend to use quadprog and want to use the active-set algorithm, call setSolver with:

```
p = p.setSolver('quadprog', 'Algorithm', 'active-set');
display(p.solverOptions.Algorithm);
active-set
```

In addition, if you want to specify any of the options for quadprog that are normally set through optimset, setSolver accepts an optimset structure as the second argument. For example, you can start with the default options for quadprog set by setSolver and then change the algorithm to trust-region-reflective with no displayed output:

```
p = Portfolio;
options = optimset('quadprog');
options = optimset(options, 'Algorithm', 'trust-region-reflective', 'Display', 'off');
p = p.setSolver('quadprog', options);
display(p.solverOptions.Algorithm);
display(p.solverOptions.Display);
trust-region-reflective
off
```


## Estimate Efficient Frontiers

In this section...<br>"Obtaining Portfolio Risks and Returns" on page 4-88<br>"Plotting the Efficient Frontier" on page 4-90

Whereas the previous section ("Estimate Efficient Portfolios" on page 4-77) focused on estimation of efficient portfolios, this section focuses on the estimation of efficient frontiers.

## Obtaining Portfolio Risks and Returns

Given any portfolio and, in particular, efficient portfolios, the methods estimatePortReturns, estimatePortRisk, and estimatePortMoments provide estimates for the return (or return proxy), risk (or the risk proxy), and, in the case of mean-variance portfolio optimization, the moments of expected portfolio returns. Each method has the same input syntax but with different combinations of outputs. Suppose you have this following portfolio optimization problem that gave you a collection of portfolios along the efficient frontier in pwgt:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('AssetMean', m, 'AssetCovar', C, 'InitPort', pwgt0);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontier;
```

Given pwgt0 and pwgt, use the portfolio risk and return estimation methods to obtain risks and returns for your initial portfolio and the portfolios on the efficient frontier:

```
[prsk0, pret0] = p.estimatePortMoments(pwgt0);
[prsk, pret] = p.estimatePortMoments(pwgt);
```

or

```
prsk0 = p.estimatePortRisk(pwgt0);
pret0 = p.estimatePortReturn(pwgt0);
prsk = p.estimatePortRisk(pwgt);
pret = p.estimatePortReturn(pwgt);
```

In either case, you obtain these risks and returns:

```
display(prsk0);
display(pret0);
display(prsk);
display(pret);
prsk0 =
    0.1103
pret0 =
    0.0870
prsk =
    0.0769
    0.0831
    0.0994
    0.1217
    0.1474
    0.1750
    0.2068
    0.2487
    0.2968
    0.3500
pret =
    0.0590
    0.0725
    0.0859
    0.0994
    0.1128
```

0.1262
0.1397
0.1531
0.1666
0.1800

Note the returns and risks are at the periodicity of the moments of asset returns so that, if you have values for AssetMean and AssetCovar in terms of monthly returns, the estimates for portfolio risk and return are in terms of monthly returns as well. In addition, the estimate for portfolio risk in the mean-variance case is the standard deviation of portfolio returns, not the variance of portfolio returns.

## Plotting the Efficient Frontier

The plotFrontier method creates a plot of the efficient frontier for a given portfolio optimization problem. This method accepts several types of inputs and generates a plot with an optional possibility to output the estimates for portfolio risks and returns along the efficient frontier. plotFrontier has four different ways that it can be used. In addition to a plot of the efficient frontier, if you have an initial portfolio in the InitPort property, plotFrontier also displays the return versus risk of the initial portfolio on the same plot. If you have a well-posed portfolio optimization problem set up in a portfolio object and you use plotFrontier, you will get a plot of the efficient frontier with the default number of portfolios on the frontier (the default number is currently 10 and is maintained in the hidden property defaultNumPorts). This example illustrates a typical use of plotFrontier to create a new plot:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('Name', 'Asset Allocation Portfolio', 'InitPort', pwgt0);
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
p.plotFrontier;
```



The Name property is displayed as the title of the efficient frontier plot if you set it in the portfolio object. Without an explicit name, the title on the plot would be "Efficient Frontier." If you want to obtain a specific number of portfolios along the efficient frontier, use plotFrontier with the number of portfolios that you want. Suppose you have the portfolio object from the previous example and you want to plot 20 portfolios along the efficient frontier and to obtain 20 risk and return values for each portfolio:

```
[prsk, pret] = p.plotFrontier(20);
display([pret, prsk]);
ans =
    0.0590 0.0769
    0.0654 0.0784
    0.0718 0.0825
    0.0781 0.0890
    0.0845 0.0973
```

| 0.0909 | 0.1071 |
| :--- | :--- |
| 0.0972 | 0.1179 |
| 0.1036 | 0.1296 |
| 0.1100 | 0.1418 |
| 0.1163 | 0.1545 |
| 0.1227 | 0.1676 |
| 0.1291 | 0.1810 |
| 0.1354 | 0.1955 |
| 0.1418 | 0.2128 |
| 0.1482 | 0.2323 |
| 0.1545 | 0.2535 |
| 0.1609 | 0.2760 |
| 0.1673 | 0.2995 |
| 0.1736 | 0.3239 |
| 0.1800 | 0.3500 |



## Plotting Existing Efficient Portfolios

If you already have efficient portfolios from any of the "estimateFrontier" methods (see "Estimate Efficient Portfolios" on page 4-77), pass them into plotFrontier directly to plot the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgtO = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('Name', 'Asset Allocation Portfolio', 'InitPort', pwgtO);
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontier(20);
p.plotFrontier(pwgt);
```



## Plotting Existing Efficient Portfolio Risks and Returns

If you already have efficient portfolio risks and returns, you can use the interface to plotFrontier to pass them into plotFrontier to obtain a plot of the efficient frontier:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('Name', 'Asset Allocation Portfolio', 'InitPort', pwgtO);
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
[prsk, pret] = p.estimatePortMoments(p.estimateFrontier(20));
p.plotFrontier(prsk, pret);
```



## Post-Processing

## In this section...

"Setting Up Tradable Portfolios" on page 4-95
"Troubleshooting" on page 4-97

After obtaining efficient portfolios or estimates for expected portfolio risks and returns, use your results to set up trades to move toward an efficient portfolio.

## Setting Up Tradable Portfolios

Suppose you set up a portfolio optimization problem and obtained portfolio on the efficient frontier. Use the dataset object from Statistics Toolbox ${ }^{\text {TM }}$ software to form a blotter that lists your portfolios with the names for each asset. For example, suppose you want to obtain five portfolios along the efficient frontier. You can set up a blotter with weights multiplied by 100 to view the allocations for each portfolio:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
    p = Portfolio('InitPort', pwgt0);
p = p.setAssetList('Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontier(5);
pnames = cell(1,5);
    for i = 1:5
        pnames{i} = sprintf('Port%d',i);
    end
    Blotter = dataset([{100*pwgt},pnames],'obsnames',p.AssetList);
    display(Blotter);
Blotter =
```

|  | Port1 | Port2 | Port3 | Port4 | Port5 |
| :--- | :--- | :--- | :--- | ---: | :---: |
| Bonds | 88.906 | 51.216 | 13.525 | 0 | 0 |
| Large-Cap Equities | 3.6875 | 24.387 | 45.086 | 27.479 | 0 |
| Small-Cap Equities | 4.0425 | 7.7088 | 11.375 | 13.759 | 0 |
| Emerging Equities | 3.364 | 16.689 | 30.014 | 58.762 | 100 |

This result indicates that you would invest primarily in bonds at the minimum-risk/minimum-return end of the efficient frontier (Port1), and that you would invest completely in emerging equity at the maximum-risk/maximum-return end of the efficient frontier (Port5). You can also select a particular efficient portfolio, for example, suppose you want a portfolio with $15 \%$ risk and you add purchase and sale weights outputs obtained from the "estimateFrontier" methods to set up a trade blotter:

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = Portfolio('InitPort', pwgt0);
p = p.setAssetList('Bonds','Large-Cap Equities','Small-Cap Equities','Emerging Equities');
p = p.setAssetMoments(m,C);
p = p.setDefaultConstraints;
[pwgt, pbuy, psell] = p.estimateFrontierByRisk(0.15);
Blotter = dataset([{100*[pwgt0, pwgt, pbuy, psell]}, ...
    {'Initial','Weight', 'Purchases','Sales'}],'obsnames',p.AssetList);
display(Blotter);
Blotter =
```

|  | Initial | Weight | Purchases | Sales |
| :--- | :--- | ---: | :---: | ---: |
| Bonds | 30 | 20.299 | 0 | 9.7007 |
| Large-Cap Equities | 30 | 41.366 | 11.366 | 0 |
| Small-Cap Equities | 20 | 10.716 | 0 | 9.2838 |
| Emerging Equities | 10 | 27.619 | 17.619 | 0 |

If you have prices for each asset (in this example, they can be ETFs), add them to your blotter and then use the tools of the dataset object to obtain shares and shares to be traded. For an example, see "Asset Allocation Example" on page 4-100.

## Troubleshooting

## Portfolio Object Destroyed When Modifying

If a portfolio object is destroyed when modifying, remember to pass an existing object into the constructor if you want to modify it, otherwise it creates a new object. See "Constructing the Portfolio Object" on page 4-22 for details.

## Optimization Fails with "Bad Pivot" Message

If the optimization fails with a "bad pivot" message from lcprog, try a larger value for tolpiv which is a tolerance for pivot selection in the lcprog algorithm (try 1.0e-7, for example) or try the interior-point-convex version of quadprog. For details, see "Choosing and Controlling the Solver" on page $4-86$, the help header for lcprog, and the quadprog documentation.

## Matrix Incompatibility and "Non-Conformable" Errors

If you get "non-conformable" or matrix incompatibility errors, the representation of data in the tools follows a specific set of basic rules described in "Conventions for Representation of Data" on page 4-20.

## Missing Data Estimation Fails

If asset return data has missing or NaN values, the method estimateAssetMoments with the 'missingdata' flag set to true may fail with either too many iterations or a singular covariance. To correct this problem, consider this:

- If you have asset return data with no missing or NaN values, you can compute a covariance matrix that may be singular without difficulties. If you have missing or NaN values in your data, the supported missing data feature requires that your covariance matrix must be positive-definite, i.e., nonsingular.
- estimateAssetMoments uses default settings for the missing data estimation procedure that may not be appropriate for all problems.

In either case, you may want to estimate the moments of asset returns separately with either the ECM estimation functions such as ecmnmle or with your own methods.

## mv_optim_transform Errors

If you obtain optimization errors such as:

```
??? Error using ==> mv_optim_transform at 212
Portfolio set appears to be empty. Check constraints, especially the turnover constraint.
Error in ==> Portfolio.estimateFrontier at 65
[A, b, f0, f, H, g, lb] = mv_optim_transform(obj);
```

or

```
??? Error using ==> mv_optim_transform at 221
Cannot obtain finite lower bounds for specified portfolio set.
Error in ==> Portfolio.estimateFrontier at 65
[A, b, fO, f, H, g, lb] = mv_optim_transform(obj);
Error in ==> Portfolio.plotFrontier at 117
pwgt = obj.estimateFrontier(NumPorts);
```

Since the portfolio optimization tools require a bounded portfolio set, these errors (and similar errors) can occur if your portfolio set is either empty and, if nonempty, unbounded. Specifically, the portfolio optimization algorithm requires that your portfolio set have at least a finite lower bound. The best way to deal with these problems is to use the validation methods in "Validating the Portfolio Problem" on page 4-73. Specifically, use estimateBounds to examine your portfolio set, and use checkFeasibility to ensure that your initial portfolio is either feasible and, if infeasible, that you have sufficient turnover to get from your initial portfolio to the portfolio set.

Note To correct this problem, try solving your problem with larger values for turnover and gradually reduce to the value that you want.

## Efficient Portfolios Do Not Make Sense

If you obtain efficient portfolios that do not seem to make sense, this can happen if you forget to set specific constraints or you set incorrect constraints. For example, if you allow portfolio weights to fall between 0 and 1 and do not set a budget constraint, you can get portfolios that are $100 \%$ invested in every asset. Although it may be hard to detect, the best thing to do is to review the constraints you have set with display of the object. If you get portfolios with $100 \%$ invested in each asset, you can review the display of your object and quickly see that no budget constraint is set. Also, you can use estimateBounds and checkFeasibility to determine if the bounds for your portfolio set make sense and to determine if the portfolios you obtained are feasible relative to an independent formulation of your portfolio set.

## Asset Allocation Example

| In this section... |
| :--- |
| "Defining the Portfolio Problem" on page 4-100 |
| "Simulating Asset Prices" on page 4-101 |
| "Setting Up the Portfolio Object" on page 4-103 |
| "Validating the Portfolio Problem" on page 4-105 |
| "Plotting the Efficient Frontier" on page 4-105 |
| "Evaluating Gross vs. Net Portfolio Returns" on page 4-106 |
| "Analyzing Descriptive Properties of the Portfolio Structures" on page 4-107 |
| "Obtaining a Portfolio at the Specified Return Level on the Efficient |
| Frontier" on page 4-108 |
| "Obtaining a Portfolio at the Specified Risk Levels on the Efficient Frontier" |
| on page 4-109 |
| "Displaying the Final Results" on page 4-112 |

The following example sets up a basic asset allocation problem to use mean-variance portfolio optimization to estimate efficient portfolios. Suppose you want to manage an asset allocation fund with four asset classes: bonds, large-cap equities, small-cap equities, and emerging equities. The fund is long-only with no borrowing or leverage, should have no more than $85 \%$ of the portfolio in equities, and no more than $35 \%$ of the portfolio in emerging equities.

The cost to trade the first three assets is 10 basis points annualized and the cost to trade emerging equities is four times higher. Finally, you want to ensure that average turnover is no more than $15 \%$. To solve this problem, you will set up a basic mean-variance portfolio optimization problem and then slowly introduce the various constraints on the problem to get to a solution.

## Defining the Portfolio Problem

To set up the portfolio optimization problem, start with basic definitions of known quantities associated with the structure of this problem. Each asset class is assumed to have a tradeable asset with a real-time price. Such assets
can be, for example, exchange-traded funds (ETFs). The initial portfolio with holdings in each asset that has a total of $\$ 7.5$ million along with an additional cash position of $\$ 60,000$. These basic quantities and the costs to trade are set up in the following variables with asset names in the cell array Asset, current prices in the vector Price, current portfolio holdings in the vector Holding, and transaction costs in the vector UnitCost.

```
Asset = { 'Bonds', 'Large-Cap Equities', 'Small-Cap Equities', 'Emerging Equities' };
Price = [ 52.4; 122.7; 35.2; 46.9 ];
Holding = [ 42938; 24449; 42612; 15991 ];
UnitCost = [ 0.001; 0.001; 0.001; 0.004 ];
```

To analyze this portfolio, you can set up a blotter in a dataset object to help track prices, holdings, weights, and so forth. In particular, you can compute the initial portfolio weights and maintain them in a new blotter field called InitPort.

```
Blotter = dataset({Price, 'Price'}, {Holding, 'InitHolding'},'obsnames',Asset);
Wealth = sum(Blotter.Price .* Blotter.InitHolding);
Blotter.InitPort = (1/Wealth)*(Blotter.Price .* Blotter.InitHolding);
Blotter.UnitCost = UnitCost;
disp(Blotter);
```

|  | Price | InitHolding | InitPort | UnitCost |
| :--- | ---: | :---: | :---: | ---: |
| Bonds | 52.4 | 42938 | 0.3 | 0.001 |
| Large-Cap Equities | 122.7 | 24449 | 0.4 | 0.001 |
| Small-Cap Equities | 35.2 | 42612 | 0.2 | 0.001 |
| Emerging Equities | 46.9 | 15991 | 0.1 | 0.004 |

## Simulating Asset Prices

Since this is a hypothetical example, to simulate asset prices from a given mean and covariance of annual asset total returns for the asset classes, portsim is used to create asset returns with the desired mean and covariance. Specifically, portsim is used to simulate 5 years of monthly total returns. The mean and covariance of annual asset total returns are maintained in the variables AssetMean and AssetCovar. The simulated asset total return prices (which are compounded total returns) are maintained in the variable Y. All initial asset total return prices are normalized to 1 in this example.

```
AssetCovar = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    O 0.0119 0.0336 0.1225 ];
X = portsim(AssetMean'/12, AssetCovar/12, 60); % monthly total returns for 5 years (60 months)
[Y, T] = ret2tick(X, [], 1/12); % form total return prices
```

This plot shows the log of the simulated total return prices:

```
plot(T, log(Y));
title('\bfSimulated Asset Class Total Return Prices');
xlabel('Year');
ylabel('Log Total Return Price');
legend(Asset,'Location','best');
```



If working with actual historical asset prices, income, and corporate actions data, you would compute total returns for your assets by other means.

## Setting Up the Portfolio Object

To explore portfolios on the efficient frontier, set up a portfolio object using these specifications:

- Portfolio weights are nonnegative and sum to 1.
- Equity allocation is no more than $85 \%$ of the portfolio.
- Emerging equity is no more than $35 \%$ of the portfolio.

These specifications are incorporated into the portfolio object $p$ in the following sequence of methods that starts with the portfolio constructor:

```
p = Portfolio('Name', 'Asset Allocation Portfolio', ...
'AssetList', Asset, 'InitPort', Blotter.InitPort);
```

The specification of the initial portfolio from Blotter gives the number of assets in your universe so you do not need to specify the NumAssets property directly. Next, set up default constraints (long-only with a budget constraint). In addition, set up the group constraint that imposes an upper bound on equities in the portfolio (equities are identified in the group matrix with 1s) and the upper bound constraint on emerging equities.

```
p = p.setDefaultConstraints;
p = p.setGroups([ 0, 1, 1, 1 ], [], 0.85);
p = p.addGroups([ 0, 0, 0, 1 ], [], 0.35);
```

Although you could have set the upper bound on emerging equities using the setBounds method, notice how you used the addGroups method to set up this constraint.

Finally, to have a fully specified mean-variance portfolio optimization problem, you must specify the mean and covariance of asset returns. Since starting with these moments in the variables AssetMean and AssetCovar, you could use the method setAssetMoments to enter these variables into your portfolio object in the following way (remember that you are assuming that your raw data are monthly returns which is why you divide your annual input moments by 12 to get monthly returns).

```
p = p.setAssetMoments(AssetMean/12, AssetCovar/12);
```

To make things more interesting, however, you can use the total return prices and use the method estimateAssetMoments with a specification that your data in $Y$ are prices, and not returns, to estimate asset return moments for your portfolio object.

```
p = p.estimateAssetMoments(Y, 'DataFormat', 'Prices');
```

Although the returns in your portfolio object are in units of monthly returns, and since subsequent costs are annualized, it is convenient to specify them as annualized total returns with this direct transformation of the AssetMean and AssetCovar properties of your object:

```
p.AssetMean = 12*p.AssetMean;
p.AssetCovar = 12*p.AssetCovar;
```

Now, the portfolio object is ready:

```
display(p);
p =
    Portfolio
    Properties:
            BuyCost: []
            SellCost: []
    RiskFreeRate: []
            AssetMean: [4x1 double]
            AssetCovar: [4\times4 double]
                Turnover: []
                    Name: 'Asset Allocation Portfolio'
            NumAssets: 4
            AssetList: {'Bonds' 'Large-Cap Equities' 'Small-Cap Equities' 'Emerging Equities'}
            InitPort: [4\times1 double]
        AInequality: []
            bInequality: []
            AEquality: []
            bEquality: []
            LowerBound: [4\times1 double]
            UpperBound: []
            LowerBudget: 1
```

```
UpperBudget: 1
GroupMatrix: [2x4 double]
LowerGroup: []
UpperGroup: [2x1 double]
    GroupA: []
    GroupB: []
LowerRatio: []
UpperRatio: []
```


## Validating the Portfolio Problem

An important step in portfolio optimization is to validate that the portfolio problem is feasible and the main test is to ensure that the set of portfolios is nonempty and bounded. Use the estimateBounds method to determine the bounds for the portfolio set:

```
[lb, ub] = p.estimateBounds;
display([lb, ub]);
ans =
\begin{tabular}{ll}
0.1500 & 1.0000 \\
0.0000 & 0.8500 \\
0.0000 & 0.8500 \\
0.0000 & 0.3500
\end{tabular}
```

Since both lb and ub are finite, the set is bounded.

## Plotting the Efficient Frontier

Given the constructed portfolio object, use the method plotFrontier to view the efficient frontier. Instead of using the default of 10 portfolios along the frontier, you can display the frontier with 40 portfolios:

```
p.plotFrontier(40);
```



Notice gross efficient portfolio returns fall between approximately $6 \%$ and $16 \%$ per years.

## Evaluating Gross vs. Net Portfolio Returns

The portfolio object p does not include transaction costs so that the portfolio optimization problem specified in $p$ uses gross portfolio return as the return proxy. To handle net returns, create a second portfolio object q that includes transaction costs:

```
q = p.setCosts(UnitCost, UnitCost);
display(q);
q =
```

    Portfolio
    Properties:
    BuyCost: [ \(4 \times 1\) double]
    ```
    SellCost: [4x1 double]
RiskFreeRate: []
    AssetMean: [4x1 double]
    AssetCovar: [4x4 double]
    Turnover: []
            Name: 'Asset Allocation Portfolio'
    NumAssets: 4
    AssetList: {'Bonds' 'Large-Cap Equities' 'Small-Cap Equities' 'Emerging Equities'}
        InitPort: [4\times1 double]
AInequality: []
bInequality: []
    AEquality: []
    bEquality: []
    LowerBound: [4x1 double]
    UpperBound: []
LowerBudget: 1
UpperBudget: 1
GroupMatrix: [2x4 double]
    LowerGroup: []
    UpperGroup: [2x1 double]
        GroupA: []
        GroupB: []
    LowerRatio: []
    UpperRatio: []
```


## Analyzing Descriptive Properties of the Portfolio Structures

To be more concrete about the ranges of efficient portfolio returns and risks, use the method estimateFrontierLimits to obtain portfolios at the endpoints of the efficient frontier. Given these portfolios, compute their moments using estimatePortMoments. The following code generates a table that lists the risk and return of the initial portfolio as well as the gross and net moments of portfolio returns for the portfolios at the endpoints of the efficient frontier:

```
[prsk0, pret0] = p.estimatePortMoments(p.InitPort);
pret = p.estimatePortReturn(p.estimateFrontierLimits);
qret = q.estimatePortReturn(q.estimateFrontierLimits);
```

```
fprintf('Annualized Portfolio Returns ...\n');
fprintf(' %6s %6s\n','Gross','Net');
fprintf('Initial Portfolio Return %6.2f %% %6.2f %%\n',100*pret0,100*pret0);
fprintf('Minimum Efficient Portfolio Return %6.2f %% %6.2f %%\n',100*pret(1),100*qret(1));
fprintf('Maximum Efficient Portfolio Return %6.2f %% %6.2f %%\n',100*pret(2),100*qret(2));
```

Annualized Portfolio Returns ...

|  | Gross | Net |
| :--- | ---: | ---: |
| Initial Portfolio Return | $9.70 \%$ | $9.70 \%$ |
| Minimum Efficient Portfolio Return | $5.90 \%$ | $5.77 \%$ |
| Maximum Efficient Portfolio Return | $13.05 \%$ | $12.86 \%$ |

This result shows that the cost to trade ranges from 14 to 19 basis points to get from the current portfolio to the efficient portfolios at the endpoints of the efficient frontier (these costs are the difference between gross and net portfolio returns.) In addition, notice that the maximum efficient portfolio return ( $13 \%$ ) is less than the maximum asset return ( $18 \%$ ) due to the constraints on equity allocations.

## Obtaining a Portfolio at the Specified Return Level on the Efficient Frontier

A common approach to select efficient portfolios is to pick a portfolio that has a desired fraction of the range of expected portfolio returns. To obtain the portfolio that is $30 \%$ of the range from the minimum to maximum return on the efficient frontier, obtain the range of net returns in qret using the portfolio object q and interpolate to obtain a $30 \%$ level with interp1 to obtain a portfolio qwgt:

```
Level = 0.3;
qret = q.estimatePortReturn(q.estimateFrontierLimits);
qwgt = q.estimateFrontierByReturn(interp1([0, 1], qret, Level));
[qrsk, qret] = q.estimatePortMoments(qwgt);
fprintf('Portfolio at %g%% return level on efficient frontier ...\n',100*Level);
fprintf('%10s %10s\n','Return','Risk');
fprintf('%10.2f %10.2f\n',100*qret,100*qrsk);
display(qwgt);
```

```
Portfolio at 30% return level on efficient frontier ...
    Return Risk
        7.90 9.09
qwgt =
    0.6252
    0.1856
    0.0695
    0.1198
```

The target portfolio that is $30 \%$ of the range from minimum to maximum net returns has a return of $7.9 \%$ and a risk of $9.1 \%$.

## Obtaining a Portfolio at the Specified Risk Levels on the Efficient Frontier

Although you could accept this result, suppose you want to target values for portfolio risk. Specifically, suppose you have a conservative target risk of $10 \%$, a moderate target risk of $15 \%$, and an aggressive target risk of $20 \%$ and you want to obtain portfolios that satisfy each risk target. Use the estimateFrontierByRisk method to obtain targeted risks specified in the variable TargetRisk. The resultant three efficient portfolios are obtained in qwgt:

```
TargetRisk = [ 0.10; 0.15; 0.20 ];
qwgt = q.estimateFrontierByRisk(TargetRisk);
display(qwgt);
qwgt =
    0.5407 0.2020 0.1500
    0.2332 0.4000 0.0318
    0.0788 0.1280 0.4682
    0.1474 0.2700 0.3500
```

Use estimatePortRisk to compute the portfolio risks for the three portfolios to confirm that the target risks have been attained:

```
display(q.estimatePortRisk(qwgt));
ans =
    0.1000
    0.1500
    0.2000
```

Suppose you want to shift from the current portfolio to the moderate portfolio. You can estimate the purchases and sales to get to this portfolio:

```
[qwgt, qbuy, qsell] = q.estimateFrontierByRisk(0.15);
```

If you average the purchases and sales for this portfolio, you can see that the average turnover is $17 \%$, which is greater than the target of $15 \%$ :

```
disp(sum(qbuy + qsell)/2)
    0.1700
```

Since you also want to ensure that average turnover is no more than $15 \%$, you can add the turnover constraint to the portfolio object:

```
q = q.setTurnover(0.15);
[qwgt, qbuy, qsell] = q.estimateFrontierByRisk(0.15);
```

You can enter the estimated efficient portfolio with purchases and sales into the Blotter:

```
qbuy(abs(qbuy) < 1.0e-5) = 0;
qsell(abs(qsell)< 1.0e-5) = 0; % zero out near 0 trade weights
Blotter.Port = qwgt;
Blotter.Buy = qbuy;
Blotter.Sell = qsell;
display(Blotter);
Blotter =
Bonds
\begin{tabular}{rllllrc} 
Price & InitHolding & InitPort & UnitCost & Port & Buy & Sell \\
52.4 & 42938 & 0.3 & 0.001 & 0.18787 & 0 & 0.11213
\end{tabular}
```

| Large-Cap Equities | 122.7 | 24449 | 0.4 | 0.001 | 0.4 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Small-Cap Equities | 35.2 | 42612 | 0.2 | 0.001 | 0.16213 | 0 | 0.037871 |
| Emerging Equities | 46.9 | 15991 | 0.1 | 0.004 | 0.25 | 0.15 | 0 |

The Buy and Sell elements of the Blotter are changes in portfolio weights that must be converted into changes in portfolio holdings to determine the trades. Since you are working with net portfolio returns, you must first compute the cost to trade from your initial portfolio to the new portfolio. This can be accomplished as follows:

```
TotalCost = Wealth * sum(Blotter.UnitCost .* (Blotter.Buy + Blotter.Sell))
TotalCost =
    5.6248e+003
```

The cost to trade is $\$ 5,625$, so that, in general, you would have to adjust your initial wealth accordingly before setting up your new portfolio weights. However, to keep the analysis simple, note that you have sufficient cash $(\$ 60,0000)$ set aside to pay the trading costs and that you will not touch the cash position to build up any positions in your portfolio. Thus, you can populate your blotter with the new portfolio holdings and the trades to get to the new portfolio without making any changes in your total invested wealth.

First, compute portfolio holding:

```
Blotter.Holding = Wealth * (Blotter.Port ./ Blotter.Price);
```

Next, compute number of shares to Buy and Sell in your Blotter:

```
Blotter.BuyShare = Wealth * (Blotter.Buy ./ Blotter.Price);
Blotter.SellShare = Wealth * (Blotter.Sell ./ Blotter.Price);
```

Notice how you used an add-hoc truncation rule to obtain unit numbers of shares to buy and sell.

Finally, clean up the blotter by removing the unit costs and the buy and sell portfolio weights:

```
Blotter.Buy = [];
Blotter.Sell = [];
Blotter.UnitCost = [];
```


## Displaying the Final Results

The final result is a blotter that contains proposed trades to get from your current portfolio to a moderate-risk portfolio. To make the trade, you would need to sell 16,049 shares of your bond asset and 8,069 shares of your small-cap equity asset and would need to purchase 23,986 shares of your emerging equities asset.

```
display(Blotter);
Blotter =
```

|  | Price | InitHolding | InitPort | Port | Holding | BuyShare | SellShare |
| :--- | ---: | :--- | :--- | :--- | :--- | ---: | ---: |
| Bonds | 52.4 | 42938 | 0.3 | 0.18787 | 26889 | 0 | 16049 |
| Large-Cap Equities | 122.7 | 24449 | 0.4 | 0.4 | 24449 | 0 | 0 |
| Small-Cap Equities | 35.2 | 42612 | 0.2 | 0.16213 | 34543 | 0 | 8068.8 |
| Emerging Equities | 46.9 | 15991 | 0.1 | 0.25 | 39977 | 23986 | 0 |

The final plot uses plotFrontier to display the efficient frontier and the initial portfolio for the fully specified portfolio optimization problem. It also adds the location of the moderate-risk or final portfolio on the efficient frontier.

```
q.plotFrontier(40);
hold on
scatter(q.estimatePortRisk(qwgt), q.estimatePortReturn(qwgt), 'filled', 'r');
h = legend('Initial Portfolio', 'Efficient Frontier', 'Final Portfolio', 'location', 'best');
set(h, 'Fontsize', 8);
hold off
```



# Investment Performance Metrics 

- "Overview of Performance Metrics" on page 5-2
- "Using the Sharpe Ratio" on page 5-6
- "Using the Information Ratio" on page 5-8
- "Tracking Error" on page 5-10
- "Risk-Adjusted Return" on page 5-11
- "Sample and Expected Lower Partial Moments" on page 5-14
- "Maximum and Expected Maximum Drawdown" on page 5-17


## Overview of Performance Metrics

In this section...<br>"Performance Metrics Classes" on page 5-2<br>"Performance Metrics Example" on page 5-3

## Performance Metrics Classes

Sharpe first proposed a ratio of excess return to total risk as an investment performance metric. Subsequent work by Sharpe, Lintner, and Mossin extended these ideas to entire asset markets in what is called the Capital Asset Pricing Model (CAPM). Since the development of the CAPM, a variety of investment performance metrics has evolved.

This chapter presents four classes of investment performance metrics:

- The first class of metrics are absolute investment performance metrics that are called "classic" metrics since they are based on the CAPM. They include the Sharpe ratio, the information ratio, and tracking error. To compute the Sharpe ratio from data, use the function sharpe to calculate the ratio for one or more asset return series. To compute the information ratio and associated tracking error, use the function inforatio to calculate these quantities for one or more asset return series.
- The second class of metrics are relative investment performance metrics to compute risk-adjusted returns. These metrics are also based on the CAPM and include Beta, Jensen's Alpha, the Security Market Line (SML), Modigliani and Modigliani Risk-Adjusted Return, and the Graham-Harvey measures. To calculate risk-adjusted alpha and return, use portalpha.
- The third class of metrics are alternative investment performance metrics based on lower partial moments. To calculate lower partial moments, use the functions lpm for sample lower partial moments and elpm for expected lower partial moments.
- The fourth class of metrics are performance metrics based on maximum drawdown and expected maximum drawdown. To calculate maximum or expected maximum drawdowns, use the functions maxdrawdown and emaxdrawdown.


## Performance Metrics Example

To illustrate the functions for investment performance metrics, you will work with three financial time series objects using performance data for:

- An actively managed, large-cap value mutual fund
- A large-cap market index
- 90-day Treasury bills

The data is monthly total return prices that cover a span of 5 years.
The following plot illustrates the performance of each series in terms of total returns to an initial $\$ 1$ invested at the start of this 5 -year period:

```
load FundMarketCash
plot(TestData)
hold all
title('\bfFive-Year Total Return Performance');
legend('Fund','Market','Cash','Location','SouthEast');
hold off
```



The mean (Mean) and standard deviation (Sigma) of returns for each series are

```
Returns = tick2ret(TestData);
Assets
Mean = mean(Returns)
Sigma = std(Returns, 1)
```

which gives the following result:

| Assets $=$ |  |  |
| :---: | :---: | :---: |
| 'Fund' | 'Market' | 'Cash' |
| Mean $=$ |  |  |
| 0.0038 <br> Sigma $=$ | 0.0030 | 0.0017 |
| 0.0229 | 0.0389 | 0.0009 |

In this chapter, you will work with this data to demonstrate that the example fund has done well in absolute, relative, and risk-adjusted terms with respect to the investment performance metrics.

Note Functions for investment performance metrics use total return price and total returns. To convert between total return price and total returns, use ret2tick and tick2ret.

## Using the Sharpe Ratio

```
In this section...
"Introduction" on page 5-6
"Sharpe Ratio Example" on page 5-6
```


## Introduction

The Sharpe ratio is the ratio of the excess return of an asset divided by the asset's standard deviation of returns. The Sharpe ratio has the form:
(Mean - Riskless) / Sigma
Here Mean is the mean of asset returns, Riskless is the return of a riskless asset, and Sigma is the standard deviation of asset returns. A higher Sharpe ratio is better than a lower Sharpe ratio. A negative Sharpe ratio indicates "anti-skill" since the performance of the riskless asset is superior.

## Sharpe Ratio Example

To compute the Sharpe ratio, the mean return of the cash asset is used as the return for the riskless asset. Thus, given asset return data and the riskless asset return, the Sharpe ratio is calculated with

```
load FundMarketCash
Returns = tick2ret(TestData);
Riskless = mean(Returns(:,3))
Sharpe = sharpe(Returns, Riskless)
```

which gives the following result:

```
Riskless =
    0.0017
Sharpe =
    0.0886 0.0315 0
```

The Sharpe ratio of the example fund is significantly higher than the Sharpe ratio of the market. As will be demonstrated with portalpha, this translates into a strong risk-adjusted return. Since the Cash asset is the same as

Riskless, it makes sense that its Sharpe ratio is 0 . The Sharpe ratio was calculated with the mean of cash returns. It can also be calculated with the cash return series as input for the riskless asset

Sharpe = sharpe(Returns, Returns(:,3))
which gives the following result:
Sharpe =
$0.0886 \quad 0.03150$

## Using the Information Ratio

In this section...<br>"Introduction" on page 5-8<br>"Information Ratio Example" on page 5-8

## Introduction

Although originally called the "appraisal ratio" by Treynor and Black, the information ratio is the ratio of relative return to relative risk (known as "tracking error"). Whereas the Sharpe ratio looks at returns relative to a riskless asset, the information ratio is based on returns relative to a risky benchmark which is known colloquially as a "bogey." Given an asset or portfolio of assets with random returns designated by Asset and a benchmark with random returns designated by Benchmark, the information ratio has the form:

```
Mean(Asset - Benchmark) / Sigma (Asset - Benchmark)
```

Here Mean(Asset - Benchmark) is the mean of Asset minus Benchmark returns, and Sigma(Asset - Benchmark) is the standard deviation of Asset minus Benchmark returns. A higher information ratio is considered better than a lower information ratio.

## Information Ratio Example

To calculate the information ratio using the example data, the mean return of the market series is used as the return of the benchmark. Thus, given asset return data and the riskless asset return, compute the information ratio with

```
load FundMarketCash
Returns = tick2ret(TestData);
Benchmark = Returns(:,2);
InfoRatio = inforatio(Returns, Benchmark)
```

which gives the following result:

```
InfoRatio =
    0.0432 NaN -0.0315
```

Since the market series has no risk relative to itself, the information ratio for the second series is undefined (which is represented as NaN in MATLAB software). Its standard deviation of relative returns in the denominator is 0 .

## Tracking Error

In this section...<br>"Introduction" on page 5-10<br>"Tracking Error Example" on page 5-10

## Introduction

Given an asset or portfolio of assets and a benchmark, the relative standard deviation of returns between the asset or portfolio of assets and the benchmark is called tracking error.

## Tracking Error Example

The function inforatio computes tracking error and returns it as a second argument

```
load FundMarketCash
Returns = tick2ret(TestData);
Benchmark = Returns(:,2);
[InfoRatio, TrackingError] = inforatio(Returns, Benchmark)
```

which gives the following results:
InfoRatio $=$
0.0432

| TrackingError |
| :--- |
| 0.0187 | NaN $\quad-0.0315$

Tracking error is a useful measure of performance relative to a benchmark since it is in units of asset returns. For example, the tracking error of $1.87 \%$ for the fund relative to the market in this example is reasonable for an actively managed, large-cap value fund.

## Risk-Adjusted Return

In this section...<br>"Introduction" on page 5-11<br>"Risk-Adjusted Return Example" on page 5-11

## Introduction

Risk-adjusted return either shifts the risk (which is the standard deviation of returns) of a portfolio to match the risk of a market portfolio or shifts the risk of a market portfolio to match the risk of a fund. According to the Capital Asset Pricing Model (CAPM), the market portfolio and a riskless asset are points on a Security Market Line (SML). The return of the resultant shifted portfolio, levered or unlevered, to match the risk of the market portfolio, is the risk-adjusted return. The SML provides another measure of risk-adjusted return, since the difference in return between the fund and the SML, return at the same level of risk.

## Risk-Adjusted Return Example

Given our example data with a fund, a market, and a cash series, you can calculate the risk-adjusted return and compare it with the fund and market's mean returns

```
load FundMarketCash
Returns = tick2ret(TestData);
Fund = Returns(:,1);
Market = Returns(:,2);
Cash = Returns(:,3);
MeanFund = mean(Fund)
MeanMarket = mean(Market)
[MM, aMM] = portalpha(Fund, Market, Cash, 'MM')
[GH1, aGH1] = portalpha(Fund, Market, Cash, 'gh1')
[GH2, aGH2] = portalpha(Fund, Market, Cash, 'gh2')
[SML, aSML] = portalpha(Fund, Market, Cash, 'sml')
```

which gives the following results:

```
MeanFund =
    0.0038
MeanMarket =
    0.0030
MM =
    0.0022
aMM =
    0.0052
GH1 =
    0.0013
aGH1 =
    0.0025
GH2 =
    0.0022
aGH2 =
    0.0052
SML =
    0.0013
aSML =
    0.0025
```

Since the fund's risk is much less than the market's risk, the risk-adjusted return of the fund is much higher than both the nominal fund and market returns.

## Sample and Expected Lower Partial Moments

In this section...<br>"Introduction" on page 5-14<br>"Sample Lower Partial Moments Example" on page 5-14<br>"Expected Lower Partial Moments Example" on page 5-15

## Introduction

Use lower partial moments to examine what is colloquially known as "downside risk." The main idea of the lower partial moment framework is to model moments of asset returns that fall below a minimum acceptable level of return. To compute lower partial moments from data, use lpm to calculate lower partial moments for multiple asset return series and for multiple moment orders. To compute expected values for lower partial moments under several assumptions about the distribution of asset returns, use elpm to calculate lower partial moments for multiple assets and for multiple orders.

## Sample Lower Partial Moments Example

The following example demonstrates lpm to compute the zero-order, first-order, and second-order lower partial moments for the three time series, where the mean of the third time series is used to compute MAR (with the so-called risk-free rate).

```
load FundMarketCash
Returns = tick2ret(TestData);
Assets
MAR = mean(Returns(:,3))
LPM = lpm(Returns, MAR, [0 1 2])
```

which gives the following results:

```
Assets =
    'Fund' 'Market' 'Cash'
MAR =
    0.0017
LPM =
```

| 0.4333 | 0.4167 | 0.6167 |
| :--- | :--- | :--- |
| 0.0075 | 0.0140 | 0.0004 |
| 0.0003 | 0.0008 | 0.0000 |

The first row of LPM contains zero-order lower partial moments of the three series. The fund and market index fall below MAR about $40 \%$ of the time and cash returns fall below its own mean about $60 \%$ of the time.

The second row contains first-order lower partial moments of the three series. The fund and market have large expected shortfall returns relative to MAR by 75 and 140 basis points per month. On the other hand, cash underperforms MAR by about only 4 basis points per month on the downside.

The third row contains second-order lower partial moments of the three series. The square root of these quantities provides an idea of the dispersion of returns that fall below the MAR. The market index has a much larger variation on the downside when compared to the fund.

## Expected Lower Partial Moments Example

To compare realized values with expected values, use elpm to compute expected lower partial moments based on the mean and standard deviations of normally distributed asset returns. The elpm function works with the mean and standard deviations for multiple assets and multiple orders

```
load FundMarketCash
Returns = tick2ret(TestData);
MAR = mean(Returns(:,3))
Mean = mean(Returns)
Sigma = std(Returns, 1)
Assets
ELPM = elpm(Mean, Sigma, MAR, [0 1 2 2])
```

which gives the following results:

```
Assets =
    'Fund' 'Market' 'Cash'
ELPM =
    0.4647 0.4874 0.5000
    0.0082 0.0149 0.0004
```

$$
\begin{array}{lll}
0.0002 & 0.0007 & 0.0000
\end{array}
$$

Based on the moments of each asset, the expected values for lower partial moments imply better than expected performance for the fund and market and worse than expected performance for cash. Note that this function works with either degenerate or nondegenerate normal random variables. For example, if cash were truly riskless, its standard deviation would be 0 . You can examine the difference in expected shortfall.

```
RisklessCash = elpm(Mean(3), 0, MAR, 1)
```

which gives the following result:

```
RisklessCash =
    0
```


## Maximum and Expected Maximum Drawdown

In this section...<br>"Introduction" on page 5-17<br>"Maximum Drawdown Example" on page 5-17<br>"Expected Maximum Drawdown Example" on page 5-21

## Introduction

Maximum drawdown is the maximum decline of a series, measured as return, from a peak to a nadir over a period of time. Although additional metrics exist that are used in the hedge fund and commodity trading communities (see Pederson and Rudholm-Alfvin [20] in Appendix A, "Bibliography"), the original definition and subsequent implementation of these metrics is not yet standardized.

It is possible to compute analytically the expected maximum drawdown for a Brownian motion with drift (see Magdon-Ismail, Atiya, Pratap, and Abu-Mostafa [16] Appendix A, "Bibliography"). These results are used to estimate the expected maximum drawdown for a series that approximately follows a geometric Brownian motion.

Use maxdrawdown and emaxdrawdown to calculate the maximum and expected maximum drawdowns.

## Maximum Drawdown Example

This example demonstrates how to compute the maximum drawdown (MaxDD) using our example data with a fund, a market, and a cash series:
load FundMarketCash
MaxDD = maxdrawdown(TestData)
which gives the following results:

```
MaxDD =
    0.1658 0.3381 0
```

The maximum drop in the given time period was of $16.58 \%$ for the fund series, and $33.81 \%$ for the market. There was no decline in the cash series, as expected, because the cash account never loses value.
maxdrawdown can also return the indices (MaxDDIndex) of the maximum drawdown intervals for each series in an optional output argument:

```
[MaxDD, MaxDDIndex] = maxdrawdown(TestData)
```

which gives the following results:

```
MaxDD =
    0.1658 0.3381
MaxDDIndex =
    2 NaN
    18 18 NaN
```

The first two series experience their maximum drawdowns from the 2nd to the 18th month in the data. The indices for the third series are NaNs because it never has a drawdown.

The $16.58 \%$ value loss from month 2 to month 18 for the fund series is verified using the reported indices:

```
Start = MaxDDIndex(1,:);
End = MaxDDIndex(2,:);
(TestData(Start(1),1) - TestData(End(1),1))/TestData(Start(1),1)
ans =
0.1658
```

Although the maximum drawdown is measured in terms of returns, maxdrawdown can measure the drawdown in terms of absolute drop in value, or in terms of log-returns. To contrast these alternatives more clearly, we work with the fund series assuming, an initial investment of 50 dollars:

```
Fund50 = 50*TestData(:,1);
```

```
plot(Fund50);
title('\bfFive-Year Fund Performance, Initial Investment 50 usd');
xlabel('Months');
ylabel('Value of Investment');
```



First, we compute the standard maximum drawdown, which coincides with the results above because returns are independent of the initial amounts invested:

```
MaxDD50Ret = maxdrawdown(Fund50)
MaxDD50Ret =
```

0.1658

Next, we compute the maximum drop in value, using the arithmetic argument:
[MaxDD50Arith, Ind50Arith] = maxdrawdown(Fund50,'arithmetic')

```
MaxDD50Arith =
    8.4285
Ind50Arith =
    2
    18
```

The value of this investment was $\$ 50.84$ in month 2 , but by month 18 the value was down to $\$ 42.41$, a drop of $\$ 8.43$. This is the largest loss in dollar value from a previous high in the given time period. In this case, the maximum drawdown period, 2nd to 18 th month, is the same independently of whether drawdown is measured as return or as dollar value loss.

Last, we compute the maximum decline based on log-returns using the geometric argument. In this example, the log-returns result in a maximum drop of $18.13 \%$, again from the 2 nd to the 18 th month, not far from the $16.58 \%$ obtained using standard returns.

```
[MaxDD50LogRet, Ind50LogRet] = maxdrawdown(Fund50,'geometric')
MaxDD50LogRet =
```

    0.1813
    Ind50LogRet $=$
2
18

Note, the last measure is equivalent to finding the arithmetic maximum drawdown for the log of the series:

```
MaxDD50LogRet2 = maxdrawdown(log(Fund50),'arithmetic')
MaxDD50LogRet2 =
```

    0.1813
    
## Expected Maximum Drawdown Example

This example demonstrates using the log-return moments of the fund to compute the expected maximum drawdown (EMaxDD) and then compare it with the realized maximum drawdown (MaxDD).

```
load FundMarketCash
logReturns = log(TestData(2:end,:) ./ TestData(1:end - 1,:));
Mu = mean(logReturns(:,1));
Sigma = std(logReturns(:,1),1);
T = size(logReturns,1);
MaxDD = maxdrawdown(TestData(:,1),'geometric')
EMaxDD = emaxdrawdown(Mu-0.5*Sigma^2, Sigma, T)
```

which gives the following results:
MaxDD =
0.1813

EMaxDD =
0.1588

The drawdown observed in this time period is above the expected maximum drawdown. There is no contradiction here. The expected maximum drawdown is not an upper bound on the maximum losses from a peak, but an estimate of their average, based on a geometric Brownian motion assumption.

## Credit Risk Analysis

- "Credit Rating" on page 6-2
- "Estimation of Transition Probabilities" on page 6-3


## Credit Rating

## In this section...

"Introduction" on page 6-2
"Example" on page 6-2

## Introduction

A credit rating is an evaluation of a potential borrower's ability to repay debt. The credit rating demo describes one approach to assigning credit ratings to companies. The demo uses the same financial ratios as Altman's z-score (see Altman [39] in Appendix A, "Bibliography") as predictors of credit quality, and then it trains a classifier using one of the statistical learning tools available in Statistics Toolbox software. The classifier used in the demo outputs a classification score to help reviewers identify companies that fall into "gray areas" between ratings that require a closer look. The demo also discusses back-testing tools to evaluate the ratings accuracy.

## Example

To run the credit rating demo at the MATLAB command line, enter:

```
echodemo demo_creditrating
```


## Estimation of Transition Probabilities

In this section...<br>"Introduction" on page 6-3<br>"Estimating Transition Probabilities" on page 6-4<br>"Estimating $t$-year Default Probabilities and Confidence Intervals" on page 6-7<br>"Removing Outliers, Estimating Subgroup Probabilities, and Aggregating Datasets" on page 6-9

## Introduction

Credit ratings rank borrowers according to their creditworthiness. Though this ranking is, in itself, useful, institutions are also interested in knowing how likely it is that borrowers in a particular rating category will be upgraded or downgraded to a different rating, and especially, how likely it is that they will default. Transition probabilities offer one way to characterize the past changes in credit quality of obligors (typically firms), and are cardinal inputs to many risk management applications. Financial Toolbox software supports the estimation of transition probabilities using both cohort and duration (also known as hazard rate or intensity) approaches using transprob and transprobbytotals.

The general workflow for estimating transition probabilities is:
1 Gather the historical credit ratings for the data input to transprob. The historical data is a cell array of size nRecords-by- 3 containing credit ratings history at the obligor level. For example, the nRecords-by- 3 cell array looks like this:

| '00010283' | '10-Nov-1984' | ' CCC' |
| :---: | :---: | :---: |
| '00010283' | '12-May-1986' | B' |
| '00010283' | '29-Jun-1988' | ' CCC' |
| '00010283' | '12-Dec-1991' | D' |
| '00013326' | '09-Feb-1985' | 'A' |
| '00013326' | ' 24-Feb-1994' | 'AA' |
| '00013326' | '10-Nov-2000' | BBB' |
| '00014413' | '23-Dec-1982' | B |

```
'00014413' '20-Apr-1988' 'BB'
'00014413' '16-Jan-1998' 'B'
```

2 Use transprob to create the transMat output. transMat is the matrix of transition probabilities, or transition matrix, in percent.

3 Use idTotals or sampleTotals structures created from a previous use of transprob, as the totals input for transprobbytotals.
transprobbytotals is useful for efficiently performing tasks such as removing outlier information, obtaining bootstrapped confidence intervals, or computing transition probability estimates for different transition intervals (1-year transitions, 2 -year transitions, etc.).

## Estimating Transition Probabilities

The file Data_TransProb.mat contains sample historical credit ratings data.

```
load Data_TransProb
data(1:10,:)
ans =
    '00010283' '10-Nov-1984' 'CCC'
    '00010283' '12-May-1986' 'B'
    '00010283' '29-Jun-1988' 'CCC'
    '00010283' '12-Dec-1991' 'D'
    '00013326' '09-Feb-1985' 'A'
    '00013326' '24-Feb-1994' 'AA'
    '00013326' '10-Nov-2000' 'BBB'
    '00014413' '23-Dec-1982' 'B'
    '00014413' '20-Apr-1988' 'BB'
    '00014413' '16-Jan-1998' 'B'
```

The historical data is formatted as a cell array with three columns. Each row contains an ID (column 1), a date (column 2), and a credit rating (column 3). The assigned credit rating corresponds to the associated ID on the associated date. All information corresponding to the same ID must be stored in contiguous rows. In this example, IDs, dates, and ratings are stored in string format, but you also can enter them in numeric format.

In this example, the simplest calling syntax for transprob passes the nRecords-by- 3 cell array as the only input argument. The default startDate and endDate are the earliest and latest dates in the data. The default estimation algorithm is the duration method and 1-year transition probabilities are estimated:

| transMat0 $=$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Usually, specific startDate and endDate are provided. Here, you can assume that the time window of interest is the 5 -year period from the end of 1995 to the end of 2000 . For comparisons, compute the estimates for this time window, first using duration algorithm (default option), and then with the cohort algorithm explicitly set.

```
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
transMat1 = transprob(data,'startDate',startDate,'endDate',endDate)
transMat2 = transprob(data,'startDate',startDate,'endDate',endDate,...
'algorithm','cohort')
transMat1 =
\begin{tabular}{rrrrrrrr}
90.6236 & 7.9051 & 1.0314 & 0.4123 & 0.0210 & 0.0020 & 0.0003 & 0.0043 \\
4.4780 & 89.5558 & 4.5298 & 1.1225 & 0.2284 & 0.0094 & 0.0009 & 0.0754 \\
0.3983 & 6.1164 & 87.0641 & 5.4801 & 0.7637 & 0.0892 & 0.0050 & 0.0832 \\
0.1029 & 0.8572 & 10.7918 & 83.0204 & 3.9971 & 0.7001 & 0.1313 & 0.3992 \\
0.1043 & 0.3745 & 2.2962 & 14.0954 & 78.9840 & 3.0013 & 0.0463 & 1.0980 \\
0.0113 & 0.0544 & 0.7055 & 3.2925 & 15.4350 & 75.5988 & 1.8166 & 3.0860 \\
0.0044 & 0.0189 & 0.1903 & 1.9743 & 6.2320 & 10.2334 & 75.9983 & 5.3484 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100.0000
\end{tabular}
```

| transMat2 $=$ |  |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  |  |  |
| 90.1554 | 8.5492 | 0.9067 | 0.3886 | 0 | 0 | 0 | 0 |
| 4.9512 | 88.5221 | 5.1763 | 1.0503 | 0.2251 | 0 | 0 | 0.0750 |
| 0.2770 | 6.6482 | 86.2188 | 6.0942 | 0.6233 | 0.0693 | 0 | 0.0693 |
| 0.0794 | 0.8737 | 11.6759 | 81.6521 | 4.3685 | 0.7943 | 0.1589 | 0.3971 |
| 0.1002 | 0.4008 | 1.9038 | 15.4309 | 77.8557 | 3.4068 | 0 | 0.9018 |
| 0 | 0 | 0.2262 | 2.4887 | 17.4208 | 74.2081 | 2.2624 | 3.3937 |
| 0 | 0 | 0.7576 | 1.5152 | 6.0606 | 10.6061 | 75.0000 | 6.0606 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

By default, the cohort algorithm internally gets yearly snapshots of the credit ratings, but the number of snapshots per year is definable using the parameter/value pair snapsPerYear. To get the estimates using quarterly snapshots:

| 'algorithm', 'cohort', 'snapsPerYear', 4) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| transMat3 = |  |  |  |  |  |  |  |
| 90.4765 | 8.0881 | 1.0072 | 0.4069 | 0.0164 | 0.0015 | 0.0002 | 0.0032 |
| 4.5949 | 89.3216 | 4.6489 | 1.1239 | 0.2276 | 0.0074 | 0.0007 | 0.0751 |
| 0.3747 | 6.3158 | 86.7380 | 5.6344 | 0.7675 | 0.0856 | 0.0040 | 0.0800 |
| 0.0958 | 0.7967 | 11.0441 | 82.6138 | 4.1906 | 0.7230 | 0.1372 | 0.3987 |
| 0.1028 | 0.3571 | 2.3312 | 14.4954 | 78.4276 | 3.1489 | 0.0383 | 1.0987 |
| 0.0084 | 0.0399 | 0.6465 | 3.0962 | 16.0789 | 75.1300 | 1.9044 | 3.0956 |
| 0.0031 | 0.0125 | 0.1445 | 1.8759 | 6.2613 | 10.7022 | 75.6300 | 5.3705 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0000 |

Both the duration and the cohort algorithms compute 1-year transition probabilities by default, but the time interval for the transitions is definable using the parameter/value pair transInterval. For example, to get the 2 -year transition probabilities using the cohort algorithm with the same snapshot periodicity and time window:

```
transMat4 = transprob(data,'startDate',startDate,'endDate',endDate,...
'algorithm','cohort','snapsPerYear',4,'transInterval',2)
transMat4 =
```

| 82.2358 | 14.6092 | 2.2062 | 0.8543 | 0.0711 | 0.0074 | 0.0011 | 0.0149 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 8.2803 | 80.4584 | 8.3606 | 2.2462 | 0.4665 | 0.0316 | 0.0030 | 0.1533 |
| 0.9604 | 11.1975 | 76.1729 | 9.7284 | 1.5322 | 0.2044 | 0.0162 | 0.1879 |
| 0.2483 | 2.0903 | 18.8440 | 69.5145 | 6.9601 | 1.2966 | 0.2329 | 0.8133 |
| 0.2129 | 0.8713 | 5.4893 | 23.5776 | 62.6438 | 4.9464 | 0.1390 | 2.1198 |
| 0.0378 | 0.1895 | 1.7679 | 7.2875 | 24.9444 | 57.1783 | 2.8816 | 5.7132 |
| 0.0154 | 0.0716 | 0.6576 | 4.2157 | 11.4465 | 16.3455 | 57.4078 | 9.8399 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Estimating $\boldsymbol{t}$-year Default Probabilities and Confidence Intervals

Commonly, a whole range of transition intervals for 1-year, 2-year, 3-year, 4 -year, and 5 -year transition probabilities are of interest. In this case, it is more efficient to call transprob once, with two output arguments, and use the second output sampleTotals to call transprobbytotals sequentially with the desired transition intervals. In this example, assume that the 1-year, 2 -year, 3 -year, 4 -year, and 5 -year probabilities are of interest:

```
load Data_TransProb
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
[transMat,sampleTotals,idTotals] = transprob(data,'startDate', ...
startDate, 'endDate',endDate);
DefProb = zeros(7,5);
for t = 1:5
    transMatTemp = transprobbytotals(sampleTotals,'transInterval',t);
    DefProb(:,t) = transMatTemp(1:7,8);
end
DefProb
DefProb =
\begin{tabular}{lllll}
0.0043 & 0.0169 & 0.0377 & 0.0666 & 0.1033 \\
0.0754 & 0.1542 & 0.2377 & 0.3265 & 0.4213 \\
0.0832 & 0.1936 & 0.3276 & 0.4819 & 0.6536 \\
0.3992 & 0.8127 & 1.2336 & 1.6566 & 2.0779 \\
1.0980 & 2.1189 & 3.0668 & 3.9468 & 4.7644
\end{tabular}
```

| 3.0860 | 5.6994 | 7.9281 | 9.8418 | 11.4963 |
| ---: | ---: | ---: | ---: | ---: |
| 5.3484 | 9.8053 | 13.5320 | 16.6599 | 19.2964 |

The sample totals contain the information on the total time spent on each credit rating, and the total number of transitions between ratings. When transprob is called, the duration algorithm parses through the data for each company, to compute the total time spent in each rating and the total transitions between ratings. This information is stored, at a company level, in the idTotals optional output argument, and it is aggregated for the whole sample in the sampleTotals optional output argument.

Using transprobbytotals is efficient because the historical data is not parsed, which is a time consuming step. Instead, the summary of information contained in sampleTotals or idTotals is directly used to finalize the computation of the transition probability estimates. The estimation of confidence intervals for the transition probability estimates provides another good example of the benefits of using transprobbytotals. You can get the confidence intervals using a bootstrapping procedure. To bootstrap, call transprob once, with three output arguments, the second being sampleTotals, and the third idTotals. Then, use bootstrp from Statistics Toolbox with transprobbytotals as the bootstrapping function and idTotals as the bootstrapping array. The following example demonstrates how to use bootstrp with transprobbytotals. As in the previous example, the focus is probabilities of default and display the estimates with an upper and lower bounds at a $95 \%$ confidence level.

```
PD = transMat(1:7,8);
bootstat = bootstrp(100,@(idTotals)transprobbytotals(idTotals),idTotals);
ci = prctile(bootstat,[2.5 97.5]); % 95% confidence
CIlower = reshape(ci(1,:),8,8);
CIupper = reshape(ci(2,:),8,8);
PD_LB = CIlower(1:7,8);
PD_UB = CIupper(1:7,8);
[PD_LB PD PD_UB]
ans =
    0.0004 0.0043 0.0106
```

| 0.0028 | 0.0754 | 0.2192 |
| :--- | :--- | :--- |
| 0.0126 | 0.0832 | 0.2180 |
| 0.1659 | 0.3992 | 0.6617 |
| 0.5703 | 1.0980 | 1.7260 |
| 1.7264 | 3.0860 | 4.7602 |
| 1.7678 | 5.3484 | 9.5055 |

## Removing Outliers, Estimating Subgroup Probabilities, and Aggregating Datasets

- "Removing Outliers" on page 6-9
- "Estimating Subgroup Probabilities" on page 6-10
- "Aggregating Datasets" on page 6-11


## Removing Outliers

The idTotals output from transprob can also be exploited to update the transition probability estimates after removing some outlier information. For example, if you knew that the credit rating migration information for the 4th and 27th companies in the data have problems, those companies can be removed and the transition probabilities can be efficiently updated as follows:

```
load Data_TransProb
startDate = '31-Dec-1995';
endDate = '31-Dec-2000';
[transMat,sampleTotals,idTotals] = transprob(data,'startDate', ...
startDate, 'endDate',endDate);
nIDs = length(idTotals);
keepInd = setdiff(1:nIDs,[4 27]);
transMatnooutlier = transprobbytotals(idTotals(keepInd))
transMatnooutlier =
\begin{tabular}{rrrrrrrr}
90.6241 & 7.9067 & 1.0290 & 0.4124 & 0.0211 & 0.0020 & 0.0003 & 0.0043 \\
4.4917 & 89.5918 & 4.4779 & 1.1240 & 0.2288 & 0.0094 & 0.0009 & 0.0756 \\
0.3990 & 6.1220 & 87.0530 & 5.4841 & 0.7643 & 0.0893 & 0.0050 & 0.0833 \\
0.1030 & 0.8576 & 10.7909 & 83.0207 & 3.9971 & 0.7001 & 0.1313 & 0.3992 \\
0.1043 & 0.3746 & 2.2960 & 14.0955 & 78.9840 & 3.0013 & 0.0463 & 1.0980
\end{tabular}
```

| 0.0113 | 0.0544 | 0.7054 | 3.2925 | 15.4350 | 75.5988 | 1.8166 | 3.0860 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.0044 | 0.0189 | 0.1903 | 1.9743 | 6.2320 | 10.2334 | 75.9983 | 5.3484 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Deciding which companies must be removed is a case-by-case situation. Reasons to remove a company can include a typo in one of the ratings history, or an unusual migration between ratings whose impact on the transition probability estimates must be measured. Note that transprob does not change the order of the companies in any way. The ordering of companies in the input data is the same as the ordering in the idTotals array.

## Estimating Subgroup Probabilities

You can use idTotals efficiently to get estimates over a subsample that corresponds to a particular subgroup of companies. For example, assume that the companies in the example are grouped into three geographic regions and that the companies were grouped by geographic regions beforehand, so that the first 340 companies correspond to the first region, the next 572 companies to the second region, and the rest to the third region. You can efficiently get transition probabilities for each region as follows:

| $\mathrm{n} 1=340$; |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| transMatG1 = transprobbytotals(idTotals(1:n1)) |  |  |  |  |  |  |  |
| transMatG3 $=$ transprobbytotals(idTotals(n1+n2+1:end)) |  |  |  |  |  |  |  |
| transMatG1 = |  |  |  |  |  |  |  |
| 90.8299 | 7.6501 | 0.3178 | 1.1700 | 0.0255 | 0.0044 | 0.0021 | 0.0002 |
| 4.3572 | 89.0262 | 5.7838 | 0.8039 | 0.0245 | 0.0029 | 0.0013 | 0.0001 |
| 0.7066 | 6.7567 | 86.6320 | 5.4950 | 0.3721 | 0.0252 | 0.0101 | 0.0023 |
| 0.0626 | 1.3688 | 10.3895 | 83.5022 | 3.6823 | 0.6466 | 0.3084 | 0.0396 |
| 0.0256 | 0.7884 | 2.6970 | 13.7857 | 78.8321 | 2.8310 | 0.0561 | 0.9842 |
| 0.0026 | 0.1095 | 0.4280 | 3.5204 | 21.1437 | 72.9230 | 1.6456 | 0.2273 |
| 0.0005 | 0.0216 | 0.0730 | 0.4574 | 4.9586 | 4.2821 | 80.3062 | 9.9006 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |
| transMatG2 = |  |  |  |  |  |  |  |
| 90.5798 | 8.4877 | 0.8202 | 0.0884 | 0.0132 | 0.0011 | 0.0000 | 0.0096 |


| 4.1999 | 90.0371 | 3.8657 | 1.4744 | 0.2144 | 0.0128 | 0.0001 | 0.1956 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.3022 | 5.9869 | 86.7128 | 5.5526 | 1.0411 | 0.1902 | 0.0015 | 0.2127 |
| 0.0204 | 0.5606 | 10.9342 | 82.9195 | 4.0123 | 0.7398 | 0.0059 | 0.8073 |
| 0.0089 | 0.3338 | 2.1185 | 16.6496 | 76.2395 | 3.1241 | 0.0261 | 1.4995 |
| 0.0013 | 0.0465 | 0.6710 | 2.4731 | 14.7281 | 76.7378 | 1.2993 | 4.0428 |
| 0.0002 | 0.0080 | 0.0681 | 0.4598 | 4.1324 | 8.4380 | 80.9092 | 5.9843 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| transMatG3 | $=$ |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| 0.5655 | 7.5408 | 1.5288 | 0.3369 | 0.0258 | 0.0015 | 0.0003 | 0.0004 |
| 4.8073 | 89.3842 | 4.4865 | 0.9582 | 0.3509 | 0.0095 | 0.0009 | 0.0025 |
| 0.3153 | 5.8771 | 87.6353 | 5.4101 | 0.7160 | 0.0322 | 0.0052 | 0.0088 |
| 0.1995 | 0.8625 | 10.8682 | 82.8717 | 4.1423 | 0.6903 | 0.1565 | 0.2090 |
| 0.2465 | 0.1091 | 2.1558 | 12.0289 | 81.5803 | 3.0057 | 0.0616 | 0.8122 |
| 0.0227 | 0.0400 | 0.9380 | 4.3175 | 12.3632 | 75.9429 | 2.5766 | 3.7991 |
| 0.0149 | 0.0180 | 0.3414 | 3.6918 | 8.1414 | 13.6010 | 70.7254 | 3.4661 |

## Aggregating Datasets

This example demonstrates how to aggregate estimates from two (or more) datasets. It is possible that two datasets, coming from two different databases, must be considered for the estimation of the transition probabilities. Also, if a dataset is large and cannot be loaded into memory, the dataset will need to be split into two (or more) datasets. In these cases, use transprob on the first dataset, then on the second dataset, and then aggregate the sampleTotals outputs and use transprobbytotals to get the final estimates.

For demonstration, artificially break the original data into two sections, but in practice the first and second calls to transprob are made on datasets that are disjoint.

Note When aggregating multiple datasets, the history of a company cannot be split across datasets.

1 Use transprob on each dataset, with two output arguments:

```
cutoff = 2099
[transMatDS1, sampleTotalsDS1] = transprob(data(1:cutoff,:),...
'startDate',startDate,'endDate',endDate);
[transMatDS2, sampleTotalsDS2] = transprob(data(cutoff+1:end,:),...
'startDate',startDate,'endDate',endDate);
```

2 Aggregate the sampleTotals information in a new structure with the same fields:

```
sampleTotAggr.totalsVec = sampleTotalsDS1.totalsVec + sampleTotalsDS2.totalsVec;
sampleTotAggr.totalsMat = sampleTotalsDS1.totalsMat + sampleTotalsDS2.totalsMat;
```

3 Compute the aggregated transition probability estimates:


As a sanity check for this example, you can verify that the aggregation procedure yields the same estimates (up to numerical differences) as estimating the probabilities directly over the entire sample:

```
aggError = max(max(abs(transMatAggr - transMat)))
aggError =
    2.8422e-014
```

Using analogous steps, the datasets can also be broken up across time, to aggregate over multiple non-overlapping time periods.

# Regression with Missing Data 

- "Multivariate Normal Regression" on page 7-2
- "Maximum Likelihood Estimation with Missing Data" on page 7-9
- "Multivariate Normal Regression Types" on page 7-17
- "Valuation with Missing Data" on page 7-34


## Multivariate Normal Regression

In this section...<br>"Introduction" on page 7-2<br>"Multivariate Normal Linear Regression" on page 7-3<br>"Maximum Likelihood Estimation" on page 7-4<br>"Special Case of a Multiple Linear Regression Model" on page 7-5<br>"Least-Squares Regression" on page 7-5<br>"Mean and Covariance Estimation" on page 7-5<br>"Convergence" on page 7-6<br>"Fisher Information" on page 7-6<br>"Statistical Tests" on page 7-7

## Introduction

This section focuses on using likelihood-based methods for multivariate normal regression. The parameters of the regression model are estimated via maximum likelihood estimation. For multiple series, this requires iteration until convergence. The complication due to the possibility of missing data is incorporated into the analysis with a variant of the EM algorithm known as the ECM algorithm.

The underlying theory of maximum likelihood estimation and the definition and significance of the Fisher information matrix can be found in Caines [1] and Cramér [2]. The underlying theory of the ECM algorithm can be found in Meng and Rubin [8] and Sexton and Swensen [9].

In addition, these two examples of maximum likelihood estimation are presented:

- "Example of Portfolios with Missing Data" on page 7-26
- "Estimation of Some Technology Stock Betas" on page 7-36


## Multivariate Normal Linear Regression

Suppose you have a multivariate normal linear regression model in the form

$$
\left[\begin{array}{c}
Z_{1} \\
\vdots \\
Z_{m}
\end{array}\right] \sim N\left(\left[\begin{array}{c}
H_{1} b \\
\vdots \\
H_{m} b
\end{array}\right],\left[\begin{array}{ccc}
C & & 0 \\
& \ddots & \\
0 & & C
\end{array}\right]\right)
$$

where the model has $m$ observations of $n$-dimensional random variables $\mathbf{Z}_{1}, \ldots$, $\mathbf{Z}_{m}$ with a linear regression model that has a $p$-dimensional model parameter vector $\mathbf{b}$. In addition, the model has a sequence of $m$ design matrices $\mathbf{H}_{1}, \ldots$, $\mathbf{H}_{m}$, where each design matrix is a known $n$-by- $p$ matrix.

Given a parameter vector $\mathbf{b}$ and a collection of design matrices, the collection of $m$ independent variables $\mathbf{Z}_{k}$ is assumed to have independent identically distributed multivariate normal residual errors $\mathbf{Z}_{k}-\mathbf{H}_{k} \mathbf{b}$ with $n$-vector mean $\mathbf{0}$ and n -by-n covariance matrix $\mathbf{C}$ for each $k=1, \ldots, m$.

A concise way to write this model is

$$
Z_{k} \square N\left(H_{k} b, C\right)
$$

for $k=1, \ldots, m$.
The goal of multivariate normal regression is to obtain maximum likelihood estimates for $\mathbf{b}$ and $\mathbf{C}$ given a collection of $m$ observations $\mathbf{z}_{1}, \ldots, \mathbf{z}_{m}$ of the random variables $\mathbf{Z}_{1}, \ldots, \mathbf{Z}_{m}$. The estimated parameters are the $p$ distinct elements of $\mathbf{b}$ and the $n(n+1) / 2$ distinct elements of $\mathbf{C}$ (the lower-triangular elements of $\mathbf{C}$ ).

Note Quasi-maximum likelihood estimation works with the same models but with a relaxation of the assumption of normally distributed residuals. In this case, however, the parameter estimates are asymptotically optimal.

## Maximum Likelihood Estimation

To estimate the parameters of the multivariate normal linear regression model using maximum likelihood estimation, it is necessary to maximize the $\log$-likelihood function over the estimation parameters given observations $\mathbf{z}_{1}$, $\ldots, \mathbf{z}_{m}$.

Given the multivariate normal model to characterize residual errors in the regression model, the log-likelihood function is

$$
\begin{aligned}
L\left(z_{1}, \ldots, z_{m} ; b, C\right)= & \frac{1}{2} m n \log (2 \pi)+\frac{1}{2} m \log (\operatorname{det}(C)) \\
& +\frac{1}{2} \sum_{k=1}^{m}\left(z_{k}-H_{k} b\right)^{T} C^{-1}\left(z_{k}-H_{k} b\right) .
\end{aligned}
$$

Although the cross-sectional residuals must be independent, you can use this log-likelihood function for quasi-maximum likelihood estimation. In this case, the estimates for the parameters $\mathbf{b}$ and $\mathbf{C}$ provide estimates to characterize the first and second moments of the residuals. See Caines [1] for details.

Except for a special case (see "Special Case of a Multiple Linear Regression Model" on page 7-5), if both the model parameters in $\mathbf{b}$ and the covariance parameters in $\mathbf{C}$ are to be estimated, the estimation problem is intractably nonlinear and a solution must use iterative methods. Denote estimates for the parameters $\mathbf{b}$ and $\mathbf{C}$ for iteration $t=0,1, \ldots$ with the superscript notation $\mathbf{b}^{(t)}$ and $\mathbf{C}^{(t)}$.

Given initial estimates $\mathbf{b}^{(0)}$ and $\mathbf{C}^{(0)}$ for the parameters, the maximum likelihood estimates for $\mathbf{b}$ and $\mathbf{C}$ are obtained using a two-stage iterative process with

$$
b^{(t+1)}=\left(\sum_{k=1}^{m} H_{k}^{T}\left(C^{(t)}\right)^{-1} H_{k}\right)^{-1}\left(\sum_{k=1}^{m} H_{k}^{T}\left(C^{(t)}\right)^{-1} z_{k}\right)
$$

and

$$
C^{(t+1)}=\frac{1}{m} \sum_{k=1}^{m}\left(z_{k}-H_{k} b^{(t+1)}\right)\left(z_{k}-H_{k} b^{(t+1)}\right)^{T}
$$

for $t=0,1, \ldots$.

## Special Case of a Multiple Linear Regression Model

The special case mentioned in "Maximum Likelihood Estimation" on page 7-4 occurs if $n=1$ so that the sequence of observations is a sequence of scalar observations. This model is known as a multiple linear regression model. In this case, the covariance matrix $\mathbf{C}$ is a 1 -by- 1 matrix that drops out of the maximum likelihood iterates so that a single-step estimate for $\mathbf{b}$ and $\mathbf{C}$ can be obtained with converged estimates $\mathbf{b}^{(1)}$ and $\mathbf{C}^{(1)}$.

## Least-Squares Regression

Another simplification of the general model is called least-squares regression. If $\mathbf{b}^{(0)}=\mathbf{0}$ and $\mathbf{C}^{(0)}=\mathbf{I}$, then $\mathbf{b}^{(1)}$ and $\mathbf{C}^{(1)}$ from the two-stage iterative process are least-squares estimates for $\mathbf{b}$ and $\mathbf{C}$, where

$$
b^{L S}=\left(\sum_{k=1}^{m} H_{k}^{T} H_{k}\right)^{-1}\left(\sum_{k=1}^{m} H_{k}^{T} z_{k}\right)
$$

and

$$
C^{L S}=\frac{1}{m} \sum_{k=1}^{m}\left(z_{k}-H_{k} b^{L S}\right)\left(z_{k}-H_{k} b^{L S}\right)^{T} .
$$

## Mean and Covariance Estimation

A final simplification of the general model is to estimate the mean and covariance of a sequence of $n$-dimensional observations $\mathbf{z}_{1}, \ldots, \mathbf{z}_{m}$. In this case, the number of series is equal to the number of model parameters with $n=p$ and the design matrices are identity matrices with $\mathbf{H}_{k}=\mathbf{I}$ for $i=1, \ldots, m$ so that $\mathbf{b}$ is an estimate for the mean and $\mathbf{C}$ is an estimate of the covariance of the collection of observations $\mathbf{z}_{1}, \ldots, \mathbf{z}_{m}$.

## Convergence

If the iterative process continues until the log-likelihood function increases by no more than a specified amount, the resultant estimates are said to be maximum likelihood estimates $\mathbf{b}^{M L}$ and $\mathbf{C}^{M L}$.

Note that if $n=1$ (which implies a single data series), convergence occurs after only one iterative step, which, in turn, implies that the least-squares and maximum likelihood estimates are identical. If, however, $n>1$, the least-squares and maximum likelihood estimates are usually distinct.

In Financial Toolbox software, both the changes in the log-likelihood function and the norm of the change in parameter estimates are monitored. Whenever both changes fall below specified tolerances (which should be something between machine precision and its square root), the toolbox functions terminate under an assumption that convergence has been achieved.

## Fisher Information

Since maximum likelihood estimates are formed from samples of random variables, their estimators are random variables; an estimate derived from such samples has an uncertainty associated with it. To characterize these uncertainties, which are called standard errors, two quantities are derived from the total log-likelihood function.

The Hessian of the total log-likelihood function is

$$
\nabla^{2} L\left(z_{1}, \ldots, z_{m} ; \theta\right)
$$

and the Fisher information matrix is

$$
I(\theta)=-E\left[\nabla^{2} L\left(z_{1}, \ldots, z_{m} ; \theta\right)\right]
$$

where the partial derivatives of the $\nabla^{2}$ operator are taken with respect to the combined parameter vector $\Theta$ that contains the distinct components of $\mathbf{b}$ and $\mathbf{C}$ with a total of $q=p+n(n+1) / 2$ parameters.

Since maximum likelihood estimation is concerned with large-sample estimates, the central limit theorem applies to the estimates and the Fisher
information matrix plays a key role in the sampling distribution of the parameter estimates. Specifically, maximum likelihood parameter estimates are asymptotically normally distributed such that

$$
\left(\theta^{(t)}-\theta\right)_{\square N}\left(0, I^{-1},\left(\theta^{(t)}\right)\right) \text { as } t \rightarrow \infty,
$$

where $\Theta$ is the combined parameter vector and $\Theta^{(t)}$ is the estimate for the combined parameter vector at iteration $t=0,1, \ldots$.

The Fisher information matrix provides a lower bound, called a Cramér-Rao lower bound, for the standard errors of estimates of the model parameters.

## Statistical Tests

Given an estimate for the combined parameter vector $\Theta$, the squared standard errors are the diagonal elements of the inverse of the Fisher information matrix

$$
s^{2}\left(\hat{\theta}_{i}\right)=\left(I^{-1}\left(\hat{\theta}_{i}\right)\right)_{i i}
$$

for $i=1, \ldots, q$.
Since the standard errors are estimates for the standard deviations of the parameter estimates, you can construct confidence intervals so that, for example, a $95 \%$ interval for each parameter estimate is approximately

$$
\hat{\theta}_{i} \pm 1.96 s\left(\hat{\theta}_{i}\right)
$$

for $i=1, \ldots, q$.
Error ellipses at a level-of-significance $\alpha \varepsilon[0,1$ ] for the parameter estimates satisfy the inequality

$$
(\theta-\hat{\theta})^{T} I(\hat{\theta})(\theta-\hat{\theta}) \leq \chi_{1-\alpha, q}^{2}
$$

and follow a $\chi^{2}$ distribution with $q$ degrees-of-freedom. Note that similar inequalities can be formed for any subcollection of the parameters.

In general, given parameter estimates, the computed Fisher information matrix, and the log-likelihood function, you can perform numerous statistical tests on the parameters, the model, and the regression.

## Maximum Likelihood Estimation with Missing Data

In this section...<br>"Introduction" on page 7-9<br>"ECM Algorithm" on page 7-10<br>"Standard Errors" on page 7-10<br>"Data Augmentation" on page 7-11<br>"Multivariate Normal Regression Functions" on page 7-12<br>"Multivariate Normal Regression Without Missing Data" on page 7-14<br>"Multivariate Normal Regression With Missing Data" on page 7-14<br>"Least-Squares Regression with Missing Data" on page 7-15<br>"Multivariate Normal Parameter Estimation with Missing Data" on page 7-15

"Support Functions" on page 7-16

## Introduction

Suppose that a portion of the sample data is missing, where missing values are represented as NaNs. If the missing values are missing-at-random and ignorable, where Little and Rubin [7] have precise definitions for these terms, it is possible to use a version of the Expectation Maximization, or EM, algorithm of Dempster, Laird, and Rubin [3] to estimate the parameters of the multivariate normal regression model. The algorithm used in Financial Toolbox software is the ECM (Expectation Conditional Maximization) algorithm of Meng and Rubin [8] with enhancements by Sexton and Swensen [9].

Each sample $\mathbf{z}_{k}$ for $k=1, \ldots, m$, is either complete with no missing values, empty with no observed values, or incomplete with both observed and missing values. Empty samples are ignored since they contribute no information.

To understand the missing-at-random and ignorabable conditions, consider an example of stock price data before an IPO. For a counterexample, censored data, in which all values greater than some cutoff are replaced with NaNs, does not satisfy these conditions.

In sample $k$, let $\mathbf{x}_{k}$ represent the missing values in $\mathbf{z}_{k}$, and $\mathbf{y}_{k}$ represent the observed values. Define a permutation matrix $\mathbf{P}_{\mathrm{k}}$ so that

$$
z_{k}=P_{k}\left[\begin{array}{l}
x_{k} \\
y_{k}
\end{array}\right]
$$

for $k=1, \ldots, m$.

## ECM Algorithm

The ECM algorithm has two steps - an E, or expectation step, and a CM, or conditional maximization, step. As with maximum likelihood estimation, the parameter estimates evolve according to an iterative process, where estimates for the parameters after $t$ iterations are denoted as $\mathbf{b}^{(t)}$ and $\mathbf{C}^{(t)}$.

The E step forms conditional expectations for the elements of missing data with

$$
\begin{aligned}
& E\left[X_{k} \mid Y_{k}=y_{k} ; b^{(t)}, C^{(t)}\right] \\
& \operatorname{cov}\left[X_{k} \mid Y_{k}=y_{k} ; b^{(t)}, C^{(t)}\right]
\end{aligned}
$$

for each sample $k \in\{1, \ldots, m\}$ that has missing data.
The CM step proceeds in the same manner as the maximum likelihood procedure without missing data. The main difference is that missing data moments are imputed from the conditional expectations obtained in the E step.

The E and CM steps are repeated until the log-likelihood function ceases to increase. One of the important properties of the ECM algorithm is that it is always guaranteed to find a maximum of the log-likelihood function and, under suitable conditions, this maximum can be a global maximum.

## Standard Errors

The negative of the expected Hessian of the log-likelihood function and the Fisher information matrix are identical if no data is missing. However, if
data is missing, the Hessian, which is computed over available samples, accounts for the loss of information due to missing data. Consequently, the Fisher information matrix provides standard errors that are a Cramér-Rao lower bound whereas the Hessian matrix provides standard errors that may be greater if there is missing data.

## Data Augmentation

The ECM functions do not "fill in" missing values as they estimate model parameters. In some cases, you may want to fill in the missing values. Although you can fill in the missing values in your data with conditional expectations, you would get optimistic and unrealistic estimates because conditional estimates are not random realizations.

Several approaches are possible, including resampling methods and multiple imputation (see Little and Rubin [7] and Shafer [10] for details). A somewhat informal sampling method for data augmentation is to form random samples for missing values based on the conditional distribution for the missing values. Given parameter estimates for $X \subset R^{n}$ and $\hat{C}$, each observation has moments

$$
E\left[Z_{k}\right]=H_{k} \hat{b}
$$

and

$$
\operatorname{cov}\left(Z_{k}\right)=H_{k} \hat{C} H_{k}^{T}
$$

for $k=1, \ldots, m$, where you have dropped the parameter dependence on the left sides for notational convenience.

For observations with missing values partitioned into missing values $\mathbf{X}_{k}$ and observed values $\mathbf{Y}_{k}=\mathbf{y}_{k}$, you can form conditional estimates for any subcollection of random variables within a given observation. Thus, given estimates $\boldsymbol{E}\left[\mathbf{Z}_{k}\right]$ and $\operatorname{cov}\left(\mathbf{Z}_{k}\right)$ based on the parameter estimates, you can create conditional estimates

$$
E\left[X_{k} \mid y_{k}\right]
$$

and

$$
\operatorname{cov}\left(X_{k} \mid y_{k}\right)
$$

using standard multivariate normal distribution theory. Given these conditional estimates, you can simulate random samples for the missing values from the conditional distribution

$$
X_{k} \square N\left(E\left[X_{k} \mid y_{k}\right], \operatorname{cov}\left(X_{k} \mid y_{k}\right)\right) .
$$

The samples from this distribution reflect the pattern of missing and nonmissing values for observations $k=1, \ldots, m$. You must sample from conditional distributions for each observation to preserve the correlation structure with the nonmissing values at each observation.

If you follow this procedure, the resultant filled-in values are random and generate mean and covariance estimates that are asymptotically equivalent to the ECM-derived mean and covariance estimates. Note, however, that the filled-in values are random and reflect likely samples from the distribution estimated over all the data and may not reflect "true" values for a particular observation.

## Multivariate Normal Regression Functions

Financial Toolbox software has a number of functions for multivariate normal regression with or without missing data. The toolbox functions solve four classes of regression problems with functions to estimate parameters, standard errors, log-likelihood functions, and Fisher information matrices. The four classes of regression problems are:

- "Multivariate Normal Regression Without Missing Data" on page 7-14
- "Multivariate Normal Regression With Missing Data" on page 7-14
- "Least-Squares Regression with Missing Data" on page 7-15
- "Multivariate Normal Parameter Estimation with Missing Data" on page 7-15

Additional support functions are also provided, see "Support Functions" on page 7-16.

In all functions, the MATLAB representation for the number of observations (or samples) is NumSamples $=m$, the number of data series is NumSeries $=$ $n$, and the number of model parameters is NumParams $=p$. Note that the moment estimation functions have NumSeries = NumParams.

The collection of observations (or samples) is stored in a MATLAB matrix Data such that

$$
\operatorname{Data}(\mathrm{k},:)=z_{k}^{T}
$$

for $\mathrm{k}=1$, ..., NumSamples, where Data is a NumSamples-by-NumSeries matrix.

For the multivariate normal regression or least-squares functions, an additional required input is the collection of design matrices that is stored as either a MATLAB matrix or a vector of cell arrays denoted as Design.

If Numseries = 1, Design can be a NumSamples-by-NumParams matrix. This is the "standard" form for regression on a single data series.

If Numseries = 1, Design can be either a cell array with a single cell or a cell array with NumSamples cells. Each cell in the cell array contains a NumSeries-by-NumParams matrix such that

$$
\operatorname{Design}\{\mathrm{k}\}=H_{k}
$$

for $k=1, \ldots$, NumSamples. If Design has a single cell, it is assumed to be the same Design matrix for each sample such that

$$
\operatorname{Design}\{1\}=H_{1}=\ldots=H_{m} .
$$

Otherwise, Design must contain individual design matrices for each and every sample.

The main distinction among the four classes of regression problems depends upon how missing values are handled and where missing values are represented as the MATLAB value NaN . If a sample is to be ignored given any missing values in the sample, the problem is said to be a problem "without missing data." If a sample is to be ignored if and only if every element of the sample is missing, the problem is said to be a problem "with missing data" since the estimation must account for possible NaN values in the data.

In general, Data may or may not have missing values and Design should have no missing values. In some cases, however, if an observation in Data is to be ignored, the corresponding elements in Design are also ignored. Consult the function reference pages for details.

## Multivariate Normal Regression Without Missing Data

You can use the following functions for multivariate normal regression without missing data.

| mvnrmle | Estimate model parameters, residuals, and the <br> residual covariance. |
| :--- | :--- |
| mvnrstd | Estimate standard errors of model and <br> covariance parameters. |
| mvnrfish | Estimate the Fisher information matrix. |
| mvnrobj | Calculate the log-likelihood function. |

The first two functions are the main estimation functions. The second two are supporting functions that can be used for more detailed analyses.

## Multivariate Normal Regression With Missing Data

You can use the following functions for multivariate normal regression with missing data.

| ecmmvnrmle | Estimate model parameters, residuals, and the <br> residual covariance. |
| :--- | :--- |
| ecmmvnrstd | Estimate standard errors of model and <br> covariance parameters. |
| ecmmvnrfish | Estimate the Fisher information matrix. |
| ecmmvnrobj | Calculate the log-likelihood function. |

The first two functions are the main estimation functions. The second two are supporting functions used for more detailed analyses.

## Least-Squares Regression with Missing Data

You can use the following functions for least-squares regression with missing data or for covariance-weighted least-squares regression with a fixed covariance matrix.

| ecmlsrmle | Estimate model parameters, residuals, and the <br> residual covariance. |
| :--- | :--- |
| ecmlsrobj | Calculate the least-squares objective function <br> (pseudo log-likelihood). |

To compute standard errors and estimates for the Fisher information matrix, the multivariate normal regression functions with missing data are used.

| ecmmvnrstd | Estimate standard errors of model and <br> covariance parameters. |
| :--- | :--- |
| ecmmvnrfish | Estimate the Fisher information matrix. |

## Multivariate Normal Parameter Estimation with Missing Data

You can use the following functions to estimate the mean and covariance of multivariate normal data.

| ecmnmle | Estimate the mean and covariance of the data. |
| :--- | :--- |
| ecmnstd | Estimate standard errors of the mean and <br> covariance of the data. |
| ecmnfish | Estimate the Fisher information matrix. |
| ecmnhess | Estimate the Fisher information matrix using <br> the Hessian. |
| ecmnobj | Calculate the log-likelihood function. |

These functions behave slightly differently from the more general regression functions since they solve a specialized problem. Consult the function reference pages for details.

## Support Functions

Two support functions are included.

| convert2sur | Convert a multivariate normal regression model <br> into an SUR model. |
| :--- | :--- |
| ecmninit | Obtain initial estimates for the mean and <br> covariance of a Data matrix. |

The convert2sur function converts a multivariate normal regression model into a seemingly unrelated regression, or SUR, model. The second function ecmninit is a specialized function to obtain initial ad hoc estimates for the mean and covariance of a Data matrix with missing data. (If there are no missing values, the estimates are the maximum likelihood estimates for the mean and covariance.)

## Multivariate Normal Regression Types

In this section...<br>"Regressions" on page 7-17<br>"Multivariate Normal Regression" on page 7-17<br>"Least-Squares Regression" on page 7-18<br>"Covariance-Weighted Least Squares" on page 7-19<br>"Feasible Generalized Least Squares" on page 7-20<br>"Seemingly Unrelated Regression" on page 7-21<br>"Mean and Covariance Parameter Estimation" on page 7-23<br>"Troubleshooting Multivariate Normal Regression" on page 7-23<br>"Slow Convergence" on page 7-24<br>"Nonrandom Residuals" on page 7-24<br>"Nonconvergence" on page 7-25<br>"Example of Portfolios with Missing Data" on page 7-26

## Regressions

Each regression function has a specific operation. This section shows how to use these functions to perform specific types of regressions. To illustrate use of the functions for various regressions, "typical" usage is shown with optional arguments kept to a minimum. For a typical regression, you estimate model parameters and residual covariance matrices with the mle functions and estimate the standard errors of model parameters with the std functions. The regressions "without missing data" essentially ignore samples with any missing values, and the regressions "with missing data" ignore samples with every value missing.

## Multivariate Normal Regression

Multivariate normal regression, or MVNR, is the "standard" implementation of the regression functions in Financial Toolbox software.

## Multivariate Normal Regression Without Missing Data

Estimate Parameters

```
[Parameters, Covariance] = mvnrmle(Data, Design);
```

Estimate Standard Errors

```
StdParameters = mvnrstd(Data, Design, Covariance);
```


## Multivariate Normal Regression with Missing Data

Estimate Parameters

```
[Parameters, Covariance] = ecmmvnrmle(Data, Design);
```


## Estimate Standard Errors

```
StdParameters = ecmmvnrstd(Data, Design, Covariance);
```


## Least-Squares Regression

Least-squares regression, or LSR, sometimes called ordinary least-squares or multiple linear regression, is the simplest linear regression model. It also enjoys the property that, independent of the underlying distribution, it is a best linear unbiased estimator (BLUE).

Given $m=$ NumSamples observations, the typical least-squares regression model seeks to minimize the objective function

$$
\sum_{k=1}^{m}\left(Z_{k}-H_{k} b\right)^{T}\left(Z_{k}-H_{k} b\right),
$$

which, within the maximum likelihood framework of the multivariate normal regression routine mvnrmle, is equivalent to a single-iteration estimation of just the parameters to obtain Parameters with the initial covariance matrix Covariance held fixed as the identity matrix. In the case of missing data, however, the internal algorithm to handle missing data requires a separate routine ecmlsrmle to do least-squares instead of multivariate normal regression.

## Least-Squares Regression Without Missing Data

Estimate Parameters

```
[Parameters, Covariance] = mvnrmle(Data, Design, 1);
```

Estimate Standard Errors

```
StdParameters = mvnrstd(Data, Design, Covariance);
```


## Least-Squares Regression with Missing Data

Estimate Parameters

```
[Parameters, Covariance] = ecmlsrmle(Data, Design);
```

Estimate Standard Errors

```
StdParameters = ecmmvnrstd(Data, Design, Covariance);
```


## Covariance-Weighted Least Squares

Given $m=$ NUMSAMPLES observations, the typical covariance-weighted least squares, or CWLS, regression model seeks to minimize the objective function

$$
\sum_{k=1}^{m}\left(Z_{k}-H_{k} b\right)^{T} C_{0}\left(Z_{k}-H_{k} b\right)
$$

with fixed covariance $C_{0}$.

In most cases, $C_{0}$ is a diagonal matrix. The inverse matrix $W=C_{0}^{-1}$ has diagonal elements that can be considered relative "weights" for each series. Thus, CWLS is a form of weighted least squares with the weights applied across series.

## Covariance-Weighted Least Squares Without Missing Data

Estimate Parameters

```
[Parameters, Covariance] = mvnrmle(Data, Design, 1, [], [], [],
    Covar0);
```

Estimate Standard Errors

```
StdParameters = mvnrstd(Data, Design, Covariance);
```


## Covariance-Weighted Least Squares with Missing Data

Estimate Parameters

```
[Parameters, Covariance] = ecmlsrmle(Data, Design, [], [], [], [],
                                    Covar0);
```

Estimate Standard Errors

```
StdParameters = ecmmvnrstd(Data, Design, Covariance);
```


## Feasible Generalized Least Squares

An ad hoc form of least squares that has surprisingly good properties for misspecified or nonnormal models is known as feasible generalized least squares, or FGLS. The basic procedure is to do least-squares regression and then to do covariance-weighted least-squares regression with the resultant residual covariance from the first regression.

```
Feasible Generalized Least Squares Without Missing Data
Estimate Parameters
    [Parameters, Covariance] = mvnrmle(Data, Design, 2, 0, 0);
or (to illustrate the FGLS process explicitly)
    [Parameters, Covar0] = mvnrmle(Data, Design, 1);
    [Parameters, Covariance] = mvnrmle(Data, Design, 1, [], [], [],
    Covar0);
```

Estimate Standard Errors
StdParameters = mvnrstd(Data, Design, Covariance);

## Feasible Generalized Least Squares with Missing Data

Estimate Parameters

```
[Parameters, Covar0] = ecmlsrmle(Data, Design);
[Parameters, Covariance] = ecmlsrmle(Data, Design, [], [], [], [],
    CovarO);
```

Estimate Standard Errors

```
StdParameters = ecmmvnrstd(Data, Design, Covariance);
```


## Seemingly Unrelated Regression

Given a multivariate normal regression model in standard form with a Data matrix and a Design array, it is possible to convert the problem into a seemingly unrelated regression (SUR) problem by a simple transformation of the Design array. The main idea of SUR is that instead of having a common parameter vector over all data series, you have a separate parameter vector associated with each separate series or with distinct groups of series that, nevertheless, share a common residual covariance. It is this ability to aggregate and disaggregate series and to perform comparative tests on each design that is the power of SUR.

To make the transformation, use the function convert2sur, which converts a standard-form design array into an equivalent design array to do SUR with a specified mapping of the series into NUMGROUPS groups. The regression functions are used in the usual manner, but with the SUR design array instead of the original design array. Instead of having NUMPARAMS elements, the SUR output parameter vector has NUMGROUPS of stacked parameter estimates, where the first NUMPARAMS elements of Parameters contain parameter estimates associated with the first group of series, the next NUMPARAMS elements of Parameters contain parameter estimates associated with the second group of series, and so on. If the model has only one series, for example, NUMSERIES $=1$, then the SUR design array is the same as the original design array since SUR requires two or more series to generate distinct parameter estimates.

Given NUMPARAMS parameters and NUMGROUPS groups with a parameter vector Parameters with NUMGROUPS * NUMPARAMS elements from any of the regression routines, the following MATLAB code fragment shows how to print a table of SUR parameter estimates with rows that correspond to each parameter and columns that correspond to each group or series:

[^0]```
    Estimates\n');
fprintf(1,' %7s ',' ');
fprintf(1,' Group(%3d) ',1:NumGroups);
fprintf(1,'\n');
for i = 1:NumParams
    fprintf(1,' %7d ',i);
    ii = i;
    for j = 1:NumGroups
        fprintf(1,'%12g ',Param(ii));
        ii = ii + NumParams;
        end
        fprintf(1,'\n');
end
fprintf(1,'\n');
```


## Seemingly Unrelated Regression Without Missing Data

Form an SUR Design

```
DesignSUR = convert2sur(Design, Group);
```

Estimate Parameters
[Parameters, Covariance] = mvnrmle(Data, DesignSUR);
Estimate Standard Errors

```
StdParameters = mvnrstd(Data, DesignSUR, Covariance);
```


## Seemingly Unrelated Regression with Missing Data

Form an SUR Design

```
DesignSUR = convert2sur(Design, Group);
```

Estimate Parameters
[Parameters, Covariance] = ecmmvnrmle(Data, DesignSUR);
Estimate Standard Errors

```
StdParameters = ecmmvnrstd(Data, DesignSUR, Covariance);
```


## Mean and Covariance Parameter Estimation

Without missing data, you can estimate the mean of your Data with the function mean and the covariance with the function cov. Nevertheless, the function ecmnmle does this for you if it detects an absence of missing values. Otherwise, it uses the ECM algorithm to handle missing values.

Estimate Parameters
[Mean, Covariance] = ecmnmle(Data);
Estimate Standard Errors

```
StdMean = ecmnstd(Data, Mean, Covariance);
```


## Troubleshooting Multivariate Normal Regression

This section provides a few pointers to handle various technical and operational difficulties that might occur.

## Biased Estimates

If samples are ignored, the number of samples used in the estimation is less than NumSamples. Clearly the actual number of samples used must be sufficient to obtain estimates. In addition, although the model parameters Parameters (or mean estimates Mean) are unbiased maximum likelihood estimates, the residual covariance estimate Covariance is biased. To convert to an unbiased covariance estimate, multiply Covariance by
Count/(Count-1),
where Count is the actual number of samples used in the estimation with Count $\leq$ NumSamples. Note that none of the regression functions perform this adjustment.

## Requirements

The regression functions, particularly the estimation functions, have several requirements. First, they must have consistent values for NumSamples, NumSeries, and NumParams. As a general rule, the multivariate normal regression functions require

$$
\text { Count } \times \text { NumSeries } \leq \max \{\text { NumParams, NumSeries } \times(\text { NumSeries }+1) / 2\}
$$

and the least-squares regression functions require
Count $\times$ NumSeries $\leq$ NumParams,
where Count is the actual number of samples used in the estimation with
Count $\leq$ NumSamples.

Second, they must have enough nonmissing values to converge. Third, they must have a nondegenerate covariance matrix.

Although some necessary and sufficient conditions can be found in the references, general conditions for existence and uniqueness of solutions in the missing-data case do not exist. Nonconvergence is usually due to an ill-conditioned covariance matrix estimate, which is discussed in greater detail in "Nonconvergence" on page 7-25.

## Slow Convergence

Since worst-case convergence of the ECM algorithm is linear, it is possible to execute hundreds and even thousands of iterations before termination of the algorithm. If you are estimating with the ECM algorithm on a regular basis with regular updates, you can use prior estimates as initial guesses for the next period's estimation. This approach often speeds things up since the default initialization in the regression functions sets the initial parameters $\mathbf{b}$ to zero and the initial covariance $\mathbf{C}$ to be the identity matrix.

Other ad hoc approaches are possible although most approaches are problem-dependent. In particular, for mean and covariance estimation, the estimation function ecmnmle uses a function ecmninit to obtain an initial estimate.

## Nonrandom Residuals

Simultaneous estimates for parameters $\mathbf{b}$ and covariances $\mathbf{C}$ require $\mathbf{C}$ to be positive-definite. Consequently, the general multivariate normal regression
routines require nondegenerate residual errors. If you are faced with a model that has exact results, the least-squares routine ecmlsrmle still works, although it provides a least-squares estimate with a singular residual covariance matrix. The other regression functions will fail.

## Nonconvergence

Although the regression functions are robust and work for most "typical" cases, they can fail to converge. The main failure mode is an ill-conditioned covariance matrix, where failures are either soft or hard. A soft failure wanders endlessly toward a nearly singular covariance matrix and can be spotted if the algorithm fails to converge after about 100 iterations. If MaxIterations is increased to 500 and display mode is initiated (with no output arguments), a typical soft failure looks like this.


This case, which is based on 20 observations of 5 assets with $30 \%$ of data missing, shows that the log-likelihood goes linearly to infinity as the likelihood
function goes to 0 . In this case, the function converges but the covariance matrix is effectively singular with a smallest eigenvalue on the order of machine precision (eps).

For the function ecmnmle, a hard error looks like this:

```
> In ecmninit at 60
    In ecmnmle at 140
??? Error using ==> ecmnmle
Full covariance not positive-definite in iteration 218.
```

From a practical standpoint, if in doubt, test your residual covariance matrix from the regression routines to ensure that it is positive-definite. This is important because a soft error has a matrix that appears to be positive-definite but actually has a near-zero-valued eigenvalue to within machine precision. To do this with a covariance estimate Covariance, use cond (Covariance), where any value greater than $1 / \mathrm{eps}$ should be considered suspect.

If either type of failure occurs, however, note that the regression routine is indicating that something is probably wrong with the data. (Even with no missing data, two time series that are proportional to one another produce a singular covariance matrix.)

## Example of Portfolios with Missing Data

This example illustrates how to use the missing data algorithms for portfolio optimization and for valuation. This example works with 5 years of daily total return data for 12 computer technology stocks, with 6 hardware and 6 software companies. The example estimates the mean and covariance matrix for these stocks, forms efficient frontiers with both a naïve approach and the ECM approach, and compares results.

You can run the example directly with ecmtechdemo.m.
1 Load the following data file:

```
load ecmtechdemo
```

This file contains these three quantities:

- Assets is a cell array of the tickers for the twelve stocks in the example.
- Data is a 1254 -by- 12 matrix of 1254 daily total returns for each of the 12 stocks.
- Dates is a 1254-by-1 column vector of the dates associated with the data.

The time period for the data extends from April 19, 2000 to April 18, 2005.
The sixth stock in Assets is Google (GOOG), which started trading on August 19, 2004. Consequently, all returns before August 20, 2004 are missing and represented as NaNs. Also, Amazon (AMZN) had a few days with missing values scattered throughout the past 5 years.

2 A naïve approach to the estimation of the mean and covariance for these 12 assets is to eliminate all days that have missing values for any of the 12 assets. Use the function ecmninit with the nanskip option to do this.
[NaNMean, NaNCovar] = ecmninit(Data,'nanskip');
3 Contrast the result of this approach with using all available data and the function ecmnmle to compute the mean and covariance. First, call ecmnmle with no output arguments to establish that enough data is available to obtain meaningful estimates.

```
ecmnmle(Data);
```

The following figure shows that, even with almost $87 \%$ of the Google data being NaN values, the algorithm converges after only four iterations.


4 Estimate the mean and covariance as computed by ecmnmle.

```
>> [ECMMean, ECMCovar] = ecmnmle(Data)
```

ECMMean =

$$
\begin{array}{r}
0.0008 \\
0.0008 \\
-0.0005 \\
0.0002 \\
0.0011 \\
0.0038 \\
-0.0003 \\
-0.0000 \\
-0.0003 \\
-0.0000 \\
-0.0003
\end{array}
$$

### 0.0004

ECMCovar =

| 0.0012 | 0.0005 | 0.0006 | 0.0005 | 0.0005 | 0.0003 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0005 | 0.0024 | 0.0007 | 0.0006 | 0.0010 | 0.0004 |
| 0.0006 | 0.0007 | 0.0013 | 0.0007 | 0.0007 | 0.0003 |
| 0.0005 | 0.0006 | 0.0007 | 0.0009 | 0.0006 | 0.0002 |
| 0.0005 | 0.0010 | 0.0007 | 0.0006 | 0.0016 | 0.0006 |
| 0.0003 | 0.0004 | 0.0003 | 0.0002 | 0.0006 | 0.0022 |
| 0.0005 | 0.0005 | 0.0006 | 0.0005 | 0.0005 | 0.0001 |
| 0.0003 | 0.0003 | 0.0004 | 0.0003 | 0.0003 | 0.0002 |
| 0.0006 | 0.0006 | 0.0008 | 0.0007 | 0.0006 | 0.0002 |
| 0.0003 | 0.0004 | 0.0005 | 0.0004 | 0.0004 | 0.0001 |
| 0.0005 | 0.0006 | 0.0008 | 0.0005 | 0.0007 | 0.0003 |
| 0.0006 | 0.0012 | 0.0008 | 0.0007 | 0.0011 | 0.0016 |

ECMCovar (continued)

| 0.0005 | 0.0003 | 0.0006 | 0.0003 | 0.0005 | 0.0006 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0005 | 0.0003 | 0.0006 | 0.0004 | 0.0006 | 0.0012 |
| 0.0006 | 0.0004 | 0.0008 | 0.0005 | 0.0008 | 0.0008 |
| 0.0005 | 0.0003 | 0.0007 | 0.0004 | 0.0005 | 0.0007 |
| 0.0005 | 0.0003 | 0.0006 | 0.0004 | 0.0007 | 0.0011 |
| 0.0001 | 0.0002 | 0.0002 | 0.0001 | 0.0003 | 0.0016 |
| 0.0009 | 0.0003 | 0.0005 | 0.0004 | 0.0005 | 0.0006 |
| 0.0003 | 0.0005 | 0.0004 | 0.0003 | 0.0004 | 0.0004 |
| 0.0005 | 0.0004 | 0.0011 | 0.0005 | 0.0007 | 0.0007 |
| 0.0004 | 0.0003 | 0.0005 | 0.0006 | 0.0004 | 0.0005 |
| 0.0005 | 0.0004 | 0.0007 | 0.0004 | 0.0013 | 0.0007 |
| 0.0006 | 0.0004 | 0.0007 | 0.0005 | 0.0007 | 0.0020 |

5 Given estimates for the mean and covariance of asset returns derived from the naïve and ECM approaches, estimate portfolios, and associated expected returns and risks on the efficient frontier for both approaches.
[ECMRisk, ECMReturn, ECMWts] = portopt(ECMMean',ECMCovar,10); [NaNRisk, NaNReturn, NaNWts] = portopt(NaNMean',NaNCovar,10);

6 Plot the results on the same graph to illustrate the differences.

```
figure(gcf)
plot(ECMRisk,ECMReturn,'-bo','MarkerFaceColor','b','MarkerSize', 3);
hold all
plot(NaNRisk,NaNReturn,'-ro','MarkerFaceColor','r','MarkerSize', 3);
title('\bfMean-Variance Efficient Frontiers under Various Assumptions');
legend('ECM','NaN','Location','SouthEast');
xlabel('\bfStd. Dev. of Returns');
ylabel('\bfMean of Returns');
hold off
```



7 Clearly, the naïve approach is optimistic about the risk-return trade-offs for this universe of 12 technology stocks. The proof, however, lies in the portfolio weights. To view the weights, enter

Assets
ECMWts
NaNWts
which generates

>> NaNWts
ans =

| -0.0000 | 0.0000 | -0.0000 | 0.1185 | 0.0000 | 0.0522 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0.0576 | -0.0000 | -0.0000 | 0.1219 | 0.0000 | 0.0854 |
| 0.1248 | -0.0000 | -0.0000 | 0.0952 | -0.0000 | 0.1195 |
| 0.1969 | -0.0000 | -0.0000 | 0.0529 | -0.0000 | 0.1551 |
| 0.2690 | -0.0000 | -0.0000 | 0.0105 | 0.0000 | 0.1906 |
| 0.3414 | 0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.2265 |
| 0.4235 | 0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.2639 |
| 0.5245 | 0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.3034 |
| 0.6269 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.3425 |
| 1.0000 | -0.0000 | -0.0000 | 0.0000 | -0.0000 | 0 |

Assets (continued)
'HPQ' 'IBM' 'INTC' 'MSFT' 'ORCL' 'YHOO'

ECMWts (continued)

| 0.0535 | 0.4676 | 0.0000 | 0.3431 | -0.0000 | 0.0000 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0.0179 | 0.3899 | -0.0000 | 0.3282 | 0.0000 | -0.0000 |
| 0 | 0.3025 | -0.0000 | 0.3074 | 0.0000 | -0.0000 |
| 0.0000 | 0.2054 | -0.0000 | 0.2806 | 0.0000 | 0.0000 |
| 0.0000 | 0.1083 | -0.0000 | 0.2538 | -0.0000 | 0.0000 |
| 0.0000 | 0.0111 | -0.0000 | 0.2271 | -0.0000 | 0.0000 |
| 0.0000 | 0.0000 | -0.0000 | 0.1255 | -0.0000 | 0.0000 |
| 0.0000 | 0 | -0.0000 | 0.0143 | -0.0000 | -0.0000 |
| 0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.0000 |
| 0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.0000 |

NaNWts (continued)

| 0.0824 | 0.1779 | 0.0000 | 0.5691 | -0.0000 | 0.0000 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 0.1274 | 0.0460 | 0.0000 | 0.5617 | -0.0000 | -0.0000 |
| 0.1674 | -0.0000 | 0.0000 | 0.4802 | 0.0129 | -0.0000 |
| 0.2056 | -0.0000 | 0.0000 | 0.3621 | 0.0274 | -0.0000 |
| 0.2438 | -0.0000 | 0.0000 | 0.2441 | 0.0419 | -0.0000 |
| 0.2782 | -0.0000 | 0.0000 | 0.0988 | 0.0551 | -0.0000 |
| 0.2788 | -0.0000 | 0.0000 | -0.0000 | 0.0337 | -0.0000 |
| 0.1721 | -0.0000 | 0.0000 | -0.0000 | -0.0000 | -0.0000 |
| 0.0306 | -0.0000 | 0.0000 | 0.0000 | 0 | -0.0000 |
| 0 | 0.0000 | 0.0000 | -0.0000 | -0.0000 | -0.0000 |

The naïve portfolios in NaNWts tend to favor Apple Computer (AAPL), which happened to do well over the period from the Google IPO to the end of the estimation period, while the ECM portfolios in ECMWts tend to underweight Apple Computer and to recommend increased weights in Google relative to the naïve weights.

8 To evaluate the impact of estimation error and, in particular, the effect of missing data, use ecmnstd to calculate standard errors. Although it is possible to estimate the standard errors for both the mean and covariance, the standard errors for the mean estimates alone are usually the main quantities of interest.

```
StdMeanF = ecmnstd(Data,ECMMean,ECMCovar,'fisher');
```

9 Calculate standard errors that use the data-generated Hessian matrix (which accounts for the possible loss of information due to missing data) with the option HESSIAN.

```
StdMeanH = ecmnstd(Data,ECMMean,ECMCovar,'hessian');
```

The difference in the standard errors shows the increase in uncertainty of estimation of asset expected returns due to missing data. This can be viewed by entering

Assets
StdMeanH ${ }^{\prime}$
StdMeanF'
StdMeanH' - StdMeanF ${ }^{\prime}$

The two assets with missing data, AMZN and GOOG, are the only assets to have differences due to missing information.

## Valuation with Missing Data

In this section...<br>"Introduction" on page 7-34<br>"Capital Asset Pricing Model" on page 7-34<br>"Estimation of the CAPM" on page 7-35<br>"Estimation with Missing Data" on page 7-36<br>"Estimation of Some Technology Stock Betas" on page 7-36<br>"Grouped Estimation of Some Technology Stock Betas" on page 7-39<br>"References" on page 7-42

## Introduction

The Capital Asset Pricing Model (CAPM) is a venerable but often maligned tool to characterize comovements between asset and market prices. Although many issues arise in CAPM implementation and interpretation, one problem that practitioners face is to estimate the coefficients of the CAPM with incomplete stock price data.

This example shows how to use the missing data regression functions to estimate the coefficients of the CAPM. You can run the example directly using CAPMdemo.m.

## Capital Asset Pricing Model

Given a host of assumptions that can be found in the references (see Sharpe [11], Lintner [6], Jarrow [5], and Sharpe, et. al. [12]), the CAPM concludes that asset returns have a linear relationship with market returns. Specifically, given the return of all stocks that constitute a market denoted as $M$ and the return of a riskless asset denoted as $C$, the CAPM states that the return of each asset $R_{\mathrm{i}}$ in the market has the expectational form

$$
E\left[R_{i}\right]=\alpha_{i}+C+\beta_{i}(E[M]-C)
$$

for assets $i=1, \ldots, n$, where $\beta_{i}$ is a parameter that specifies the degree of comovement between a given asset and the underlying market. In other
words, the expected return of each asset is equal to the return on a riskless asset plus a risk-adjusted expected market return net of riskless asset returns. The collection of parameters $\beta_{1}, \ldots, \beta_{n}$ is called asset betas.

Note that the beta of an asset has the form

$$
\beta_{i}=\frac{\operatorname{cov}\left(R_{i}, M\right)}{\operatorname{var}(M)},
$$

which is the ratio of the covariance between asset and market returns divided by the variance of market returns. If an asset has a beta $=1$, the asset is said to move with the market; if an asset has a beta $>1$, the asset is said to be more volatile than the market. Conversely, if an asset has a beta $<1$, the asset is said to be less volatile than the market.

## Estimation of the CAPM

The standard CAPM model is a linear model with additional parameters for each asset to characterize residual errors. For each of $n$ assets with $m$ samples of observed asset returns $R_{k, i}$, market returns $M_{k}$, and riskless asset returns $C_{k}$, the estimation model has the form

$$
R_{k, i}=\alpha_{i}+C_{k}+\beta_{i}\left(M_{k}-C_{k}\right)+V_{k, i}
$$

for samples $k=1, \ldots, m$ and assets $i=1, \ldots, n$, where $a_{i}$ is a parameter that specifies the nonsystematic return of an asset, $\beta_{i}$ is the asset beta, and $V_{k, i}$ is the residual error for each asset with associated random variable $V_{i}$.

The collection of parameters $a_{1}, \ldots, a_{n}$ are called asset alphas. The strict form of the CAPM specifies that alphas must be zero and that deviations from zero are the result of temporary disequilibria. In practice, however, assets may have nonzero alphas, where much of active investment management is devoted to the search for assets with exploitable nonzero alphas.

To allow for the possibility of nonzero alphas, the estimation model generally seeks to estimate alphas and to perform tests to determine if the alphas are statistically equal to zero.

The residual errors $V_{i}$ are assumed to have moments

$$
E\left[V_{i}\right]=0
$$

and

$$
E\left[V_{i} V_{j}\right]=S_{i j}
$$

for assets $i, j=1, \ldots, n$, where the parameters $S_{11}, \ldots, S_{n n}$ are called residual or nonsystematic variances/covariances.

The square root of the residual variance of each asset, for example, $\operatorname{sqrt}\left(S_{i i}\right)$ for $i=1, \ldots, n$, is said to be the residual or nonsystematic risk of the asset since it characterizes the residual variation in asset prices that are not explained by variations in market prices.

## Estimation with Missing Data

Although betas can be estimated for companies with sufficiently long histories of asset returns, it is difficult to estimate betas for recent IPOs. However, if a collection of sufficiently observable companies exists that can be expected to have some degree of correlation with the new company's stock price movements, that is, companies within the same industry as the new company, it is possible to obtain imputed estimates for new company betas with the missing-data regression routines.

## Estimation of Some Technology Stock Betas

To illustrate how to use the missing-data regression routines, estimate betas for 12 technology stocks, where a single stock (GOOG) is an IPO.

1 Load dates, total returns, and ticker symbols for the 12 stocks from the MAT-file CAPMuniverse.

```
load CAPMuniverse
whos Assets Data Dates
```

| Name | Size | Bytes | Class |
| :--- | ---: | ---: | :--- |
|  |  |  |  |
| Assets | $1 \times 14$ | 952 | cell array |
| Data | $1471 \times 14$ | 164752 | double array |

```
    Dates 1471x1 11768 double array
Grand total is 22135 elements using 177472 bytes
```

The assets in the model have the following symbols, where the last two series are proxies for the market and the riskless asset:

```
Assets(1:7)
Assets(8:14)
ans =
    'AAPL' 'AMZN' 'CSCO' 'DELL' 'EBAY' 'GOOG' 'HPQ'
ans =
    'IBM' 'INTC' 'MSFT' 'ORCL' 'YHOO' 'MARKET' 'CASH'
```

The data covers the period from January 1, 2000 to November 7, 2005 with daily total returns. Two stocks in this universe have missing values that are represented by NaNs. One of the two stocks had an IPO during this period and, consequently, has significantly less data than the other stocks.

2 Compute separate regressions for each stock, where the stocks with missing data will have estimates that reflect their reduced observability.

```
[NumSamples, NumSeries] = size(Data);
NumAssets = NumSeries - 2;
StartDate = Dates(1);
EndDate = Dates(end);
fprintf(1,'Separate regressions with ');
fprintf(1,'daily total return data from %s to %s ...\n', ...
    datestr(StartDate,1), datestr(EndDate,1));
fprintf(1,' %4s %-20s %-20s %-20s\n','','Alpha','Beta','Sigma');
fprintf(1,' ---- -------------------------
fprintf(1,'------------------ --------------------------- ');
for i = 1:NumAssets
```

```
% Set up separate asset data and design matrices
    TestData = zeros(NumSamples,1);
    TestDesign = zeros(NumSamples,2);
    TestData(:) = Data(:,i) - Data(:,14);
    TestDesign(:,1) = 1.0;
    TestDesign(:,2) = Data(:,13) - Data(:,14);
% Estimate CAPM for each asset separately
    [Param, Covar] = ecmmvnrmle(TestData, TestDesign);
    % Estimate ideal standard errors for covariance parameters
    [StdParam, StdCovar] = ecmmvnrstd(TestData, TestDesign, ...
        Covar, 'fisher');
% Estimate sample standard errors for model parameters
    StdParam = ecmmvnrstd(TestData, TestDesign, Covar, 'hessian');
% Set up results for output
    Alpha = Param(1);
    Beta = Param(2);
    Sigma = sqrt(Covar);
    StdAlpha = StdParam(1);
    StdBeta = StdParam(2);
    StdSigma = sqrt(StdCovar);
% Display estimates
    fprintf(' %4s %9.4f (%8.4f) %9.4f (%8.4f) %9.4f (%8.4f)\n', ...
        Assets{i},Alpha(1),abs(Alpha(1)/StdAlpha(1)), ...
        Beta(1),abs(Beta(1)/StdBeta(1)),Sigma(1),StdSigma(1));
end
```

This code fragment generates the following table.

```
Separate regressions with daily total return data from 03-Jan-2000
to 07-Nov-2005 ...
    Alpha Beta Sigma
AAPL 0.0012 ( 1.3882) 1.2294 ( 17.1839) 0.0322 ( 0.0062)
```

| AMZN | 0.0006 | 0.5326) | 1.3661 ( 13.6579) | 0.0449 | $0.0086)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CSCO | -0.0002 | 0.2878) | 1.5653 ( 23.6085) | 0.0298 | $0.0057)$ |
| DELL | -0.0000 | 0.0368) | 1.2594 ( 22.2164) | 0.0255 | 0.0049) |
| EBAY | 0.0014 | 1.4326) | 1.3441 ( 16.0732) | 0.0376 | $0.0072)$ |
| GOOG | 0.0046 | ( 3.2107) | 0.3742 ( 1.7328) | 0.0252 | 0.0071) |
| HPQ | 0.0001 | ( 0.1747) | 1.3745 ( 24.2390) | 0.0255 | 0.0049) |
| IBM | -0.0000 | 0.0312) | 1.0807 ( 28.7576) | 0.0169 | 0.0032) |
| INTC | 0.0001 | ( 0.1608) | 1.6002 ( 27.3684) | 0.0263 | $0.0050)$ |
| MSFT | -0.0002 | 0.4871) | 1.1765 ( 27.4554) | 0.0193 | 0.0037) |
| ORCL | 0.0000 | ( 0.0389) | 1.5010 ( 21.1855) | 0.0319 | 0.0061) |
| YHOO | 0.0001 | ( 0.1282) | 1.6543 ( 19.3838) | 0.0384 | $0.0074)$ |

The Alpha column contains alpha estimates for each stock that are near zero as expected. In addition, the t-statistics (which are enclosed in parentheses) generally reject the hypothesis that the alphas are nonzero at the $99.5 \%$ level of significance.

The Beta column contains beta estimates for each stock that also have t-statistics enclosed in parentheses. For all stocks but GOOG, the hypothesis that the betas are nonzero is accepted at the $99.5 \%$ level of significance. It seems, however, that GOOG does not have enough data to obtain a meaningful estimate for beta since its t-statistic would imply rejection of the hypothesis of a nonzero beta.

The Sigma column contains residual standard deviations, that is, estimates for nonsystematic risks. Instead of t -statistics, the associated standard errors for the residual standard deviations are enclosed in parentheses.

## Grouped Estimation of Some Technology Stock Betas

To estimate stock betas for all 12 stocks, set up a joint regression model that groups all 12 stocks within a single design. (Since each stock has the same design matrix, this model is actually an example of seemingly unrelated regression.) The routine to estimate model parameters is ecmmvnrmle, and the routine to estimate standard errors is ecmmvnrstd.

Because GOOG has a significant number of missing values, a direct use of the missing data routine ecmmvnrmle takes 482 iterations to converge. This can take a long time to compute. For the sake of brevity, the parameter and
covariance estimates after the first 480 iterations are contained in a MAT-file and are used as initial estimates to compute stock betas.

## load CAPMgroupparam <br> whos Param0 Covaro

| Name | Size | Bytes | Class |
| :--- | :--- | ---: | :--- |
| Covar0 | $12 \times 12$ | 1152 | double array |
| Param0 | $24 \times 1$ | 192 | double array |

Grand total is 168 elements using 1344 bytes
Now estimate the parameters for the collection of 12 stocks.

```
fprintf(1,'\n');
fprintf(1,'Grouped regression with ');
fprintf(1,'daily total return data from %s to %S ...\n', ...
        datestr(StartDate,1),datestr(EndDate,1));
fprintf(1,' %4s %-20s %-20s %-20s\n','','Alpha','Beta','Sigma');
fprintf(1,' ---- -------------------- ');
fprintf(1,'------------------- ----------------------\n');
NumParams = 2 * NumAssets;
% Set up grouped asset data and design matrices
TestData = zeros(NumSamples, NumAssets);
TestDesign = cell(NumSamples, 1);
Design = zeros(NumAssets, NumParams);
for k = 1:NumSamples
    for i = 1:NumAssets
            TestData(k,i) = Data(k,i) - Data(k,14);
            Design(i,2*i - 1) = 1.0;
            Design(i,2*i) = Data(k,13) - Data(k,14);
        end
        TestDesign{k} = Design;
end
% Estimate CAPM for all assets together with initial parameter
```

```
% estimates
[Param, Covar] = ecmmvnrmle(TestData, TestDesign, [], [], [],...
    Param0, CovarO);
% Estimate ideal standard errors for covariance parameters
[StdParam, StdCovar] = ecmmvnrstd(TestData, TestDesign, Covar,...
    'fisher');
% Estimate sample standard errors for model parameters
StdParam = ecmmvnrstd(TestData, TestDesign, Covar, 'hessian');
% Set up results for output
Alpha = Param(1:2:end-1);
Beta = Param(2:2:end);
Sigma = sqrt(diag(Covar));
StdAlpha = StdParam(1:2:end-1);
StdBeta = StdParam(2:2:end);
StdSigma = sqrt(diag(StdCovar));
% Display estimates
for i = 1:NumAssets
    fprintf(' %4s %9.4f (%8.4f) %9.4f (%8.4f) %9.4f (%8.4f)\n', ...
    Assets{i},Alpha(i),abs(Alpha(i)/StdAlpha(i)), ...
    Beta(i),abs(Beta(i)/StdBeta(i)),Sigma(i),StdSigma(i));
end
```

This code fragment generates the following table.

Grouped regression with daily total return data from 03-Jan-2000 to 07-Nov-2005 ...

Alpha Beta Sigma

| AAPL | 0.0012 | 1.3882) | 1.2294 | ( 17.1839) | 0.0322 | 0.0062) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AMZN | 0.0007 | $0.6086)$ | 1.3673 | ( 13.6427) | 0.0450 | 0.0086) |
| CSCO | -0.0002 | 0.2878) | 1.5653 | ( 23.6085) | 0.0298 | 0.0057) |
| DELL | -0.0000 | 0.0368) | 1.2594 | ( 22.2164) | 0.0255 | 0.0049) |
| EBAY | 0.0014 | 1.4326) | 1.3441 | ( 16.0732) | 0.0376 | 0.0072) |
| GOOG | 0.0041 | 2.8907) | 0.6173 | ( 3.1100) | 0.0337 | $0.0065)$ |
| HPQ | 0.0001 | 0.1747) | 1.3745 | ( 24.2390) | 0.0255 | 0.004 |


| IBM | $-0.0000($ | $(0.0312)$ | $1.0807(28.7576)$ | $0.0169($ | $0.0032)$ |  |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| INTC | $0.0001($ | $($ | $0.1608)$ | $1.6002(27.3684)$ | $0.0263($ | $0.0050)$ |
| MSFT | $-0.0002($ | $(0.4871)$ | $1.1765(27.4554)$ | $0.0193($ | $0.0037)$ |  |
| ORCL | $0.0000(0.0389)$ | $1.5010(21.1855)$ | $0.0319($ | $0.0061)$ |  |  |
| YHOO | $0.0001($ | $(0.1282)$ | $1.6543(19.3838)$ | $0.0384($ | $0.0074)$ |  |

Although the results for complete-data stocks are the same, note that the beta estimates for AMZN and GOOG (the two stocks with missing values) are different from the estimates derived for each stock separately. Since AMZN has few missing values, the differences in the estimates are small. With GOOG, however, the differences are more pronounced.

The $t$-statistic for the beta estimate of GOOG is now significant at the $99.5 \%$ level of significance. Note, however, that the $t$-statistics for beta estimates are based on standard errors from the sample Hessian which, in contrast to the Fisher information matrix, accounts for the increased uncertainty in an estimate due to missing values. If the $t$-statistic is obtained from the more optimistic Fisher information matrix, the t -statistic for GOOG is 8.25. Thus, despite the increase in uncertainty due to missing data, GOOG nonetheless has a statistically significant estimate for beta.

Finally, note that the beta estimate for GOOG is 0.62 -a value that may require some explanation. Although the market has been volatile over this period with sideways price movements, GOOG has steadily appreciated in value. Consequently, it is less tightly correlated with the market, implying that it is less volatile than the market (beta < 1).

## References

[1] Caines, Peter E. Linear Stochastic Systems. John Wiley \& Sons, Inc., 1988.
[2] Cramér, Harald. Mathematical Methods of Statistics. Princeton University Press, 1946.
[3] Dempster, A.P, N.M. Laird, and D.B Rubin. "Maximum Likelihood from Incomplete Data via the EM Algorithm,"Journal of the Royal Statistical Society, Series B, Vol. 39, No. 1, 1977, pp. 1-37.
[4] Greene, William H. Econometric Analysis, 5th ed., Pearson Education, Inc., 2003.
[5] Jarrow, R.A. Finance Theory, Prentice-Hall, Inc., 1988.
[6] Lintner, J. "The Valuation of Risk Assets and the Selection of Risky Investments in Stocks," Review of Economics and Statistics, Vol. 14, 1965, pp. 13-37.
[7] Little, Roderick J. A and Donald B. Rubin. Statistical Analysis with Missing Data, 2nd ed., John Wiley \& Sons, Inc., 2002.
[8] Meng, Xiao-Li and Donald B. Rubin. "Maximum Likelihood Estimation via the ECM Algorithm," Biometrika, Vol. 80, No. 2, 1993, pp. 267-278.
[9] Sexton, Joe and Anders Rygh Swensen. "ECM Algorithms that Converge at the Rate of EM," Biometrika, Vol. 87, No. 3, 2000, pp. 651-662.
[10] Shafer, J. L. Analysis of Incomplete Multivariate Data, Chapman \& Hall/CRC, 1997.
[11] Sharpe, W. F. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, Vol. 19, 1964, pp. 425-442.
[12] Sharpe, W. F., G. J. Alexander, and J. V. Bailey. Investments, 6th ed., Prentice-Hall, Inc., 1999.

## Solving Sample Problems

- "Introduction" on page 8-2
- "Common Problems in Finance" on page 8-3
- "Producing Graphics with the Toolbox" on page 8-21


## Introduction

This section shows how Financial Toolbox functions solve real-world problems. The examples ship with the toolbox as MATLAB files. Try them by entering the commands directly or by executing the code.

This chapter contains two major topics:

- "Common Problems in Finance" on page 8-3

Shows how the toolbox solves real-world financial problems, specifically:

- "Sensitivity of Bond Prices to Changes in Interest Rates" on page 8-3
- "Constructing a Bond Portfolio to Hedge Against Duration and Convexity" on page 8-6
- "Sensitivity of Bond Prices to Parallel Shifts in the Yield Curve" on page 8-9
- "Constructing Greek-Neutral Portfolios of European Stock Options" on page 8-14
- "Term Structure Analysis and Interest Rate Swap Pricing" on page 8-18
- "Producing Graphics with the Toolbox" on page 8-21

Shows how the toolbox produces presentation-quality graphics by solving these problems:

- "Plotting an Efficient Frontier" on page 8-21
- "Plotting Sensitivities of an Option" on page 8-24
- "Plotting Sensitivities of a Portfolio of Options" on page 8-26


## Common Problems in Finance

In this section...<br>"Sensitivity of Bond Prices to Changes in Interest Rates" on page 8-3<br>"Constructing a Bond Portfolio to Hedge Against Duration and Convexity" on page 8-6<br>"Sensitivity of Bond Prices to Parallel Shifts in the Yield Curve" on page 8-9<br>"Sensitivity of Bond Prices to Nonparallel Shifts in the Yield Curve" on page 8-12<br>"Constructing Greek-Neutral Portfolios of European Stock Options" on page 8-14<br>"Term Structure Analysis and Interest Rate Swap Pricing" on page 8-18

## Sensitivity of Bond Prices to Changes in Interest Rates

Macaulay and modified duration measure the sensitivity of a bond's price to changes in the level of interest rates. Convexity measures the change in duration for small shifts in the yield curve, and thus measures the second-order price sensitivity of a bond. Both measures can gauge the vulnerability of a bond portfolio's value to changes in the level of interest rates.

Alternatively, analysts can use duration and convexity to construct a bond portfolio that is partly hedged against small shifts in the term structure. If you combine bonds in a portfolio whose duration is zero, the portfolio is insulated, to some extent, against interest rate changes. If the portfolio convexity is also zero, this insulation is even better. However, since hedging costs money or reduces expected return, you need to know how much protection results from hedging duration alone compared to hedging both duration and convexity.

This example demonstrates a way to analyze the relative importance of duration and convexity for a bond portfolio using some of the SIA-compliant bond functions in Financial Toolbox software. Using duration, it constructs a first-order approximation of the change in portfolio price to a level shift in interest rates. Then, using convexity, it calculates a second-order approximation. Finally, it compares the two approximations with the true
price change resulting from a change in the yield curve. The example is ftspex1.m.

Step 1. Define three bonds using values for the settlement date, maturity date, face value, and coupon rate. For simplicity, accept default values for the coupon payment periodicity (semiannual), end-of-month payment rule (rule in effect), and day-count basis (actual/actual). Also, synchronize the coupon payment structure to the maturity date (no odd first or last coupon dates). Any inputs for which defaults are accepted are set to empty matrices ([ ]) as placeholders where appropriate.

```
Settle = '19-Aug-1999';
Maturity = ['17-Jun-2010'; '09-Jun-2015'; '14-May-2025'];
Face = [100; 100; 1000];
CouponRate = [0.07; 0.06; 0.045];
```

Also, specify the yield curve information.

```
Yields = [0.05; 0.06; 0.065];
```

Step 2. Use Financial Toolbox functions to calculate the price, modified duration in years, and convexity in years of each bond.

The true price is quoted (clean) price plus accrued interest.

```
[CleanPrice, AccruedInterest] = bndprice(Yields, CouponRate,...
Settle, Maturity, 2, 0, [], [], [], [], [], Face);
Durations = bnddury(Yields, CouponRate, Settle, Maturity, 2, 0,\ldots.
[], [], [], [], [], Face);
Convexities = bndconvy(Yields, CouponRate, Settle, Maturity, 2, 0,...
[], [], [], [], [], Face);
Prices = CleanPrice + AccruedInterest;
```

Step 3. Choose a hypothetical amount by which to shift the yield curve (here, 0.2 percentage point or 20 basis points).

```
dY = 0.002;
```

Weight the three bonds equally, and calculate the actual quantity of each bond in the portfolio, which has a total value of $\$ 100,000$.

```
PortfolioPrice = 100000;
PortfolioWeights = ones(3,1)/3;
PortfolioAmounts = PortfolioPrice * PortfolioWeights ./ Prices;
```

Step 4. Calculate the modified duration and convexity of the portfolio. Note that the portfolio duration or convextity is a weighted average of the durations or convexities of the individual bonds. Calculate the first- and second-order approximations of the percent price change as a function of the change in the level of interest rates.

```
PortfolioDuration = PortfolioWeights' * Durations;
PortfolioConvexity = PortfolioWeights' * Convexities;
PercentApprox1 = -PortfolioDuration * dY * 100;
PercentApprox2 = PercentApprox1 + ...
PortfolioConvexity*dY^2*100/2.0;
```

Step 5. Estimate the new portfolio price using the two estimates for the percent price change.

```
PriceApprox1 = PortfolioPrice + ...
PercentApprox1 * PortfolioPrice/100;
PriceApprox2 = PortfolioPrice + ...
PercentApprox2 * PortfolioPrice/100;
```

Step 6. Calculate the true new portfolio price by shifting the yield curve.

```
[CleanPrice, AccruedInterest] = bndprice(Yields + dY,...
CouponRate, Settle, Maturity, 2, 0, [], [], [], [], [],...
Face);
NewPrice = PortfolioAmounts' * (CleanPrice + AccruedInterest);
```

Step 7. Compare the results. The analysis results are as follows (verify these results by running the example ftspex1.m:

- The original portfolio price was $\$ 100,000$.
- The yield curve shifted up by 0.2 percentage point or 20 basis points.
- The portfolio duration and convexity are 10.3181 and 157.6346 , respectively. These will be needed for "Constructing a Bond Portfolio to Hedge Against Duration and Convexity" on page 8-6.
- The first-order approximation, based on modified duration, predicts the new portfolio price (PriceApprox1) will be $\$ 97,936.37$.
- The second-order approximation, based on duration and convexity, predicts the new portfolio price (PriceApprox2) will be $\$ 97,967.90$.
- The true new portfolio price (NewPrice) for this yield curve shift is \$97,967.51.
- The estimate using duration and convexity is quite good (at least for this fairly small shift in the yield curve), but only slightly better than the estimate using duration alone. The importance of convexity increases as the magnitude of the yield curve shift increases. Try a larger shift (dY) to see this effect.

The approximation formulas in this example consider only parallel shifts in the term structure, because both formulas are functions of dY , the change in yield. The formulas are not well-defined unless each yield changes by the same amount. In actual financial markets, changes in yield curve level typically explain a substantial portion of bond price movements. However, other changes in the yield curve, such as slope, may also be important and are not captured here. Also, both formulas give local approximations whose accuracy deteriorates as dY increases in size. You can demonstrate this by running the program with larger values of $d Y$.

## Constructing a Bond Portfolio to Hedge Against Duration and Convexity

This example constructs a bond portfolio to hedge the portfolio of "Sensitivity of Bond Prices to Changes in Interest Rates" on page 8-3 It assumes a long position in (holding) the portfolio, and that three other bonds are available for hedging. It chooses weights for these three other bonds in a new portfolio so that the duration and convexity of the new portfolio match those of the original portfolio. Taking a short position in the new portfolio, in an amount equal to the value of the first portfolio, partially hedges against parallel shifts in the yield curve.

Recall that portfolio duration or convexity is a weighted average of the durations or convexities of the individual bonds in a portfolio. As in the previous example, this example uses modified duration in years and convexity in years. The hedging problem therefore becomes one of solving a system of linear equations, which is an easy to do in MATLAB software. The file for this example is ftspex2.m.

Step 1. Define three bonds available for hedging the original portfolio. Specify values for the settlement date, maturity date, face value, and coupon rate. For simplicity, accept default values for the coupon payment periodicity (semiannual), end-of-month payment rule (rule in effect), and day-count basis (actual/actual). Also, synchronize the coupon payment structure to the maturity date (that is, no odd first or last coupon dates). Set any inputs for which defaults are accepted to empty matrices ([]) as placeholders where appropriate. The intent is to hedge against duration and convexity and constrain total portfolio price.

```
Settle = '19-Aug-1999';
Maturity = ['15-Jun-2005'; '02-Oct-2010'; '01-Mar-2025'];
Face = [500; 1000; 250];
CouponRate = [0.07; 0.066; 0.08];
```

Also, specify the yield curve for each bond.

```
Yields = [0.06; 0.07; 0.075];
```

Step 2. Use Financial Toolbox functions to calculate the price, modified duration in years, and convexity in years of each bond.

The true price is quoted (clean price plus accrued interest.

```
[CleanPrice, AccruedInterest] = bndprice(Yields,CouponRate,...
Settle, Maturity, 2, 0, [], [], [], [], [], Face);
Durations = bnddury(Yields, CouponRate, Settle, Maturity,...
2, O, [], [], [], [], [], Face);
Convexities = bndconvy(Yields, CouponRate, Settle,...
Maturity, 2, 0, [], [], [], [], [], Face);
Prices = CleanPrice + AccruedInterest;
```

Step 3. Set up and solve the system of linear equations whose solution is the weights of the new bonds in a new portfolio with the same duration and convexity as the original portfolio. In addition, scale the weights to sum to 1 ; that is, force them to be portfolio weights. You can then scale this unit portfolio to have the same price as the original portfolio. Recall that the original portfolio duration and convexity are 10.3181 and 157.6346, respectively. Also, note that the last row of the linear system ensures that the sum of the weights is unity.

```
A = [Durations'
    Convexities'
    1 1 1];
b = [ 10.3181
    157.6346
        1];
Weights = A\b;
```

Step 4. Compute the duration and convexity of the hedge portfolio, which should now match the original portfolio.

```
PortfolioDuration = Weights' * Durations;
PortfolioConvexity = Weights' * Convexities;
```

Step 5. Finally, scale the unit portfolio to match the value of the original portfolio and find the number of bonds required to insulate against small parallel shifts in the yield curve.

```
PortfolioValue = 100000;
HedgeAmounts = Weights ./ Prices * PortfolioValue;
```

Step 6. Compare the results. Verify the analysis results by running the example ftspex2.m.

- As required, the duration and convexity of the new portfolio are 10.3181 and 157.6346, respectively.
- The hedge amounts for bonds 1,2 , and 3 are -57.37, 71.70, and 216.27, respectively.

Notice that the hedge matches the duration, convexity, and value ( $\$ 100,000$ ) of the original portfolio. If you are holding that first portfolio, you can hedge by taking a short position in the new portfolio.

Just as the approximations of the first example are appropriate only for small parallel shifts in the yield curve, the hedge portfolio is appropriate only for reducing the impact of small level changes in the term structure.

## Sensitivity of Bond Prices to Parallel Shifts in the Yield Curve

Often bond portfolio managers want to consider more than just the sensitivity of a portfolio's price to a small shift in the yield curve, particularly if the investment horizon is long. This example shows how MATLAB software can help you to visualize the price behavior of a portfolio of bonds over a wide range of yield curve scenarios, and as time progresses toward maturity.

This example uses Financial Toolbox bond pricing functions to evaluate the impact of time-to-maturity and yield variation on the price of a bond portfolio. It plots the portfolio value and shows the behavior of bond prices as yield and time vary. The file for this example is $\mathrm{ftspex} 3 . \mathrm{m}$.

Step 1. Specify values for the settlement date, maturity date, face value, coupon rate, and coupon payment periodicity of a four-bond portfolio. For simplicity, accept default values for the end-of-month payment rule (rule in effect) and day-count basis (actual/actual). Also, synchronize the coupon payment structure to the maturity date (no odd first or last coupon dates). Any inputs for which defaults are accepted are set to empty matrices ([ ]) as placeholders where appropriate.

```
Settle = '15-Jan-1995';
Maturity = datenum(['03-Apr-2020'; '14-May-2025'; ...
    '09-Jun-2019'; '25-Feb-2019']);
Face = [1000; 1000; 1000; 1000];
CouponRate = [0; 0.05; 0; 0.055];
Periods = [0; 2; 0; 2];
```

Also, specify the points on the yield curve for each bond.

```
Yields = [0.078; 0.09; 0.075; 0.085];
```

Step 2. Use Financial Toolbox functions to calculate the true bond prices as the sum of the quoted price plus accrued interest.

```
[CleanPrice, AccruedInterest] = bndprice(Yields,...
CouponRate,Settle, Maturity, Periods,...
[], [], [], [], [], [], Face);
Prices = CleanPrice + AccruedInterest;
```

Step 3. Assume the value of each bond is $\$ 25,000$, and determine the quantity of each bond such that the portfolio value is $\$ 100,000$.

```
BondAmounts = 25000 ./ Prices;
```

Step 4. Compute the portfolio price for a rolling series of settlement dates over a range of yields. The evaluation dates occur annually on January 15, beginning on 15-Jan-1995 (settlement) and extending out to 15-Jan-2018. Thus, this step evaluates portfolio price on a grid of time of progression (dT) and interest rates (dY).

```
dy = -0.05:0.005:0.05; % Yield changes
D = datevec(Settle); % Get date components
dt = datenum(D(1):2018, D(2), D(3)); % Get evaluation dates
[dT, dY] = meshgrid(dt, dy); % Create grid
NumTimes = length(dt); % Number of time steps
NumYields = length(dy); % Number of yield changes
NumBonds = length(Maturity); % Number of bonds
% Preallocate vector
Prices = zeros(NumTimes*NumYields, NumBonds);
```

Now that the grid and price vectors have been created, compute the price of each bond in the portfolio on the grid one bond at a time.

```
for i = 1:NumBonds
    [CleanPrice, AccruedInterest] = bndprice(Yields(i)+...
```

```
dY(:), CouponRate(i), dT(:), Maturity(i), Periods(i),...
[], [], [], [], [], [], Face(i));
Prices(:,i) = CleanPrice + AccruedInterest;
```

end

Scale the bond prices by the quantity of bonds.

```
Prices = Prices * BondAmounts;
```

Reshape the bond values to conform to the underlying evaluation grid.

```
Prices = reshape(Prices, NumYields, NumTimes);
```

Step 5. Plot the price of the portfolio as a function of settlement date and a range of yields, and as a function of the change in yield (dY). This plot illustrates the interest rate sensitivity of the portfolio as time progresses (dT), under a range of interest rate scenarios. With the following graphics commands, you can visualize the three-dimensional surface relative to the current portfolio value (that is, $\$ 100,000$ ).

```
figure % Open a new figure window
surf(dt, dy, Prices) % Draw the surface
```

Add the base portfolio value to the existing surface plot.

```
hold on % Add the current value for reference
basemesh = mesh(dt, dy, 100000*ones(NumYields, NumTimes));
```

Make it transparent, plot it so the price surface shows through, and draw a box around the plot.

```
set(basemesh, 'facecolor', 'none');
set(basemesh, 'edgecolor', 'm');
set(gca, 'box', 'on');
```

Plot the $x$-axis using two-digit year (YY format) labels for ticks.

```
dateaxis('x', 11);
```

Add axis labels and set the three-dimensional viewpoint. MATLAB produces the figure.

```
xlabel('Evaluation Date (YY Format)');
ylabel('Change in Yield');
zlabel('Portfolio Price');
hold off
view(-25,25);
```



MATLAB three-dimensional graphics allow you to visualize the interest rate risk experienced by a bond portfolio over time. This example assumed parallel shifts in the term structure, but it might similarly have allowed other components to vary, such as the level and slope.

## Sensitivity of Bond Prices to Nonparallel Shifts in the Yield Curve

Key rate duration enables you to determine the sensitivity of the price of a bond to nonparallel shifts in the yield curve. This example uses bndkrdur to
construct a portfolio to hedge the interest rate risk of a U.S. Treasury bond maturing in 20 years. For more information on this bond, see .

```
Settle = datenum('2-Dec-2008');
CouponRate = 5.500/100;
Maturity = datenum('15-Aug-2028');
Price = 128.68;
```

The interest rate risk of this bond is hedged with the following four on-the-run Treasury bonds:

The 30 -year bond. For more information, see .

```
Maturity_30 = datenum('15-May-2038');
Coupon_30 = .045;
Price_30 = 124.69;
```

The ten-year note. For more information, see .

```
Maturity_10 = datenum('15-Nov-2018');
Coupon_10 = .0375;
Price_10 = 109.35;
```

The five-year note. For more information, see .

```
Maturity_05 = datenum('30-Nov-2013');
Coupon_05 = .02;
Price_05 = 101.67;
```

The two-year note. For more information, see .

```
Maturity_02 = datenum('30-Nov-2010');
Coupon_02 = .01250;
Price_02 = 100.72;
```

You can get the Treasury spot or zero curve from: .

```
ZeroDates = daysadd(Settle,[30 90 180 360 360*2 360*3 360*5 ...
360*7 360*10 360*20 360*30]);
ZeroRates = ([0.09 0.07 0.44 0.81 0.90 1.16 1.71 2.13 2.72 3.51 3.22]/100)';
```

Step 1. Compute the key rate durations for both the bond and the hedging portfolio:

```
BondKRD = bndkrdur([ZeroDates ZeroRates], CouponRate, Settle,...
Maturity,'keyrates',[2 5 10 20]);
HedgeMaturity = [Maturity_02;Maturity_05;Maturity_10;Maturity_30];
HedgeCoupon = [Coupon_02;Coupon_05;Coupon_10;Coupon_30];
HedgeKRD = bndkrdur([ZeroDates ZeroRates], HedgeCoupon,...
Settle, HedgeMaturity, 'keyrates',[2 5 10 20]);
```

Step 2. Compute the dollar durations for each of the instruments and each of the key rates (assuming holding 100 bonds):

```
PortfolioDD = 100*Price* BondKRD;
HedgeDD = bsxfun(@times, HedgeKRD,[Price_30;Price_10;Price_05;Price_02]);
```

Step 3. Compute the number of bonds to sell short to obtain a key rate duration that is 0 for the entire portfolio:

```
NumBonds = PortfolioDD/HedgeDD;
NumBonds =
    3.8973 6.1596 23.0282 80.0522
```

These results indicate selling 4, 6, 23 and 80 bonds respectively of the 2 -, 5 -, $10-$, and 30 -year bonds achieves a portfolio that is neutral with respect to the 2 -, $5-, 10$-, and 30 -year spot rates.

## Constructing Greek-Neutral Portfolios of European Stock Options

The option sensitivity measures familiar to most option traders are often referred to as the greeks: delta, gamma, vega, lambda, rho, and theta. Delta is the price sensitivity of an option with respect to changes in the price of the underlying asset. It represents a first-order sensitivity measure analogous to duration in fixed income markets. Gamma is the sensitivity of an option's delta to changes in the price of the underlying asset, and represents a second-order price sensitivity analogous to convexity in fixed income markets. Vega is the price sensitivity of an option with respect to changes in the volatility of the underlying asset. See "Pricing and Analyzing

Equity Derivatives" on page 2-39 or the "Glossary" on page Glossary-1 for other definitions.

The greeks of a particular option are a function of the model used to price the option. However, given enough different options to work with, a trader can construct a portfolio with any desired values for its greeks. For example, to insulate the value of an option portfolio from small changes in the price of the underlying asset, one trader might construct an option portfolio whose delta is zero. Such a portfolio is then said to be "delta neutral." Another trader may want to protect an option portfolio from larger changes in the price of the underlying asset, and so might construct a portfolio whose delta and gamma are both zero. Such a portfolio is both delta and gamma neutral. A third trader may want to construct a portfolio insulated from small changes in the volatility of the underlying asset in addition to delta and gamma neutrality. Such a portfolio is then delta, gamma, and vega neutral.

Using the Black-Scholes model for European options, this example creates an equity option portfolio that is simultaneously delta, gamma, and vega neutral. The value of a particular greek of an option portfolio is a weighted average of the corresponding greek of each individual option. The weights are the quantity of each option in the portfolio. Hedging an option portfolio thus involves solving a system of linear equations, an easy process in MATLAB. The file for this example is ftspex4.m.

Step 1. Create an input data matrix to summarize the relevant information. Each row of the matrix contains the standard inputs to Financial Toolbox Black-Scholes suite of functions: column 1 contains the current price of the underlying stock; column 2 the strike price of each option; column 3 the time to-expiry of each option in years; column 4 the annualized stock price volatility; and column 5 the annualized dividend rate of the underlying asset. Note that rows 1 and 3 are data related to call options, while rows 2 and 4 are data related to put options.

| DataMatrix $=[100.000$ | 100 | 0.2 | 0.3 | 0 | \% Call |
| ---: | :--- | ---: | :--- | :--- | :--- |
| 119.100 | 125 | 0.2 | 0.2 | 0.025 | \% Put |
| 87.200 | 85 | 0.1 | 0.23 | 0 | \% Call |
|  | 301.125 | 315 | 0.5 | 0.25 | $0.0333]$ | | \% Put |
| :--- |

Also, assume the annualized risk-free rate is $10 \%$ and is constant for all maturities of interest.

```
RiskFreeRate = 0.10;
```

For clarity, assign each column of DataMatrix to a column vector whose name reflects the type of financial data in the column.

```
StockPrice = DataMatrix(:,1);
StrikePrice = DataMatrix(:,2);
ExpiryTime = DataMatrix(:,3);
Volatility = DataMatrix(:,4);
DividendRate = DataMatrix(:,5);
```

Step 2. Based on the Black-Scholes model, compute the prices, and the delta, gamma, and vega sensitivity greeks of each of the four options. Note that the functions blsprice and blsdelta have two outputs, while blsgamma and blsvega have only one. The price and delta of a call option differ from the price and delta of an otherwise equivalent put option, in contrast to the gamma and vega sensitivities, which are valid for both calls and puts.

```
[CallPrices, PutPrices] = blsprice(StockPrice, StrikePrice,...
RiskFreeRate, ExpiryTime, Volatility, DividendRate);
[CallDeltas, PutDeltas] = blsdelta(StockPrice,...
StrikePrice, RiskFreeRate, ExpiryTime, Volatility,...
DividendRate);
Gammas = blsgamma(StockPrice, StrikePrice, RiskFreeRate,...
    ExpiryTime, Volatility , DividendRate)';
Vegas = blsvega(StockPrice, StrikePrice, RiskFreeRate,...
    ExpiryTime, Volatility , DividendRate)';
```

Extract the prices and deltas of interest to account for the distinction between call and puts.

```
Prices = [CallPrices(1) PutPrices(2) CallPrices(3)...
PutPrices(4)];
Deltas = [CallDeltas(1) PutDeltas(2) CallDeltas(3)...
PutDeltas(4)];
```

Step 3. Now, assuming an arbitrary portfolio value of $\$ 17,000$, set up and solve the linear system of equations such that the overall option portfolio is simultaneously delta, gamma, and vega-neutral. The solution computes the value of a particular greek of a portfolio of options as a weighted average of the corresponding greek of each individual option in the portfolio. The system of equations is solved using the back slash ( $\backslash$ ) operator discussed in "Solving Simultaneous Linear Equations" on page 1-15.

```
A = [Deltas; Gammas; Vegas; Prices];
b = [0; 0; 0; 17000];
OptionQuantities = A\b; % Quantity (number) of each option.
```

Step 4. Finally, compute the market value, delta, gamma, and vega of the overall portfolio as a weighted average of the corresponding parameters of the component options. The weighted average is computed as an inner product of two vectors.

```
PortfolioValue = Prices * OptionQuantities;
PortfolioDelta = Deltas * OptionQuantities;
PortfolioGamma = Gammas * OptionQuantities;
PortfolioVega = Vegas * OptionQuantities;
```

The example ftspex4.m performs these computations and displays the output on the screen.

| Option | Price | Delta | Gamma | Vega | Quantity |
| :---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 6.3441 | 0.5856 | 0.0290 | 17.4293 | 22332.6131 |
| 2 | 6.6035 | -0.6255 | 0.0353 | 20.0347 | 6864.0731 |
| 3 | 4.2993 | 0.7003 | 0.0548 | 9.5837 | -15654.8657 |
| 4 | 22.7694 | -0.4830 | 0.0074 | 83.5225 | -4510.5153 |

```
Portfolio Value: $17000.00
Portfolio Delta: 0.00
Portfolio Gamma: -0.00
Portfolio Vega : 0.00
```

You can verify that the portfolio value is $\$ 17,000$ and that the option portfolio is indeed delta, gamma, and vega neutral, as desired. Hedges based on these measures are effective only for small changes in the underlying variables.

## Term Structure Analysis and Interest Rate Swap Pricing

This example illustrates some of the term-structure analysis functions found in Financial Toolbox software. Specifically, it illustrates how to derive implied zero (spot) and forward curves from the observed market prices of coupon-bearing bonds. The zero and forward curves implied from the market data are then used to price an interest rate swap agreement.

In an interest rate swap, two parties agree to a periodic exchange of cash flows. One of the cash flows is based on a fixed interest rate held constant throughout the life of the swap. The other cash flow stream is tied to some variable index rate. Pricing a swap at inception amounts to finding the fixed rate of the swap agreement. This fixed rate, appropriately scaled by the notional principal of the swap agreement, determines the periodic sequence of fixed cash flows.

In general, interest rate swaps are priced from the forward curve such that the variable cash flows implied from the series of forward rates and the periodic sequence of fixed-rate cash flows have the same current value. Thus, interest rate swap pricing and term structure analysis are intimately related.

Step 1. Specify values for the settlement date, maturity dates, coupon rates, and market prices for 10 U.S. Treasury Bonds. This data allows you to price a five-year swap with net cash flow payments exchanged every six months. For simplicity, accept default values for the end-of-month payment rule (rule in effect) and day-count basis (actual/actual). To avoid issues of accrued interest, assume that all Treasury Bonds pay semiannual coupons and that settlement occurs on a coupon payment date.

```
Settle = datenum('15-Jan-1999');
BondData = {'15-Jul-1999' 0.06000 99.93
    '15-Jan-2000' 0.06125 99.72
    '15-Jul-2000' 0.06375 99.70
    '15-Jan-2001' 0.06500 99.40
    '15-Jul-2001' 0.06875 99.73
    '15-Jan-2002' 0.07000 99.42
    '15-Jul-2002' 0.07250 99.32
    '15-Jan-2003' 0.07375 98.45
    '15-Jul-2003' 0.07500 97.71
```

$$
\text { '15-Jan-2004' } 0.08000 \quad 98.15\} ;
$$

BondData is an instance of a MATLAB cell array, indicated by the curly braces (\{\}).

Next assign the date stored in the cell array to Maturity, CouponRate, and Prices vectors for further processing.

```
Maturity = datenum(char(BondData{:,1}));
CouponRate = [BondData{:,2}]';
Prices = [BondData{:,3}]';
Period = 2; % semiannual coupons
```

Step 2. Now that the data has been specified, use the term structure function zbtprice to bootstrap the zero curve implied from the prices of the coupon-bearing bonds. This implied zero curve represents the series of zero-coupon Treasury rates consistent with the prices of the coupon-bearing bonds such that arbitrage opportunities will not exist.

```
ZeroRates = zbtprice([Maturity CouponRate], Prices, Settle);
```

The zero curve, stored in ZeroRates, is quoted on a semiannual bond basis (the periodic, six-month, interest rate is doubled to annualize). The first element of ZeroRates is the annualized rate over the next six months, the second element is the annualized rate over the next 12 months, and so on.

Step 3. From the implied zero curve, find the corresponding series of implied forward rates using the term structure function zero2fwd.

```
ForwardRates = zero2fwd(ZeroRates, Maturity, Settle);
```

The forward curve, stored in ForwardRates, is also quoted on a semiannual bond basis. The first element of ForwardRates is the annualized rate applied to the interval between settlement and six months after settlement, the second element is the annualized rate applied to the interval from six months to 12 months after settlement, and so on. This implied forward curve is also consistent with the observed market prices such that arbitrage activities will be unprofitable. Since the first forward rate is also a zero rate, the first element of ZeroRates and ForwardRates are the same.

Step 4. Now that you have derived the zero curve, convert it to a sequence of discount factors with the term structure function zero2disc.

```
DiscountFactors = zero2disc(ZeroRates, Maturity, Settle);
```

Step 5. From the discount factors, compute the present value of the variable cash flows derived from the implied forward rates. For plain interest rate swaps, the notional principle remains constant for each payment date and cancels out of each side of the present value equation. The next line assumes unit notional principle.

```
PresentValue = sum((ForwardRates/Period) .* DiscountFactors);
```

Step 6. Compute the swap's price (the fixed rate) by equating the present value of the fixed cash flows with the present value of the cash flows derived from the implied forward rates. Again, since the notional principle cancels out of each side of the equation, it is simply assumed to be 1 .

```
SwapFixedRate = Period * PresentValue / sum(DiscountFactors);
```

The example ftspex5.m performs these computations and displays the output on the screen.

| Zero Rates | Forward Rates |
| :---: | :---: |
| 0.0614 | 0.0614 |
| 0.0642 | 0.0670 |
| 0.0660 | 0.0695 |
| 0.0684 | 0.0758 |
| 0.0702 | 0.0774 |
| 0.0726 | 0.0846 |
| 0.0754 | 0.0925 |
| 0.0795 | 0.1077 |
| 0.0827 | 0.1089 |
| 0.0868 | 0.1239 |

Swap Price (Fixed Rate) $=0.0845$
All rates are in decimal format. The swap price, $8.45 \%$, would likely be the mid-point between a market-maker's bid/ask quotes.

## Producing Graphics with the Toolbox

In this section...<br>"Introduction" on page 8-21<br>"Plotting an Efficient Frontier" on page 8-21<br>"Plotting Sensitivities of an Option" on page 8-24<br>"Plotting Sensitivities of a Portfolio of Options" on page 8-26

## Introduction

Financial Toolbox and MATLAB graphics functions work together to produce presentation quality graphics, as these examples show. The examples ship with the toolbox as MATLAB files. Try them by entering the commands directly or by executing the code. For help using MATLAB plotting functions, see "Creating Line Plots" in the MATLAB documentation.

## Plotting an Efficient Frontier

This example plots the efficient frontier of a hypothetical portfolio of three assets. It illustrates how to specify the expected returns, standard deviations, and correlations of a portfolio of assets, how to convert standard deviations and correlations into a covariance matrix, and how to compute and plot the efficient frontier from the returns and covariance matrix. The example also illustrates how to randomly generate a set of portfolio weights, and how to add the random portfolios to an existing plot for comparison with the efficient frontier. The file for this example is ftgex1.m.

First, specify the expected returns, standard deviations, and correlation matrix for a hypothetical portfolio of three assets.
$\left.\begin{array}{rl}\begin{array}{l}\text { Returns } \\ \text { STDs }\end{array} & =\left[\begin{array}{lll}0.1 & 0.15 & 0.12\end{array}\right] ; \\ \text { Correlations }= & {\left[\begin{array}{ccc}0.2 & 0.25 & 0.18\end{array}\right] ;} \\ & 0.3 \\ 0.3 & 0.4 \\ & 0.4 \\ 0.3 & 1\end{array}\right] ;$

Convert the standard deviations and correlation matrix into a variance-covariance matrix with the Financial Toolbox function corr2cov.

```
Covariances = corr2cov(STDs, Correlations);
```

Evaluate and plot the efficient frontier at 20 points along the frontier, using the function portopt and the expected returns and corresponding covariance matrix. Although rather elaborate constraints can be placed on the assets in a portfolio, for simplicity accept the default constraints and scale the total value of the portfolio to 1 and constrain the weights to be positive (no short-selling).

```
portopt(Returns, Covariances, 20)
```



Now that the efficient frontier is displayed, randomly generate the asset weights for 1000 portfolios starting from the MATLAB initial state.

```
rand('state', 0)
Weights = rand(1000, 3);
```

The previous line of code generates three columns of uniformly distributed random weights, but does not guarantee they sum to 1 . The following code segment normalizes the weights of each portfolio so that the total of the three weights represent a valid portfolio.

```
Total = sum(Weights, 2); % Add the weights
Total = Total(:,ones(3,1)); % Make size-compatible matrix
Weights = Weights./Total; % Normalize so sum = 1
```

Given the 1000 random portfolios just created, compute the expected return and risk of each portfolio associated with the weights.

```
[PortRisk, PortReturn] = portstats(Returns, Covariances, ...
    Weights);
```

Finally, hold the current graph, and plot the returns and risks of each portfolio on top of the existing efficient frontier for comparison. After plotting, annotate the graph with a title and return the graph to default holding status (any subsequent plots will erase the existing data). The efficient frontier appears in blue, while the 1000 random portfolios appear as a set of red dots on or below the frontier.

```
hold on
plot (PortRisk, PortReturn, '.r')
title('Mean-Variance Efficient Frontier and Random Portfolios')
hold off
```



## Plotting Sensitivities of an Option

This example creates a three-dimensional plot showing how gamma changes relative to price for a Black-Scholes option. Recall that gamma is the second derivative of the option price relative to the underlying security price. The plot shows a three-dimensional surface whose $z$-value is the gamma of an option as price ( $x$-axis) and time ( $y$-axis) vary. It adds yet a fourth dimension by showing option delta (the first derivative of option price to security price) as the color of the surface. The file for this example is ftgex2.m.

First set the price range of the options, and set the time range to one year divided into half-months and expressed as fractions of a year.

```
Range = 10:70;
Span = length(Range);
j = 1:0.5:12;
Newj = j(ones(Span,1),:)'/12;
```

For each time period create a vector of prices from 10 to 70 and create a matrix of all ones.

```
JSpan = ones(length(j),1);
NewRange = Range(JSpan,:);
Pad = ones(size(Newj));
```

Calculate the toolbox gamma and delta sensitivity functions (greeks). (Recall that gamma is the second derivative of the option price with respect to the stock price, and delta is the first derivative of the option price with respect to the stock price.) The exercise price is $\$ 40$, the risk-free interest rate is $10 \%$, and volatility is 0.35 for all prices and periods.

```
ZVal = blsgamma(NewRange, 40*Pad, 0.1*Pad, Newj, 0.35*Pad);
Color = blsdelta(NewRange, 40*Pad, 0.1*Pad, Newj, 0.35*Pad);
```

Display the greeks as a function of price and time. Gamma is the $z$-axis; delta is the color.

```
mesh(Range, j, ZVal, Color);
xlabel('Stock Price ($)');
ylabel('Time (months)');
zlabel('Gamma');
title('Call Option Price Sensitivity');
axis([10 70 1 12 -inf inf]);
view(-40, 50);
colorbar('horiz');
```



## Plotting Sensitivities of a Portfolio of Options

This example plots gamma as a function of price and time for a portfolio of 10 Black-Scholes options. The plot shows a three-dimensional surface. For each point on the surface, the height ( $z$-value) represents the sum of the gammas for each option in the portfolio weighted by the amount of each option. The $x$-axis represents changing price, and the $y$-axis represents time. The plot adds a fourth dimension by showing delta as surface color. This has applications in hedging.

The file for this example is $\mathrm{ftgex} 3 . \mathrm{m}$.
First set up the portfolio with arbitrary data. Current prices range from $\$ 20$ to $\$ 90$ for each option. Set corresponding exercise prices for each option.

```
Range = 20:90;
PLen = length(Range);
ExPrice = [75 70 50 55 75 50 40 75 60 35];
```

Set all risk-free interest rates to $10 \%$, and set times to maturity in days. Set all volatilities to 0.35 . Set the number of options of each instrument, and allocate space for matrices.

```
Rate = 0.1*ones(10,1);
Time = [[lllllllllllll}36\mp@code{36
Sigma = 0.35*ones(10,1);
NumOpt = 1000*[[4 8 3 5 5.5 2 4.8 3 4.8 2.5];
ZVal = zeros(36, PLen);
Color = zeros(36, PLen);
```

For each instrument, create a matrix (of size Time by PLen) of prices for each period.

```
for i = 1:10
    Pad = ones(Time(i),PLen);
    NewR = Range(ones(Time(i),1),:);
```

Create a vector of time periods 1 to Time; and a matrix of times, one column for each price.

```
T = (1:Time(i))';
NewT = T(:,ones(PLen,1));
```

Call the toolbox gamma and delta sensitivity functions to compute gamma and delta.

```
    ZVal(36-Time(i)+1:36,:) = ZVal(36-Time(i)+1:36,:) ...
    + NumOpt(i) * blsgamma(NewR, ExPrice(i)*Pad, ...
    Rate(i)*Pad, NewT/36, Sigma(i)*Pad);
Color(36-Time(i)+1:36,:) = Color(36-Time(i)+1:36,:) ...
    + NumOpt(i) * blsdelta(NewR, ExPrice(i)*Pad, ...
    Rate(i)*Pad, NewT/36, Sigma(i)*Pad);
end
```

Draw the surface as a mesh, set the viewpoint, and reverse the $x$-axis because of the viewpoint. The axes range from 20 to 90,0 to 36 , and $-\infty$ to $\infty$.

```
mesh(Range, 1:36, ZVal, Color);
view(60,60);
```

```
set(gca, 'xdir','reverse', 'tag', 'mesh_axes_3');
axis([20 90 0 36 -inf inf]);
```

Add a title and axis labels and draw a box around the plot. Annotate the colors with a bar and label the colorbar.

```
title('Call Option Portfolio Sensitivity');
xlabel('Stock Price ($)');
ylabel('Time (months)');
zlabel('Gamma');
set(gca, 'box', 'on');
colorbar('horiz');
```



# Financial Time Series Analysis 

- "Analyzing Financial Time Series" on page 9-2
- "Creating Financial Time Series Objects" on page 9-3
- "Visualizing Financial Time Series Objects" on page 9-18


## Analyzing Financial Time Series

Financial Toolbox software provides a collection of tools for the analysis of time series data in the financial markets. The toolbox contains a financial time series object constructor and several methods that operate on and analyze the object. Financial engineers working with time series data, such as equity prices or daily interest fluctuations, can use these tools for more intuitive data management than by using regular vectors or matrices.

This chapter discusses how to create a financial time series object in one of two ways:

- "Using the Constructor" on page 9-3
- "Transforming a Text File" on page 9-14

The chapter also discusses chartfts, a graphical tool for visualizing financial time series objects. You can find this discussion in "Visualizing Financial Time Series Objects" on page 9-18.

## Creating Financial Time Series Objects

In this section...<br>"Introduction" on page 9-3<br>"Using the Constructor" on page 9-3<br>"Transforming a Text File" on page 9-14

## Introduction

Financial Toolbox software provides two ways to create a financial time series object:

- At the command line using the object constructor fints
- From a text data file through the function ascii2fts

The structure of the object minimally consists of a description field, a frequency indicator field, the date vector field, and at least one data series vector. The names for the fields are fixed for the first three fields: desc, freq, and dates. You can specify names of your choice for any data series vectors. If you do not specify names, the object uses the default names series1, series2, series3, and so on.

If time-of-day information is incorporated in the date vector, the object contains an additional field named times.

## Using the Constructor

The object constructor function fints has five different syntaxes. These forms exist to simplify object construction. The syntaxes vary according to the types of input arguments presented to the constructor. The syntaxes are

- Single Matrix Input
- See "Time-of-Day Information Excluded" on page 9-4.
- See "Time-of-Day Information Included" on page 9-7.
- Separate Vector Input
- See "Time-of-Day Information Excluded" on page 9-8.
- See "Time-of-Day Information Included" on page 9-9.
- See "Data Name Input" on page 9-10.
- See "Frequency Indicator Input" on page 9-12.
- See "Description Field Input" on page 9-14.


## Single Matrix Input

The date information provided with this syntax must be in serial date number format. The date number may or may not include time-of-day information.

Note If you are unfamiliar with the concepts of date strings and serial date numbers, consult "Handling and Converting Dates" on page 2-4.

## Time-of-Day Information Excluded.

```
fts = fints(dates_and_data)
```

In this simplest form of syntax, the input must be at least a two-column matrix. The first column contains the dates in serial date format; the second column is the data series. The input matrix can have more than two columns, each additional column representing a different data series or set of observations.

If the input is a two-column matrix, the output object contains four fields: desc, freq, dates, and series1. The description field, desc, defaults to blanks ' ' , and the frequency indicator field, freq, defaults to 0 . The dates field, dates, contains the serial dates from the first column of the input matrix, while the data series field, series1, has the data from the second column of the input matrix.

The first example makes two financial time series objects. The first one has only one data series, while the other has more than one. A random vector provides the values for the data series. The range of dates is arbitrarily chosen using the today function:

```
date_series = (today:today+100)';
```

```
data_series = exp(randn(1, 101))';
dates_and_data = [date_series data_series];
fts1 = fints(dates_and_data);
```

Examine the contents of the object fts 1 create. The actual date series you observe will vary according to the day when you run the example (the value of today). Also, your values in series 1 will differ from those shown, depending upon the sequence of random numbers generated:

```
fts1 =
    desc: (none)
    freq: Unknown (0)
    'dates: (101)' 'series1: (101)'
    '12-Jul-1999' [ 0.3124]
    '13-Jul-1999' [ 3.2665]
    '14-Jul-1999' [ 0.9847]
    '15-Jul-1999' [ 1.7095]
    '16-Jul-1999' [ 0.4885]
    '17-Jul-1999' [ 0.5192]
    '18-Jul-1999' [ 1.3694]
    '19-Jul-1999' [ 1.1127]
    '20-Jul-1999' [ 6.3485]
    '21-Jul-1999' [ 0.7595]
    '22-Jul-1999' [ 9.1390]
    '23-Jul-1999' [ 4.5201]
    '24-Jul-1999' [ 0.1430]
    '25-Jul-1999' [ 0.1863]
    '26-Jul-1999' [ 0.5635]
    '27-Jul-1999' [ 0.8304]
    '28-Jul-1999' [ 1.0090]...
```

The output is truncated for brevity. There are actually 101 data points in the object.

Note that the desc field displays as (none) instead of ' ' , and that the contents of the object display as cell array elements. Although the object displays as such, it should be thought of as a MATLAB structure containing the default field names for a single data series object: desc, freq, dates, and series1.

Now create an object with more than one data series in it:

```
date_series = (today:today+100)';
data_series1 = exp(randn(1, 101))';
data_series2 = exp(randn(1, 101))';
dates_and_data = [date_series data_series1 data_series2];
fts2 = fints(dates_and_data);
```

Now look at the object created (again in abbreviated form):

```
fts2 =
    desc: (none)
    freq: Unknown (0)
    'dates: (101)' 'series1: (101)' 'series2: (101)'
    '12-Jul-1999' [ 0.5816] [ 1.2816]
    '13-Jul-1999' [ 5.1253] [ 0.9262]
    '14-Jul-1999' [ 2.2824] [ 5.6869]
    '15-Jul-1999' [ 1.2596] [ 5.0631]
    '16-Jul-1999' [ 1.9574] [ 1.8709]
    '17-Jul-1999' [ 0.6017] [ 1.0962]
    '18-Jul-1999' [ 2.3546] [ 0.4459]
    '19-Jul-1999' [ 1.3080] [ 0.6304]
    '20-Jul-1999' [ 1.8682] [ 0.2451]
    '21-Jul-1999' [ 0.3509] [ 0.6876]
    '22-Jul-1999' [ 4.6444] [ 0.6244]
    '23-Jul-1999' [ 1.5441] [ 5.7621]
    '24-Jul-1999' [ 0.1470] [ 2.1238]
    '25-Jul-1999' [ 1.5999] [ 1.0671]
    '26-Jul-1999' [ 3.5764] [ 0.7462]
    '27-Jul-1999' [ 1.8937] [ 1.0863]
    '28-Jul-1999' [ 3.9780] [ 2.1516]...
```

The second data series name defaults to series2, as expected.
Before you can perform any operations on the object, you must set the frequency indicator field freq to the valid frequency of the data series contained in the object. You can leave the description field desc blank.

To set the frequency indicator field to a daily frequency, enter
fts2.freq $=1$, or
fts2.freq = 'daily'
See the fints function description in Chapter 15, "Function Reference" or Chapter 17, "Functions - Alphabetical List".

Time-of-Day Information Included. The serial date number used with this form of the fints function can incorporate time-of-day information. When time-of-day information is present, the output of the function contains a field times that indicates the time of day.

If you recode the previous example to include time-of-day information, you can see the additional column present in the output object:

```
time_series = (now:now+100)';
data_series = exp(randn(1, 101))';
times_and_data = [time_series data_series];
fts1 = fints(times_and_data);
fts1 =
    desc: (none)
    freq: Unknown (0)
    'dates: (101)' 'times: (101)' 'series1: (101)'
    '29-Nov-2001' '14:57' [ 0.5816]
    '30-Nov-2001' '14:57' [ 5.1253]
    '01-Dec-2001' '14:57' [ 2.2824]
    '02-Dec-2001' '14:57' [ 1.2596]...
```


## Separate Vector Input

The date information provided with this syntax can be in serial date number or date string format. The date information may or may not include time-of-day information.

## Time-of-Day Information Excluded.

```
fts = fints(dates, data)
```

In this second syntax the dates and data series are entered as separate vectors to fints, the financial time series object constructor function. The dates vector must be a column vector, while the data series data can be a column vector (if there is only one data series) or a column-oriented matrix (for multiple data series). A column-oriented matrix, in this context, indicates that each column is a set of observations. Different columns are different sets of data series.

Here is an example:

```
dates = (today:today+100)';
data_series1 = exp(randn(1, 101))';
data_series2 = exp(randn(1, 101))';
data = [data_series1 data_series2];
fts = fints(dates, data)
fts =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{|c|c|c|c|c|}
\hline 'dates: (101) & 'series1: & (101) & 'series2: & (101) \({ }^{\prime}\) \\
\hline '12-Jul-1999' & [ & 0.5816] & [ & 1.2816] \\
\hline '13-Jul-1999' & [ & 5.1253] & [ & \(0.9262]\) \\
\hline '14-Jul-1999' & [ & \(2.2824]\) & [ & \(5.6869]\) \\
\hline '15-Jul-1999' & [ & \(1.2596]\) & [ & \(5.0631]\) \\
\hline '16-Jul-1999' & [ & \(1.9574]\) & [ & \(1.8709]\) \\
\hline '17-Jul-1999' & [ & \(0.6017]\) & [ & \(1.0962]\) \\
\hline '18-Jul-1999' & [ & \(2.3546]\) & [ & \(0.4459]\) \\
\hline '19-Jul-1999' & [ & \(1.3080]\) & [ & \(0.6304]\) \\
\hline '20-Jul-1999' & [ & \(1.8682]\) & [ & \(0.2451]\) \\
\hline '21-Jul-1999' & [ & \(0.3509]\) & [ & \(0.6876]\) \\
\hline '22-Jul-1999' & [ & \(4.6444]\) & [ & \(0.6244]\) \\
\hline '23-Jul-1999' & [ & 1.5441] & [ & \(5.7621]\) \\
\hline '24-Jul-1999' & [ & 0.1470 ] & [ & \(2.1238]\) \\
\hline '25-Jul-1999' & [ & 1.5999] & [ & 1.0671 ] \\
\hline '26-Jul-1999' & [ & \(3.5764]\) & [ & \(0.7462]\) \\
\hline
\end{tabular}
```

| '27-Jul-1999' | $[$ | $1.8937]$ | $[$ | $1.0863]$ |
| :--- | :--- | :--- | :--- | :--- |
| $' 28-J u l-1999 '$ | $[$ | $3.9780]$ | $[$ | $2.1516] \ldots$ |

The result is exactly the same as the first syntax. The only difference between the first and second syntax is the way the inputs are entered into the constructor function.

Time-of-Day Information Included. With this form of the function you can enter the time-of-day information either as a serial date number or as a date string. If more than one serial date and time are present, the entry must be in the form of a column-oriented matrix. If more than one string date and time are present, the entry must be a column-oriented cell array of dates and times.

With date string input the dates and times can initially be separate column-oriented date and time series, but you must concatenate them into a single column-oriented cell array before entering them as the first input to fints.

For date string input the allowable formats are

- 'ddmmmyy hh:mm' or 'ddmmmyyyy hh:mm'
- 'mm/dd/yy hh:mm' or 'mm/dd/yyyy hh:mm'
- 'dd-mmm-yy hh:mm' or 'dd-mmm-yyyy hh:mm'
- 'mmm.dd,yy hh:mm' or 'mmm.dd, yyyy hh:mm'

The next example shows time-of-day information input as serial date numbers in a column-oriented matrix:

```
f = fints([now;now+1],(1:2)')
f =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{llll} 
'dates: (2)' & 'times: & (2)' & 'series1: \\
'29-Nov-2001' & (2)' \(15: 22^{\prime}\) & & {\([\)} \\
'30-Nov-2001' & \(' 15: 22^{\prime}\) & & {\([\)}
\end{tabular}
```

If the time-of-day information is in date string format, you must provide it to fints as a column-oriented cell array:

```
f = fints({'01-Jan-2001 12:00';'02-Jan-2001 12:00'},(1:2)')
f =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{lllll} 
'dates: (2)' & 'times: & (2)' & 'series1: & (2)' \\
'01-Jan-2001' & '12:00' & {\([\)} & \(1]\) \\
'02-Jan-2001' & '12:00' & {\([\)} & \(2]\)
\end{tabular}
```

If the dates and times are in date string format and contained in separate matrices, you must concatenate them before using the date and time information as input to fints:

```
dates = ['01-Jan-2001'; '02-Jan-2001'; '03-Jan-2001'];
times = ['12:00';'12:00';'12:00'];
dates_time = cellstr([dates,repmat(' ',size(dates,1),1),times]);
f = fints(dates_time,(1:3)')
f =
    desc: (none)
    freq: Unknown (0)
    'dates: (3)' 'times: (3)' 'series1: (3)'
    '01-Jan-2001' '12:00' [ 1]
    '02-Jan-2001' '12:00' [ 2]
    '03-Jan-2001' '12:00' [ 3]
```


## Data Name Input

```
fts = fints(dates, data, datanames)
```

The third syntax lets you specify the names for the data series with the argument datanames. The datanames argument can be a MATLAB string
for a single data series. For multiple data series names, it must be a cell array of strings.

Look at two examples, one with a single data series and a second with two. The first example sets the data series name to the specified name First:

```
dates = (today:today+100)';
data = exp(randn(1, 101))';
fts1 = fints(dates, data, 'First')
fts1 =
    desc: (none)
    freq: Unknown (0)
    'dates: (101)' 'First: (101)'
    '12-Jul-1999' [ 0.4615]
    '13-Jul-1999' [ 1.1640]
    '14-Jul-1999' [ 0.7140]
    '15-Jul-1999' [ 2.6400]
    '16-Jul-1999' [ 0.8983]
    '17-Jul-1999' [ 2.7552]
    '18-Jul-1999' [ 0.6217]
    '19-Jul-1999' [ 1.0714]
    '20-Jul-1999' [ 1.4897]
    '21-Jul-1999' [ 3.0536]
    '22-Jul-1999' [ 1.8598]
    '23-Jul-1999' [ 0.7500]
    '24-Jul-1999' [ 0.2537]
    '25-Jul-1999' [ 0.5037]
    '26-Jul-1999' [ 1.3933]
    '27-Jul-1999' [ 0.3687]...
```

The second example provides two data series named First and Second:

```
dates = (today:today+100)';
data_series1 = exp(randn(1, 101))';
data_series2 = exp(randn(1, 101))';
data = [data_series1 data_series2];
fts2 = fints(dates, data, {'First', 'Second'})
```

```
fts2 =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{|c|c|c|c|c|}
\hline ates: (101) & First: & (101) & 'Second: & (101) \\
\hline '12-Jul-1999' & [ & 1.2305] & [ & \(0.7396]\) \\
\hline '13-Jul-1999' & [ & 1.2473 ] & [ & \(2.6038]\) \\
\hline '14-Jul-1999' & [ & \(0.3657]\) & [ & \(0.5866]\) \\
\hline '15-Jul-1999' & [ & \(0.6357]\) & [ & 0.4061 ] \\
\hline '16-Jul-1999' & [ & \(4.0530]\) & [ & \(0.4096]\) \\
\hline '17-Jul-1999' & [ & \(0.6300]\) & [ & \(1.3214]\) \\
\hline '18-Jul-1999' & [ & 1.0333] & [ & \(0.4744]\) \\
\hline '19-Jul-1999' & [ & \(2.2228]\) & [ & 4.9702] \\
\hline '20-Jul-1999' & [ & \(2.4518]\) & [ & 1.7758 ] \\
\hline '21-Jul-1999' & [ & \(1.1479]\) & [ & 1.3780 ] \\
\hline '22-Jul-1999' & [ & 0.1981 ] & [ & \(0.8595]\) \\
\hline '23-Jul-1999' & [ & \(0.1927]\) & [ & \(1.3713]\) \\
\hline '24-Jul-1999' & [ & 1.5353] & [ & \(3.8332]\) \\
\hline '25-Jul-1999' & [ & \(0.4784]\) & [ & 0.1067 ] \\
\hline '26-Jul-1999' & [ & 1.7593] & [ & \(3.6434]\) \\
\hline '27-Jul-1999' & [ & \(0.2505]\) & [ & \(0.6849]\) \\
\hline '28-Jul-1999' & [ & 1.5845] & [ & \(1.0025]\) \\
\hline
\end{tabular}
```

Note Data series names must be valid MATLAB variable names. The only allowed nonalphanumeric character is the underscore ( $\_$) character.

Because freq for fts2 has not been explicitly indicated, the frequency indicator for fts2 is set to Unknown. Set the frequency indicator field freq before you attempt any operations on the object. You will not be able to use the object until the frequency indicator field is set to a valid indicator.

## Frequency Indicator Input

```
fts = fints(dates, data, datanames, freq)
```

With the fourth syntax you can set the frequency indicator field when you create the financial time series object. The frequency indicator field freq is set as the fourth input argument. You will not be able to use the financial time series object until freq is set to a valid indicator. Valid frequency indicators are

```
UNKNOWN, Unknown, unknown, U, u,O
DAILY, Daily, daily, D, d,1
WEEKLY, Weekly, weekly, W, w,2
MONTHLY, Monthly, monthly, M, m,3
QUARTERLY, Quarterly, quarterly, Q, q,4
SEMIANNUAL, Semiannual, semiannual, S, s,5
ANNUAL, Annual, annual, A, a,6
```

The previous example contained sets of daily data. The freq field displayed as Unknown ( 0 ) because the frequency indicator was not explicitly set. The command

```
fts = fints(dates, data, {'First', 'Second'}, 1);
```

sets the freq indicator to Daily(1) when creating the financial time series object:

```
fts =
desc: (none)
freq: Daily (1)
'dates: (101)' 'First: (101)' 'Second: (101)'
'12-Jul-1999' [ 1.2305] [ 0.7396]
'13-Jul-1999' [ 1.2473] [ 2.6038]
'14-Jul-1999' [ 0.3657] [ 0.5866]
'15-Jul-1999' [ 0.6357] [ 0.4061]
'16-Jul-1999' [ 4.0530] [ 0.4096]
'17-Jul-1999' [ 0.6300] [ 1.3214]
'18-Jul-1999' [ 1.0333] [ 0.4744]...
```

When you create the object using this syntax, you can use the other valid frequency indicators for a particular frequency. For a daily data set you can
use DAILY, Daily, daily, D, or d. Similarly, with the other frequencies, you can use the valid string indicators or their numeric counterparts.

## Description Field Input

```
fts = fints(dates, data, datanames, freq, desc)
```

With the fifth syntax, you can explicitly set the description field as the fifth input argument. The description can be anything you want. It is not used in any operations performed on the object.

This example sets the desc field to 'Test TS'.

```
dates = (today:today+100)';
data_series1 = exp(randn(1, 101))';
data_series2 = exp(randn(1, 101))';
data = [data_series1 data_series2];
fts = fints(dates, data, {'First', 'Second'}, 1, 'Test TS')
fts =
    desc: Test TS
    freq: Daily (1)
    'dates: (101)' 'First: (101)' 'Second: (101)'
    '12-Jul-1999' [ 0.5428] [ 1.2491]
    '13-Jul-1999' [ 0.6649] [ 6.4969]
    '14-Jul-1999' [ 0.2428] [ 1.1163]
    '15-Jul-1999' [ 1.2550] [ 0.6628]
    '16-Jul-1999' [ 1.2312] [ 1.6674]
    '17-Jul-1999' [ 0.4869] [ 0.3015]
    '18-Jul-1999' [ 2.1335] [ 0.9081]...
```

Now the description field is filled with the specified string 'Test TS' when the constructor is called.

## Transforming a Text File

The function ascii2fts creates a financial time series object from a text (ASCII) data file provided that the data file conforms to a general format. The general format of the text data file is as follows:

- Can contain header text lines.
- Can contain column header information. The column header information must immediately precede the data series columns unless the skiprows argument (see below) is specified.
- Leftmost column must be the date column.
- Dates must be in a valid date string format.
- 'ddmmmyy' or 'ddmmmyyyy'
- 'mm/dd/yy' or 'mm/dd/yyyy'
- 'dd-mmm-yy' or 'dd-mmm-yyyy'
- 'mmm.dd,yy' or 'mmm.dd,yyyy'
- Each column must be separated either by spaces or a tab.

Several example text data files are included with the toolbox. These files are in the ftsdata subdirectory within the directory matlabroot/toolbox/finance.

The syntax of the function

```
fts = ascii2fts(filename, descrow, colheadrow, skiprows);
```

takes in the data file name (filename), the row number where the text for the description field is (descrow), the row number of the column header information (colheadrow), and the row numbers of rows to be skipped (skiprows). For example, rows need to be skipped when there are intervening rows between the column head row and the start of the time series data.

Look at the beginning of the ASCII file disney.dat in the ftsdata subdirectory:
Walt Disney Company (DIS)

| Daily prices ( $3 / 29 / 96$ to | $3 / 29 / 99$ ) |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| DATE | OPEN | HIGH | LOW | CLOSE | VOLUME |
| $3 / 29 / 99$ | 33.0625 | 33.188 | 32.75 | 33.063 | 6320500 |
| $3 / 26 / 99$ | 33.3125 | 33.375 | 32.75 | 32.938 | 5552800 |
| $3 / 25 / 99$ | 33.5 | 33.625 | 32.875 | 33.375 | 7936000 |
| $3 / 24 / 99$ | 33.0625 | 33.25 | 32.625 | 33.188 | $6025400 \ldots$ |

The command line

```
disfts = ascii2fts('disney.dat', 1, 3, 2)
```

uses disney. dat to create time series object disfts. This example

- Reads the text data file disney.dat
- Uses the first line in the file as the content of the description field
- Skips the second line
- Parses the third line in the file for column header (or data series names)
- Parses the rest of the file for the date vector and the data series values

The resulting financial time series object looks like this.

| desc: Walt Disney Company (DIS) <br> freq: Unknown (0) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 'dates: (782)' |  | 'OPEN: (782)' |  | 'HIGH: (782)' | 'LOW: (782) ${ }^{\prime}$ |
| ' 29-Mar-1996' | [ | 21.1938 ] | [ | 21.6250] [ | $21.2920]$ |
| '01-Apr-1996' | [ | $21.1120]$ | [ | 21.6250] [ | $21.4170]$ |
| '02-Apr-1996' | [ | $21.3165]$ | [ | 21.8750 ] [ | $21.6670]$ |
| '03-Apr-1996' | [ | $21.4802]$ | [ | 21.8750 ] [ | $21.7500]$ |
| '04-Apr-1996' | [ | $21.4393]$ | [ | 21.8750] [ | $21.5000]$ |
| '05-Apr-1996' | [ | $\mathrm{NaN}]$ | [ | $\mathrm{NaN}]$ [ | $\mathrm{NaN}]$ |
| '09-Apr-1996' | [ | $21.1529]$ | [ | 21.5420] [ | $21.2080]$ |
| '10-Apr-1996' | [ | 20.7387] | [ | 21.1670] [ | 20.2500] |
| '11-Apr-1996' | [ | 20.0829] | [ | 20.5000] [ | $20.0420]$ |
| '12-Apr-1996' | [ | 19.9189] | [ | 20.5830] [ | $20.0830]$ |
| '15-Apr-1996' | [ | 20.2878 ] | [ | 20.7920] [ | $20.3750]$ |
| '16-Apr-1996' | [ | 20.3698] | [ | 20.9170] [ | 20.1670] |
| '17-Apr-1996' | [ | 20.4927 ] | [ | 20.9170] [ | $20.7080]$ |
| '18-Apr-1996' | [ | 20.4927] | [ | 21.0420] [ | $20.7920]$ |

There are 782 data points in this object. Only the first few lines are shown here. Also, this object has two other data series, the CLOSE and VOLUME data
series, that are not shown here. Note that in creating the financial time series object, ascii2fts sorts the data into ascending chronological order.

The frequency indicator field, freq, is set to 0 for Unknown frequency. You can manually reset it to the appropriate frequency using structure syntax disfts.freq $=1$ for Daily frequency.

With a slightly different syntax, the function ascii2fts can create a financial time series object when time-of-day data is present in the ASCII file. The new syntax has the form

```
fts = ascii2fts(filename, timedata, descrow, colheadrow,
skiprows);
```

Set timedata to ' $T$ ' when time-of-day data is present and to ' $N T$ ' when there is no time data. For an example using this function with time-of-day data, see the reference page for ascii2fts.

# Visualizing Financial Time Series Objects 

In this section...<br>"Introduction" on page 9-18<br>"Using chartfts" on page 9-18<br>"Zoom Tool" on page 9-21<br>"Combine Axes Tool" on page 9-24

## Introduction

Financial Toolbox software contains the function chartfts, which provides a visual representation of a financial time series object. chartfts is an interactive charting and graphing utility for financial time series objects. With this function, you can observe time series values on the entire range of dates covered by the time series.

Note Interactive charting is also available from the Graphs menu of the graphical user interface. See "Interactive Chart" on page 12-17 for additional information.

## Using chartfts

chartfts requires a single input argument, tsobj, where tsobj is the name of the financial time series object you want to explore. Most equity financial time series objects contain four price series, such as opening, closing, highest, and lowest prices, plus an additional series containing the volume traded. However, chartfts is not limited to a time series of equity prices and volume traded. It can be used to display any time series data you may have.

To illustrate the use of chartfts, use the equity price and volume traded data for the Walt Disney Corporation (NYSE: DIS) provided in the file disney.mat:
load disney.mat
whos

| Name | Size | Bytes | Class |
| :--- | :---: | ---: | :--- |
|  |  |  |  |
| dis | $782 \times 5$ | 39290 | fints object |
| dis_CLOSE | $782 \times 1$ | 6256 | double array |
| dis_HIGH | $782 \times 1$ | 6256 | double array |
| dis_LOW | $782 \times 1$ | 6256 | double array |
| dis_OPEN | $782 \times 1$ | 6256 | double array |
| dis_VOLUME | $782 \times 1$ | 6256 | double array |
| dis_nv | $782 \times 4$ | 32930 | fints object |
| q_dis | $13 \times 4$ | 2196 fints object |  |

For charting purposes look only at the objects dis (daily equity data including volume traded) and dis_nv (daily data without volume traded). Both objects contain the series OPEN, HIGH, LOW, and CLOSE, but only dis contains the additional VOLUME series.

Use chartfts(dis) to observe the values.


The chart contains five plots, each representing one of the series in the time series object. Boxes indicate the value of each individual plot. The date box is always on the left. The number of data boxes on the right depends upon the number of data series in the time series object, five in this case. The order in which these boxes are arranged (left to right) matches the plots from top to bottom. With more than eight data series in the object, the scroll bar on the right is activated so that additional data from the other series can be brought into view.

Slide the mouse cursor over the chart. A vertical bar appears across all plots. This bar selects the set of data shown in the boxes below. Move this bar horizontally and the data changes accordingly.


Click the plot. A small information box displays the data at the point where you click the mouse button.


## Zoom Tool

The zoom feature of chartfts enables a more detailed look at the data during a selected time frame. The Zoom tool is found under the Chart Tools menu.


Note Due to the specialized nature of this feature, do not use the MATLAB zoom command or Zoom In and Zoom Out from the Tools menu.

When the feature is turned on, you will see two inactive buttons (ZOOM In and Reset ZOOM) above the boxes. The buttons become active later after certain actions have been performed.


The window title bar displays the status of the chart tool that you are using. With the Zoom tool turned on, you see Zoom ON in the title bar in addition to the name of the time series you are working with. When the tool is off, no status is displayed.


To zoom into the chart, you need to define the starting and ending dates. Define the starting date by moving the cursor over the chart until the desired date appears at the bottom-left box and click the mouse button. A blue vertical line indicates the starting date you have selected. Next, again move the cursor over the chart until the desired ending date appears in the box and click the mouse once again. This time, a red vertical line appears and the ZOOM In button is activated.


To zoom into the chart, click the ZOOM In button.


The chart is zoomed in. Note that the Reset ZOOM button now becomes active while the ZOOM In button becomes inactive again. To return the chart
to its original state (not zoomed), click the Reset ZOOM button. To zoom into the chart even further, repeat the steps above for zooming into the chart.

Turn the Zoom tool off by going back to the Chart Tools menu and choosing Zoom Off.


With the tool turned off, the chart stays at the last state that it was in. If you turn it off when the chart is zoomed in, the chart stays zoomed in. If you reset the zoom before turning it off, the chart becomes the original (not zoomed).

## Combine Axes Tool

The Combine Axes tool allows you to combine all axes or specific axes into one. With axes combined, you can visually spot any trends that can occur among the data series in a financial time series object.

To illustrate this tool, use dis_nv, the financial time series object that does not contain volume traded data:

```
chartfts(dis_nv)
```



To combine axes, choose the Chart Tools menu, followed by Combine Axes and $\mathbf{O n}$.


When the Combine Axes tool is on, check boxes appear beside each individual plot. An additional check box enables the combination of all plots.


## Combining All Axes

To combine all plots, select the Select all plots check box.


Now click the Combine Selected Graphs button to combine the chosen plots. In this case, all plots are combined.


The combined plots have a single plot axis with all data series traced. The background of each data box has changed to the color corresponding to the color of the trace that represents the data series. After the axes are combined, the tool is turned off.

## Combining Selected Axes

You can choose any combination of the available axes to combine. For example, combine the HIGH and LOW price series of the Disney time series. Click the check boxes next to the corresponding plots. The Combine Selected Graphs button appears and is active.


Click the Combine Selected Graphs button. The chart with the combined plots looks like the next figure.


The plot with the combined axes is located at the top of the chart while the remaining plots follow it. The data boxes have also been changed. The boxes that correspond to the combined axes are relocated to the beginning, and the background colors are set to the color of the respective traces. The data boxes for the remaining axes retain their original formats.

## Resetting Axes

If you have altered the chart by combining axes, you must reset the axes before you can visualize additional combinations. Reset the axes with the Reset Axes menu item under Chart Tools > Combine Axes. Note that now the On and Off features are turned off.


With axes reset, the interactive chart appears in its original format, and you can proceed with additional axes combinations.

| - Interactive Chart: Walt Disney Company (DIS) - No volume |  |  |  |  | - [a] $\times$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Eile Edit Yiew Insert Iools Window Help Chart Tools |  |  |  |  |  |
| Walt Disney Company (DIS) - No VOLUME |  |  |  |  |  |
| z芫00 |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| Dates:28-Jun-1996 | $\begin{aligned} & \text { OPEN: } \\ & 20.8206 \end{aligned}$ | $\left\lvert\, \begin{aligned} & \text { HIGH: } \\ & 21.25 \end{aligned}\right.$ | $\left\lvert\, \begin{aligned} & \text { LOW: } \\ & 20.875 \end{aligned}\right.$ | $\left\lvert\, \begin{aligned} & \text { CLOSE: } \\ & 20.958 \end{aligned}\right.$ | $\triangle$ |
|  |  |  |  |  | $-$ |

# Using Financial Time Series 

- "Introduction" on page 10-2
- "Working with Financial Time Series Objects" on page 10-3
- "Demonstration Program" on page 10-25


## Introduction

This chapter discusses how to manipulate and analyze financial time series data. The major topics discussed include

- "Financial Time Series Object Structure" on page 10-3
- "Data Extraction" on page 10-4
- "Object-to-Matrix Conversion" on page 10-6
- "Indexing a Financial Time Series Object" on page 10-8
- "Operations" on page 10-15
- "Data Transformation and Frequency Conversion" on page 10-19

Much of this information is summarized in the "Demonstration Program" on page 10-25.

## Working with Financial Time Series Objects

In this section...<br>"Introduction" on page 10-3<br>"Financial Time Series Object Structure" on page 10-3<br>"Data Extraction" on page 10-4<br>"Object-to-Matrix Conversion" on page 10-6<br>"Indexing a Financial Time Series Object" on page 10-8<br>"Operations" on page 10-15<br>"Data Transformation and Frequency Conversion" on page 10-19

## Introduction

A financial time series object is designed to be used as if it were a MATLAB structure. (See the MATLAB documentation for a description of MATLAB structures or how to use MATLAB in general.)

This part of the tutorial assumes that you know how to use MATLAB and are familiar with MATLAB structures. The terminology is similar to that of a MATLAB structure. The financial time series object term component is interchangeable with the MATLAB structure term field.

## Financial Time Series Object Structure

A financial time series object always contains three component names: desc (description field), freq (frequency indicator field), and dates (date vector). If you build the object using the constructor fints, the default value for the description field is a blank string (' ' ). If you build the object from a text data file using ascii2fts, the default is the name of the text data file. The default for the frequency indicator field is 0 (Unknown frequency). Objects created from operations can default the setting to 0 . For example, if you decide to pick out values selectively from an object, the frequency of the new object might not be the same as that of the object from which it came.

The date vector dates does not have a default set of values. When you create an object, you have to supply the date vector. You can change the date vector afterward but, at object creation time, you must provide a set of dates.

The final component of a financial time series object is one or more data series vectors. If you do not supply a name for the data series, the default name is series1. If you have multiple data series in an object and do not supply the names, the default is the name series followed by a number, for example, series1, series2, and series3.

## Data Extraction

Here is an exercise on how to extract data from a financial time series object. As mentioned before, you can think of the object as a MATLAB structure. Highlight each line in the exercise in the MATLAB Help browser, press the right mouse button, and select Evaluate Selection to execute it.

To begin, create a financial time series object called myfts:

```
dates = (datenum('05/11/99'):datenum('05/11/99')+100)';
data_series1 = exp(randn(1, 101))';
data_series2 = exp(randn(1, 101))';
data = [data_series1 data_series2];
myfts = fints(dates, data);
```

The myfts object looks like this:

```
myfts =
    desc: (none)
    freq: Unknown (0)
    'dates: (101)' 'series1: (101)' 'series2: (101)'
    '11-May-1999' [ 2.8108] [ 0.9323]
    '12-May-1999' [ 0.2454] [ 0.5608]
    '13-May-1999' [ 0.3568] [ 1.5989]
    '14-May-1999' [ 0.5255] [ 3.6682]
    '15-May-1999' [ 1.1862] [ 5.1284]
    '16-May-1999' [ 3.8376] [ 0.4952]
    '17-May-1999' [ 6.9329] [ 2.2417]
```

| '18-May-1999' | [ | $2.0987]$ | $[$ | $0.3579]$ |
| :--- | :--- | :--- | :--- | :--- |
| '19-May-1999' | [ | $2.2524]$ | $[$ | $3.6492]$ |
| '20-May-1999' | [ | $0.8669]$ | $[$ | $1.0150]$ |
| '21-May-1999' | [ | $0.9050]$ | $[$ | $1.2445]$ |
| '22-May-1999' | [ | $0.4493]$ | $[$ | $5.5466]$ |
| '23-May-1999' | [ | $1.6376]$ | $[$ | $0.1251]$ |
| '24-May-1999' | [ | $3.4472]$ | $[$ | $1.1195]$ |
| '25-May-1999' | [ | $3.6545]$ | $[$ | $0.3374] \ldots$ |

There are more dates in the object; only the first few lines are shown here.

Note The actual data in your series1 and series2 will differ from the above because of the use of random numbers.

Now create another object with only the values for series2:

```
srs2 = myfts.series2
srs2 =
    desc: (none)
    freq: Unknown (0)
    dates: (101)' 'series2: (101)'
    '11-May-1999' [ 0.9323]
    '12-May-1999' [ 0.5608]
    '13-May-1999' [ 1.5989]
    '14-May-1999' [ 3.6682]
    '15-May-1999' [ 5.1284]
    '16-May-1999' [ 0.4952]
    '17-May-1999' [ 2.2417]
    '18-May-1999' [ 0.3579]
    '19-May-1999' [ 3.6492]
    '20-May-1999' [ 1.0150]
    '21-May-1999' [ 1.2445]
    '22-May-1999' [ 5.5466]
    '23-May-1999' [ 0.1251]
    '24-May-1999' [ 1.1195]
```

$$
\text { '25-May-1999' } \quad[\quad 0.3374] . .
$$

The new object srs2 contains all the dates in myfts, but the only data series is series2. The name of the data series retains its name from the original object, myfts.

Note The output from referencing a data series field or indexing a financial time series object is always another financial time series object. The exceptions are referencing the description, frequency indicator, and dates fields, and indexing into the dates field.

## Object-to-Matrix Conversion

The function fts2mat extracts the dates and/or the data series values from an object and places them into a vector or a matrix. The default behavior extracts just the values into a vector or a matrix. Look at the next example:

```
srs2_vec = fts2mat(myfts.series2)
srs2_vec =
    0.9323
    0.5608
    1.5989
    3.6682
    5.1284
    0.4952
    2.2417
    0.3579
    3.6492
    1.0150
    1.2445
    5.5466
    0.1251
    1.1195
    0.3374...
```

If you want to include the dates in the output matrix, provide a second input argument and set it to 1 . This results in a matrix whose first column is a vector of serial date numbers:

```
format long g
srs2_mtx = fts2mat(myfts.series2, 1)
srs2_mtx =
    730251 0.932251754559576
    730252 0.560845677519876
    730253 1.59888712183914
    730254 3.6681500883527
    730255 5.12842215360269
    730256 0.49519254119977
    730257 2.24174134286213
    730258 0.357918065917634
    730259 3.64915665824198
    730260 1.01504236943148
    730261 1.24446420606078
    730262 5.54661849025711
    730263 0.12507959735904
    730264 1.11953883096805
    730265 0.337398214166607
```

The vector srs2_vec contains just series2 values. The matrix srs2_mtx contains dates in the first column and the values of the series2 data series in the second. Dates in the first column are in serial date format. Serial date format is a representation of the date string format (for example, serial date $=1$ is equivalent to 01-Jan-0000). (The serial date vector can include time-of-day information.)

The long g display format displays the numbers without exponentiation. (To revert to the default display format, use format short. (See the format command in the MATLAB documentation for a description of MATLAB display formats.) Remember that both the vector and the matrix have 101 rows of data as in the original object myfts but are shown truncated here.

## Indexing a Financial Time Series Object

You can also index into the object as with any other MATLAB variable or structure. A financial time series object lets you use a date string, a cell array of date strings, a date string range, or normal integer indexing. You cannot, however, index into the object using serial dates. If you have serial dates, you must first use the MATLAB datestr command to convert them into date strings.

When indexing by date string, note that

- Each date string must contain the day, month, and year. Valid formats are
- 'ddmmmyy hh:mm' or 'ddmmmyyyy hh:mm'
- 'mm/dd/yy hh:mm' or 'mm/dd/yyyy hh:mm'
- 'dd-mmm-yy hh:mm' or 'dd-mmm-yyyy hh:mm'
- 'mmm.dd,yy hh:mm' or 'mmm.dd,yyyy hh:mm'
- All data falls at the end of the indicated time period, that is, weekly data falls on Fridays, monthly data falls on the end of each month, and so on, whenever the data has gone through a frequency conversion.


## Indexing with Date Strings

With date string indexing you get the values in a financial time series object for a specific date using a date string as the index into the object. Similarly, if you want values for multiple dates in the object, you can put those date strings into a cell array and use the cell array as the index to the object. Here are some examples.

This example extracts all values for May 11, 1999 from myfts:

```
format short
myfts('05/11/99')
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (1)' 'series1: (1)' 'series2: (1)'
```

```
'11-May-1999' [ 2.8108] [ 0.9323]
```

The next example extracts only series2 values for May 11, 1999 from myfts:

```
myfts.series2('05/11/99')
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (1)' 'series2: (1)'
    '11-May-1999' [ 0.9323]
```

The third example extracts all values for three different dates:

```
myfts({'05/11/99', '05/21/99', '05/31/99'})
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (3)' 'series1: (3)' 'series2: (3)
    '11-May-1999' [ 2.8108] [ 0.9323]
    '21-May-1999' [ 0.9050] [ 1.2445]
    '31-May-1999' [ 1.4266] [ 0.6470]
```

The next example extracts only series2 values for the same three dates:

```
myfts.series2({'05/11/99', '05/21/99', '05/31/99'})
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (3)' 'series2: (3)'
    '11-May-1999' [ 0.9323]
    '21-May-1999' [ 1.2445]
```

$$
\text { '31-May-1999' [ } 0.6470]
$$

## Indexing with Date String Range

A financial time series is unique because it allows you to index into the object using a date string range. A date string range consists of two date strings separated by two colons (: :). In MATLAB this separator is called the double-colon operator. An example of a MATLAB date string range is '05/11/99::05/31/99'. The operator gives you all data points available between those dates, including the start and end dates.

Here are some date string range examples:

```
myfts ('05/11/99::05/15/99')
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'series1: (5)' 'series2: (5)'
    '11-May-1999' [ 2.8108] [ 0.9323]
    '12-May-1999' [ 0.2454] [ 0.5608]
    '13-May-1999' [ 0.3568] [ 1.5989]
    '14-May-1999' [ 0.5255] [ 3.6682]
    '15-May-1999' [ 1.1862] [ 5.1284]
myfts.series2('05/11/99::05/15/99')
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'series2: (5)'
    '11-May-1999' [ 0.9323]
    '12-May-1999' [ 0.5608]
    '13-May-1999' [ 1.5989]
    '14-May-1999' [ 3.6682]
    '15-May-1999' [ 5.1284]
```

As with any other MATLAB variable or structure, you can assign the output to another object variable:

```
nfts = myfts.series2('05/11/99::05/20/99');
```

nfts is the same as ans in the second example.
If one of the dates does not exist in the object, an error message indicates that one or both date indexes are out of the range of the available dates in the object. You can either display the contents of the object or use the command ftsbound to determine the first and last dates in the object.

## Indexing with Integers

Integer indexing is the normal form of indexing in MATLAB. Indexing starts at 1 (not 0 ); index $=1$ corresponds to the first element, index $=2$ to the second element, index $=3$ to the third element, and so on. Here are some examples with and without data series reference.

Get the first item in series2:

```
myfts.series2(1)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (1)' 'series2: (1)'
    '11-May-1999' [ 0.9323]
```

Get the first, third, and fifth items in series2:

```
myfts.series2([1, 3, 5])
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (3)' 'series2: (3)'
```

| '11-May-1999' | $[$ | $0.9323]$ |
| :--- | :--- | :--- |
| '13-May-1999' | $[$ | $1.5989]$ |
| $15-$ May-1999' | $[$ | $5.1284]$ |

Get items 16 through 20 in series2:

```
myfts.series2(16:20)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'series2: (5)'
    '26-May-1999' [ 0.2105]
    '27-May-1999' [ 1.8916]
    '28-May-1999' [ 0.6673]
    '29-May-1999' [ 0.6681]
    '30-May-1999' [ 1.0877]
```

Get items 16 through 20 in the financial time series object myfts:

```
myfts(16:20)
ans =
    desc: (none)
    freq: Unknown (0)
    'dates: (5)' 'series1: (5)' 'series2: (5)'
    '26-May-1999' [ 0.7571] [ 0.2105]
    '27-May-1999' [ 1.2425] [ 1.8916]
    '28-May-1999' [ 1.8790] [ 0.6673]
    '29-May-1999' [ 0.5778] [ 0.6681]
    '30-May-1999' [ 1.2581] [ 1.0877]
```

Get the last item in myfts:
myfts(end)

```
ans =
desc: (none)
freq: Unknown (0)
'dates: (1)' 'series1: (1)' 'series2: (1)
'19-Aug-1999' [ 1.4692] [ 3.4238]
```

This example uses the MATLAB special variable end, which points to the last element of the object when used as an index. The example returns an object whose contents are the values in the object myfts on the last date entry.

## Indexing When Time-of-Day Data Is Present

Both integer and date string indexing are permitted when time-of-day information is present in the financial time series object. You can index into the object with both date and time specifications, but not with time of day alone. To show how indexing works with time-of-day data present, create a financial time series object called timeday containing a time specification:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
    times]);
timeday = fints(dates_times,(1:6)',{'Data1'},1,'My first FINTS')
timeday =
    desc: My first FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|}
\hline dates: (6)' & times: (6)' & 'Data1: (6)' \\
\hline '01-Jan-2001' & '11:00' & \(1]\) \\
\hline ' " & '12:00' & \(2]\) \\
\hline '02-Jan-2001' & '11:00' & \(3]\) \\
\hline ' " ' & '12:00' & \(4]\) \\
\hline '03-Jan-2001' & '11:00' & 5] \\
\hline ' " ' & '12:00' & \(6]\) \\
\hline
\end{tabular}
```

Use integer indexing to extract the second and third data items from timeday:

```
timeday(2:3)
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (2)' 'times: (2)' 'Data1: (2)
    '01-Jan-2001' '12:00' [ 2]
    '02-Jan-2001' '11:00' [ 3]
```

For date string indexing, enclose the date and time string in one pair of quotation marks. If there is one date with multiple times, indexing with only the date returns the data for all the times for that specific date. For example, the command timeday ('01-Jan-2001') returns the data for all times on January 1, 2001:

```
ans =
    desc: My first FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|}
\hline dates: (2)' & 'times: (2)' & 'Data1: (2)' \\
\hline '01-Jan-2001' & '11:00' & 1] \\
\hline ' " & '12:00' & 2] \\
\hline
\end{tabular}
```

You can also indicate a specific date and time:

```
timeday('01-Jan-2001 12:00')
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (1)' 'times: (1)' 'Data1: (1)'
    '01-Jan-2001' '12:00' [ 2]
```

Use the double-colon operator :: to specify a range of dates and times:

```
timeday('01-Jan-2001 12:00::03-Jan-2001 11:00')
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (4)' 'times: (4)' 'Data1: (4)
    '01-Jan-2001' '12:00' [ 2]
    '02-Jan-2001' '11:00' [ 3]
    ' " ' '12:00' [ 4]
    '03-Jan-2001' '11:00' [ 5]
```

Treat timeday as a MATLAB structure if you want to obtain the contents of a specific field. For example, to find the times of day included in this object, enter

```
datestr(timeday.times)
ans =
11:00 AM
12:00 PM
11:00 AM
12:00 PM
11:00 AM
12:00 PM
```


## Operations

Several MATLAB functions have been overloaded to work with financial time series objects. The overloaded functions include basic arithmetic functions such as addition, subtraction, multiplication, and division and other functions such as arithmetic average, filter, and difference. Also, specific methods have been designed to work with the financial time series object. For a list of functions grouped by type, refer to Chapter 15, "Function Reference" or enter

```
help ftseries
```

at the MATLAB command prompt.

## Basic Arithmetic

Financial time series objects permit you to do addition, subtraction, multiplication, and division, either on the entire object or on specific object fields. This is a feature that MATLAB structures do not allow. You cannot do arithmetic operations on entire MATLAB structures, only on specific fields of a structure.

You can perform arithmetic operations on two financial time series objects as long as they are compatible. (All contents are the same except for the description and the values associated with the data series.)

Note Compatible time series are not the same as equal time series. Two time series objects are equal when everything but the description fields is the same.

Here are some examples of arithmetic operations on financial time series objects.

Load a MAT-file that contains some sample financial time series objects:

```
load dji30short
```

One of the objects in dji30short is called myfts1:

```
myfts1 =
desc: DJI30MAR94.dat
freq: Daily (1)
'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close: (20)'
'04-Mar-1994' [ 3830.90] [ 3868.04] [ 3800.50] [ 3832.30]
'07-Mar-1994' [ 3851.72] [ 3882.40] [ 3824.71] [ 3856.22]
'08-Mar-1994' [ 3858.48] [ 3881.55] [ 3822.45] [ 3851.72]
'09-Mar-1994' [ 3853.97] [ 3874.52] [ 3817.95] [ 3853.41]
'10-Mar-1994' [ 3852.57] [ 3865.51] [ 3801.63] [ 3830.62]...
```

Create another financial time series object that is identical to myfts1:

```
newfts = fints(myfts1.dates, fts2mat(myfts1)/100,...
{'Open','High','Low', 'Close'}, 1, 'New FTS')
newfts =
desc: New FTS
freq: Daily (1)
'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close:(20)'
'04-Mar-1994' [ 38.31] [ 38.68] [ 38.01] [ 38.32]
'07-Mar-1994' [ 38.52] [ 38.82] [ 38.25] [ 38.56]
'08-Mar-1994' [ 38.58] [ 38.82] [ 38.22] [ 38.52]
'09-Mar-1994' [ 38.54] [ 38.75] [ 38.18] [ 38.53]
'10-Mar-1994' [ 38.53] [ 38.66] [ 38.02] [ 38.31]...
```

Perform an addition operation on both time series objects:

```
addup = myfts1 + newfts
addup =
desc: DJI30MAR94.dat
freq: Daily (1)
'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close: (20)'
'04-Mar-1994' [ 3869.21] [ 3906.72] [ 3838.51] [ 3870.62]
'07-Mar-1994' [ 3890.24] [ 3921.22] [ 3862.96] [ 3894.78]
'08-Mar-1994' [ 3897.06] [ 3920.37] [ 3860.67] [ 3890.24]
'09-Mar-1994' [ 3892.51] [ 3913.27] [ 3856.13] [ 3891.94]
'10-Mar-1994' [ 3891.10] [ 3904.17] [ 3839.65] [ 3868.93]...
```

Now, perform a subtraction operation on both time series objects:

```
subout = myfts1 - newfts
subout =
desc: DJI30MAR94.dat
freq: Daily (1)
```

```
'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close: (20)'
'04-Mar-1994' [ 3792.59] [ 3829.36] [ 3762.49] [ 3793.98]
'O7-Mar-1994' [ 3813.20] [ 3843.58] [ 3786.46] [ 3817.66]
'O8-Mar-1994' [ 3819.90] [ 3842.73] [ 3784.23] [ 3813.20]
09-Mar-1994' [ 3815.43] [ 3835.77] [ 3779.77] [ 3814.88]
'10-Mar-1994' [ 3814.04] [ 3826.85] [ 3763.61] [ 3792.31]...
```


## Operations with Objects and Matrices

You can also perform operations involving a financial time series object and a matrix or scalar:

```
addscalar = myfts1 + 10000
addscalar =
desc: DJI30MAR94.dat
freq: Daily (1)
'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close: (20)'
'04-Mar-1994' [ 13830.90] [ 13868.04] [ 13800.50] [ 13832.30]
'07-Mar-1994' [ 13851.72] [ 13882.40] [ 13824.71] [ 13856.22]
'08-Mar-1994' [ 13858.48] [ 13881.55] [ 13822.45] [ 13851.72]
'09-Mar-1994' [ 13853.97] [ 13874.52] [ 13817.95] [ 13853.41]
'10-Mar-1994' [ 13852.57] [ 13865.51] [ 13801.63] [ 13862.70]...
```

For operations with both an object and a matrix, the size of the matrix must match the size of the object. For example, a matrix to be subtracted from myfts 1 must be 20 -by- 4 , since myfts 1 has 20 dates and four data series:

```
submtx = myfts1 - randn(20, 4)
submtx =
desc: DJI30MAR94.dat
freq: Daily (1)
'dates: (20)' 'Open: (20)' 'High: (20)' 'Low: (20)' 'Close: (20)'
'04-Mar-1994' [ 3831.33] [ 3867.75] [ 3802.10] [ 3832.63]
'07-Mar-1994' [ 3853.39] [ 3883.74] [ 3824.45] [ 3857.06]
```

| '08-Mar-1994' | $[3858.35]$ | $[3880.84]$ | $[3823.51]$ | $[3851.22]$ |
| :--- | :--- | :--- | :--- | :--- |
| ${ }^{\prime}$ 09-Mar-1994' | $[3853.68]$ | $[3872.90]$ | $[3816.53]$ | $[3851.92]$ |
| ${ }^{\prime} 10-$ Mar-1994' | $[3853.72]$ | $[3866.20]$ | $[3802.44]$ | $[3831.17] \ldots$ |

## Arithmetic Operations with Differing Data Series Names

Arithmetic operations on two objects that have the same size but contain different data series names require the function fts2mat. This function extracts the values in an object and puts them into a matrix or vector, whichever is appropriate.

To see an example, create another financial time series object the same size as myfts1 but with different values and data series names:

```
newfts2 = fints(myfts1.dates, fts2mat(myfts1/10000),...
{'Rat1','Rat2', 'Rat3','Rat4'}, 1, 'New FTS')
```

If you attempt to add (or subtract, and so on) this new object to myfts1, an error indicates that the objects are not identical. Although they contain the same dates, number of dates, number of data series, and frequency, the two time series objects do not have the same data series names. Use fts2mat to bypass this problem:

```
addother = myfts1 + fts2mat(newfts2);
```

This operation adds the matrix that contains the contents of the data series in the object newfts2 to myfts1. You should carefully consider the effects on your data before deciding to combine financial time series objects in this manner.

## Other Arithmetic Operations

In addition to the basic arithmetic operations, several other mathematical functions operate directly on financial time series objects. These functions include exponential (exp), natural logarithm (log), common logarithm (log10), and many more. See Chapter 15, "Function Reference" for more details.

## Data Transformation and Frequency Conversion

The data transformation and the frequency conversion functions convert a data series into a different format.

## Data Transformation Functions

| Function | Purpose |
| :--- | :--- |
| boxcox | Box-Cox transformation |
| diff | Differencing |
| fillts | Fill missing values |
| filter | Filter |
| lagts | Lag time series object |
| leadts | Lead time series object |
| peravg | Periodic average |
| smoothts | Smooth data |
| tsmovavg | Moving average |

## Frequency Conversion Functions

| Function | New Frequency |
| :--- | :--- |
| convertto | As specified |
| resamplets | As specified |
| toannual | Annual |
| todaily | Daily |
| tomonthly | Monthly |
| toquarterly | Quarterly |
| tosemi | Semiannually |
| toweekly | Weekly |

As an example look at boxcox, the Box-Cox transformation function. This function transforms the data series contained in a financial time series object into another set of data series with relatively normal distributions.

First create a financial time series object from the supplied whirlpool.dat data file.

```
whrl = ascii2fts('whirlpool.dat', 1, 2, []);
```

Fill any missing values denoted with NaNs in whrl with values calculated using the linear method:

```
f_whrl = fillts(whrl);
```

Transform the nonnormally distributed filled data series f_whrl into a normally distributed one using Box-Cox transformation:

```
bc_whrl = boxcox(f_whrl);
```

Compare the result of the Close data series with a normal (Gaussian) probability distribution function and the nonnormally distributed f_whrl:

```
subplot(2, 1, 1);
hist(f_whrl.Close);
grid; title('Nonnormally Distributed Data');
subplot(2, 1, 2);
hist(bc_whrl.Close);
grid; title('Box-Cox Transformed Data');
```



## Box-Cox Transformation

The bar chart on the top represents the probability distribution function of the filled data series, f_whrl, which is the original data series whrl with the missing values interpolated using the linear method. The distribution is skewed toward the left (not normally distributed). The bar chart on the bottom is less skewed to the left. If you plot a Gaussian probability distribution function (PDF) with similar mean and standard deviation, the distribution of the transformed data is very close to normal (Gaussian).

When you examine the contents of the resulting object bc_whrl, you find an identical object to the original object whrl but the contents are the transformed data series. If you have the Statistics Toolbox software, you can generate a Gaussian PDF with mean and standard deviation equal to those of the transformed data series and plot it as an overlay to the second bar chart. In the next figure, you can see that it is an approximately normal distribution.


## Overlay of Gaussian PDF

The next example uses the smoothts function to smooth a time series.
To begin, transform ibm9599.dat, a supplied data file, into a financial time series object:

```
ibm = ascii2fts('ibm9599.dat', 1, 3, 2);
```

Fill the missing data for holidays with data interpolated using the fillts function and the Spline fill method:
f_ibm = fillts(ibm, 'Spline');

Smooth the filled data series using the default Box (rectangular window) method:

```
sm_ibm = smoothts(f_ibm);
```

Now, plot the original and smoothed closing price series for IBM stock:

```
plot(f_ibm.CLOSE('11/01/97::02/28/98'), 'r')
datetick('x', 'mmmyy')
hold on
plot(sm_ibm.CLOSE('11/01/97::02/28/98'), 'b')
hold off
datetick('x', 'mmmyy')
legend('Filled', 'Smoothed')
```



## Smoothed Data Series

These examples give you an idea of what you can do with a financial time series object. This toolbox provides some MATLAB functions that have been overloaded to work directly with the these objects. The overloaded functions are those most commonly needed to work with time series data.

## Demonstration Program

In this section...<br>"Overview" on page 10-25<br>"Loading the Data" on page 10-26<br>"Create Financial Time Series Objects" on page 10-26<br>"Create Closing Prices Adjustment Series" on page 10-27<br>"Adjust Closing Prices and Make Them Spot Prices" on page 10-28<br>"Create Return Series" on page 10-28<br>"Regress Return Series Against Metric Data" on page 10-28<br>"Plot the Results" on page 10-29<br>"Calculate the Dividend Rate" on page 10-30

## Overview

This example demonstrates a practical use of financial time series objects, predicting the return of a stock from a given set of data. The data is a series of closing stock prices, a series of dividend payments from the stock, and an explanatory series (in this case a market index). Additionally, the example calculates the dividend rate from the stock data provided.

Note You can find a file for this demonstration program in the directory matlabroot/toolbox/finance/ftsdemos on your MATLAB path. The file is named predict_ret.m.

To perform these computations:
1 Load the data.
2 Create financial time series objects from the loaded data.
3 Create the series from dividend payment for adjusting the closing prices.
4 Adjust the closing prices and make them the spot prices.

5 Create the return series.
6 Regress the return series against the metric data (for example, a market index) using the MATLAB \operator.

7 Plot the results.
8 Calculate the dividend rate.

## Loading the Data

The data for this demonstration is found in the MAT-file predict_ret_data.mat:
load predict_ret_data.mat
The MAT-file contains six vectors:

- Dates corresponding to the closing stock prices, sdates
- Closing stock prices, sdata
- Dividend dates, divdates
- Dividend paid, divdata
- Dates corresponding to the metric data, expdates
- Metric data, expdata

Use the whos command to see the variables in your MATLAB workspace.

## Create Financial Time Series Objects

It is useful to work with financial time series objects rather than with the vectors now in the workspace. By using objects, you can easily keep track of the dates. Also, you can easily manipulate the data series based on dates because the object keeps track of the administration of time series for you.

Use the object constructor fints to construct three financial time series objects.

```
t0 = fints(sdates, sdata, {'Close'}, 'd', 'Inc');
dO = fints(divdates, divdata, {'Dividends'}, 'u', 'Inc');
```

```
x0 = fints(expdates, expdata, {'Metric'}, 'w', 'Index');
```

The variables t 0 , d0, and x 0 are financial time series objects containing the stock closing prices, dividend payments, and the explanatory data, respectively. To see the contents of an object, type its name at the MATLAB command prompt and press Enter. For example:

```
d0
d0 =
    'desc:' 'Inc'
    'freq:' 'Unknown (0)'
    'dates: (4)' 'Dividends: (4)'
    '04/15/99' '0.2000'
    '06/30/99' '0.3500'
    '10/02/99' '0.2000'
    '12/30/99' '0.1500'
```


## Create Closing Prices Adjustment Series

The price of a stock is affected by the dividend payment. On the day before the dividend payment date, the stock price reflects the amount of dividend to be paid the next day. On the dividend payment date, the stock price is decreased by the amount of dividend paid. Create a time series that reflects this adjustment factor:

```
dadj1 = d0;
dadj1.dates = dadj1.dates-1;
```

Now create the series that adjust the prices at the day of dividend payment; this is an adjustment of 0 . You also need to add the previous dividend payment date since the stock price data reflect the period subsequent to that day; the previous dividend date was December 31, 1998:

```
dadj2 = d0;
dadj2.Dividends = 0;
dadj2 = fillts(dadj2,'linear','12/31/98');
dadj2('12/31/98') = 0;
```

Combining the two objects above gives the data needed to adjust the prices. However, since the stock price data is daily data and the effect of the dividend
is linearly divided during the period, use the fillts function to make a daily time series from the adjustment data. Use the dates from the stock price data to make the dates of the adjustment the same:

```
dadj3 = [dadj1; dadj2];
dadj3 = fillts(dadj3, 'linear', t0.dates);
```


## Adjust Closing Prices and Make Them Spot Prices

The stock price recorded already reflects the dividend effect. To obtain the "correct" price, subtract the dividend amount from the closing prices. Put the result inside the same object t0 with the data series name Spot.

To make sure that adjustments correspond, index into the adjustment series using the dates from the stock price series to. Use the datestr command because to.dates returns the dates in serial date format. Also, since the data series name in the adjustment series dadj3 does not match the one in to, use the function fts2mat:

```
t0.Spot = t0.Close - fts2mat(dadj3(datestr(t0.dates)));
```


## Create Return Series

Now calculate the return series from the stock price data. A stock return is calculated by dividing the difference between the current closing price and the previous closing price by the previous closing price.

```
tret = (t0.Spot - lagts(t0.Spot, 1)) ./ lagts(t0.Spot, 1);
tret = chfield(tret, 'Spot', 'Return');
```

Ignore any warnings you receive during this sequence. Since the operation on the first line above preserves the data series name Spot, it has to be changed with the chfield command to reflect the contents correctly.

## Regress Return Series Against Metric Data

The explanatory (metric) data set is a weekly data set while the stock price data is a daily data set. The frequency needs to be the same. Use todaily to convert the weekly series into a daily series. The constant needs to be included here to get the constant factor from the regression:

```
x1 = todaily(x0);
```

```
x1.Const = 1;
```

Get all the dates common to the return series calculated above and the explanatory (metric) data. Then combine the contents of the two series that have dates in common into a new time series:

```
dcommon = intersect(tret.dates, x1.dates);
regts0 = [tret(datestr(dcommon)), x1(datestr(dcommon))];
```

Remove the contents of the new time series that are not finite:

```
finite_regts0 = find(all(isfinite( fts2mat(regts0)), 2));
regts1 = regts0( finite_regts0 );
```

Now, place the data to be regressed into a matrix using the function fts2mat. The first column of the matrix corresponds to the values of the first data series in the object, the second column to the second data series, and so on. In this case, the first column is regressed against the second and third column:

```
DataMatrix = fts2mat(regts1);
XCoeff = DataMatrix(:, 2:3) \ DataMatrix(:, 1);
```

Using the regression coefficients, calculate the predicted return from the stock price data. Put the result into the return time series tret as the data series PredReturn:

```
RetPred = DataMatrix(:,2:3) * XCoeff;
tret.PredReturn(datestr(regts1.dates)) = RetPred;
```


## Plot the Results

Plot the results in a single figure window. The top plot in the window has the actual closing stock prices and the dividend-adjusted stock prices (spot prices). The bottom plot shows the actual return of the stock and the predicted stock return through regression:

```
subplot(2, 1, 1);
plot(t0);
title('Spot and Closing Prices of Stock');
subplot(2, 1, 2);
plot(tret);
```

```
title('Actual and Predicted Return of Stock');
```



## Closing Prices and Returns

## Calculate the Dividend Rate

The last part of the task is to calculate the dividend rate from the stock price data. Calculate the dividend rate by dividing the dividend payments by the corresponding closing stock prices.

First check to see if you have the stock price data on all the dividend dates:

```
datestr(d0.dates, 2)
ans =
04/15/99
06/30/99
10/02/99
12/30/99
t0(datestr(d0.dates))
ans =
    'desc:' 'Inc'
```

```
'freq:' ,, 'Daily (1)', ''
'dates: (3)' 'Close: (3)' 'Spot: (3)'
'04/15/99' '10.3369' '10.3369
'06/30/99' '11.4707' '11.4707'
'12/30/99' '11.2244' '11.2244'
```

Note that stock price data for October 2, 1999 does not exist. The fillts function can overcome this situation; fillts allows you to insert a date and interpolate a value for the date from the existing values in the series. There are a number of interpolation methods. See fillts in Chapter 15, "Function Reference" for details.

Use fillts to create a new time series containing the missing date from the original data series. Then set the frequency indicator to daily:

```
t1 = fillts(t0,'nearest',d0.dates);
t1.freq = 'd';
```

Calculate the dividend rate:

```
tdr = d0./fts2mat(t1.Close(datestr(d0.dates)))
tdr =
    'desc:' 'Inc'
    'freq:' 'Unknown (0)'
    'dates: (4)' 'Dividends: (4)'
    '04/15/99' '0.0193'
    '06/30/99' '0.0305'
    '10/02/99' '0.0166'
    '12/30/99' '0.0134'
```


# Financial Time Series Tool (FTSTool) 

- "What Is the Financial Time Series Tool?" on page 11-2
- "Getting Started with FTSTool" on page 11-4
- "Loading Data with FTSTool" on page 11-5
- "Using FTSTool for Supported Tasks" on page 11-10
- "Using FTSTool with Other Time Series GUIs" on page 11-18


## What Is the Financial Time Series Tool?

The Financial Time Series Tool (ftstool) provides a graphical user interface to create and manage financial time series (fints) objects. ftstool interoperates with the Financial Time Series Graphical User Interface (ftsgui) and Interactive Chart (chartfts). In addition, you can use Datafeed Toolbox ${ }^{\mathrm{TM}}$ or Database Toolbox ${ }^{\mathrm{TM}}$ software to connect to external data sources.

A financial time series object minimally consists of:

- Desc, which is the description field.
- Freq, which is a frequency indicator field.
- Dates, which is a date vector field. If the date vector incorporates time-of-day information, the object contains an additional field named times.
- In addition, you can have at least one data series vector. You can specify names for any data series vectors. If you do not specify names, the object uses the default names series1, series2, series3, and so on.

In general, the workflow for using FTSTool is:
1 Acquire data.
2 Create a variable.
3 Convert the variable to fints.
4 Convert fints to a MATLAB double object.
To obtain the data for ftstool, you need to use a MATLAB double object or a financial time series (fints) object. You can use previously stored internal data on your computer or you can connect to external data sources using Datafeed Toolbox or Database Toolbox software.

[^1]After creating a financial time series object, you can use ftstool to change the characteristics of the time series object, including merging with other financial time series objects, removing rows or columns, and changing the frequency. You can also use ftstool to generate various forms of plotted output and you can reconvert a fints object to a MATLAB double-precision matrix.

## Getting Started with FTSTool

To start the Financial Time Series Tool:
1 At the MATLAB command prompt, enter

## ftstool

The Financial Time Series Tool opens.
2 If you plan to load data from Database Toolbox or Datafeed Toolbox software, ensure you have a license. For more information on using these toolboxes, see the Database Toolbox User's Guide and Datafeed Toolbox User's Guide documentation.


## Loading Data with FTSTool

```
In this section...
"Overview" on page 11-5
"Obtaining External Data" on page 11-5
"Obtaining Internal Data" on page 11-7
"Viewing the MATLAB Workspace" on page 11-8
```


## Overview

The Data source pane in the Financial Time Series Tool window lets you do the following:

- Obtain live data from various external data servers using either Datafeed Toolbox or Database Toolbox software.
- Load data you previously obtained and stored in a file.
- View data contained within the MATLAB workspace.


## Obtaining External Data

You can obtain external data using Datafeed Toolbox or Database Toolbox software. Datafeed Toolbox software lets you obtain data from several financial data servers, including:

- Bloomberg ${ }^{\circledR}$
- FactSet ${ }^{\text {® }}$
- Federal Reserve Economic Data
- Haver Analytics financial data
- Interactive Data Pricing and Reference Data
- Kx Systems ${ }^{\circledR}$, Inc. kdb+ database
- Reuters ${ }^{\circledR}$
- Thomson ${ }^{\circledR}$ Datastream ${ }^{\circledR}$
- Yahoo! ${ }^{\circledR}$

Except for Federal Reserve Economic Data and Yahoo!, these data servers require that you obtain a license from the vendor before you can access their data.

Tip If you open Datafeed Toolbox or Database Toolbox software before starting FTSTool, FTSTool is unable to recognize the toolboxes. When working with FTSTool, select File > Load to open these toolboxes.

## Obtaining External Data with Datafeed Toolbox Software

1 From the Financial Time Series Tool window, select File > Load > Datafeed Toolbox to open the toolbox.

2 Click the Connection tab in Datafeed Toolbox software to select the data source you want to load into FTSTool.

3 Click the Data tab in Datafeed Toolbox software to select the security and the associated data that you want to load into FTSTool.

4 After using Datafeed Toolbox software to define the connection, security, data, and MATLAB variable name, click Get Data and then, using FTSTool, click Refresh variable list. The Data source field in FTSTool displays the name of the security you selected from the Data tab in Datafeed Toolbox software. The FTSTool Active variable field indicates the name of the MATLAB workspace variable you chose for this security.

5 Click Close to exit Datafeed Toolbox software. FTSTool clears the Data source and Active variable fields.

## Obtaining External Data with Database Toolbox Software

1 From the Financial Time Series Tool window, select File > Load > Database Toolbox to open the toolbox.

2 From the Visual Query Builder window, select the data you want to load into FTSTool.

3 After using Database Toolbox software to select data and name the MATLAB workspace variable, click Execute and then, using FTSTool, click Refresh variable list. The Data source field in FTSTool displays the name of the highlighted data source that you selected from the Data list box in the Visual Query Builder window. The FTSTool Active variable field indicates the name of the MATLAB workspace variable you chose for the security in the Visual Query Builder window.

4 From the Database Toolbox software, select Query > Close Visual Query Builder, FTSTool clears the Data source and Active variable fields.

## Obtaining Internal Data

You can use FTSTool to load data from files previously stored on your computer. The types of data files you can load are as follows:

- MATLAB .mat files
- ASCII text files (.dat or .txt suffixes)
- Excel .xls files

To obtain internal data:
1 From the Financial Time Series Tool window, select File $>$ Load $>$ File to open the Load a MAT, ASCII, .XLS File dialog box.

2 Select the data you want to load into FTSTool.

- If you load a MATLAB MAT-file, the variables in the file are placed into the MATLAB workspace. The MATLAB Workspace Variables list box shows the variables that have been added to the workspace. For example, if you load the file disney.mat, which is distributed with the toolbox, the MATLAB Workspace Variables list box displays the variables in that MAT-file.

Note FTSTool automatically generates a line plot for each workspace variables unless you disable this feature by resetting the default action under File > Preferences > Generate line plot on load.

- If you load a .dat or an ASCII .txt file, the ASCII File Parameters dialog box opens. Use this dialog box to transform a text data file into a MATLAB financial time series fints object. (See the reference page for ascii2fts for further explanation of the fields in the ASCII File Parameters dialog box.
- If you load an Excel .xls file, the Excel File Parameters dialog box opens. Use this dialog box to transform Excel worksheet data into a MATLAB financial time series (fints) object.

3 From the Financial Time Series Tool window, select File > Save to save the data you gave loaded from an internal file.

## Viewing the MATLAB Workspace

The MATLAB Workspace Variables list box displays all existing MATLAB workspace variables. Double-click any variable to display the data in the Data Table. You can only display financial time series (fints) objects, MATLAB doubles, and cell arrays of double data in the Data Table.


In addition, you can click Refresh variable list to refresh the MATLAB Workspace Variables list box. You need to refresh this list periodically because it is refreshed automatically only for operations performed with FTSTool, not for operations performed within MATLAB itself.

Click Remove variable(s) to remove variable from the MATLAB Workspace Variables list and from the MATLAB workspace.

## Using FTSTool for Supported Tasks

In this section...<br>"Creating a Financial Time Series Object" on page 11-10<br>"Merging Financial Time Series Objects" on page 11-11<br>"Converting a Financial Time Series Object to a MATLAB Double-Precision Matrix" on page 11-12<br>"Plotting the Output in Several Formats" on page 11-12<br>"Viewing Data for a Financial Time Series Object in the Data Table" on page 11-13<br>"Modifying Data for a Financial Time Series Object in the Data Table" on page 11-15<br>"Viewing and Modifying the Properties for a FINTS Object" on page 11-17

## Creating a Financial Time Series Object

Using the Create tab in the FINTS Objects and Outputs pane for FTSTool, you can create a financial time series (fints) object from one or more selected variables.

Note When you first start FTSTool, the Create tab appears on top, unless you reset the default using File $>$ Preferences $>$ Show Create tab when ftstool starts.

To create a financial time series (fints) object from one or more selected variables:

1 Load data into FTSTool from either an external data source using Datafeed Toolbox or Database Toolbox software or an internal data source using File > Load > File.

2 Select one or more variables from the MATLAB Workspace Variables list.

3 Click the Create tab and then click Active variable.
When combining multiple variables, you can type a new variable name for the combined variables in the MATLAB workspace variable box. The new variable name is added to the MATLAB Workspace Variables list. (If you do not choose a name for the MATLAB workspace variable, FTSTool uses the default name myFts.)

4 Click Create FINTS object to display the result in the Data Table.

## Merging Financial Time Series Objects

Using the Create tab in the FINTS Objects and Outputs pane for FTSTool, you can create a new financial time series object by merging (joining) multiple existing financial time series objects.

Note When you first start FTSTool, the Create tab appears on top, unless you reset the default using File > Preferences.

To create a financial time series (fints) object by merging multiple existing financial time series objects:

1 Load data into FTSTool from either an external data source using Datafeed Toolbox or Database Toolbox software or an internal data source using File > Load > File.

2 To merge multiple existing financial time series objects, click the Create tab, click Components, and then select a value for the Time vector source and one or more items from the Data sources list.

Note You can merge at once multiple financial time series objects. For more information on merging fints objects, see merge.

3 Click Create FINTS object to display the result in the Data Table.

## Converting a Financial Time Series Object to a MATLAB Double-Precision Matrix

Using the Convert tab in the FINTS Objects and Outputs pane for FTSTool, you can convert a financial time series (fints) object to a MATLAB double-precision matrix.

To create a financial time series object from one or more selected variables:
1 Load data into FTSTool from either an external data source using Datafeed Toolbox or Database Toolbox software or an internal data source using File > Load > File.

2 Select a variable from the MATLAB Workspace Variables list box.
3 Click the Convert tab and then determine whether to include or exclude dates in the conversion by clicking Include dates or Exclude dates.

4 Type a variable name in the Output variable name box. (If you do not choose a variable name, FTSTool uses the default name myDbl.)

5 Click Convert FINTS to double matrix. (This operation is equivalent to performing fts2mat on a financial time series object.)

## Plotting the Output in Several Formats

Using the Plot tab in the FINTS Objects and Outputs pane for FTSTool, you can create several forms of plotted output by using a selection list. You can create four types of bar charts, candle plots, high-low plots, line plots, and interactive charts (the latter is created by using the interoperation of FTSTool with the function chartfts).

The set of plots supported by FTSTool are identical to the set provided by the Graphs menu of the Financial Time Series GUI. (See "Graphs Menu" on page 12-15.) You can find more detailed information for the supported plots by consulting the reference page for each individual type of plot.

To create a plotted output:

1 Load data into FTSTool from either an external data source using Datafeed Toolbox or Database Toolbox software or an internal data source using File > Load > File.

2 Select a variable from the MATLAB Workspace Variables list box or select data from the Data Table.

3 Click the Plot tab and indicate whether you are plotting based on a workspace variable or data from the Data Table.

4 From the Type drop-down list, select the type of plot.
5 Click Plot. The plot is displayed.

Note If the selected workspace variable that you are plotting is not a fints object, a fints object is created when you click Plot. The new fints object uses the name designated by the MATLAB workspace variable box on the Create tab.

## Viewing Data for a Financial Time Series Object in the Data Table

Once a financial time series (fints) object is created, the FTSTool Data Table displays user-designated data, including financial time series objects, MATLAB double-precision variables, and cell arrays of doubles. (Cell arrays of doubles is often the resulting format when using Database Toolbox software.)

When displaying double variables (or a cell array of doubles) in the Data Table, the column headings for a double variable or cell array of doubles displayed in the Data Table are labeled A, B, C, and so on.

## Overwriting Data in the Data Table Display

If you use the command line to overwrite data previously retrieved using Datafeed Toolbox or Database Toolbox software, two events could occur:

- If the new data contains the same number of columns as before, the headers remain unchanged when you attempt to create a financial time series (fints) object using the modified data.
- If the data contains a different number of columns, a warning dialog box appears.

For example, assume that you use Datafeed Toolbox software to obtain Close, High, Low, and Volume data for the equity GlaxoSmithkline. You store the data in the MATLAB workspace with the variable name glaxo. From the command line, if you redefine the variable glaxo, eliminating the second column (Close)

```
glaxo(:,2) = [ ]
```

and then return to FTSTool and attempt to create a financial time series object, a warning dialog box appears.

## Financial Time Series Tool Warning

Incompatible series name(s) exist in memory for the variable 'glaxo'.

There are:
Too many DATANAMES for the number of data series.
incompatble series namersy, Close, High, Law, Open, Volume

A FWTS object will be constructed with ciefaut series name(s).

Wismatched series names will be removed from memory.
inform me of future FINTS object creation errors?

C No, do not ask me again. © Yes, ask me each time.

OK

## Modifying Data for a Financial Time Series Object in the Data Table

FTSTool lets you update your data displayed in the Data Table by adding or removing rows or columns.

Note Modifying data in the Data Table will not update the MATLAB workspace variable. To update the workspace variable after modifying the Data Table, click Update workspace variable.

## Adding and Removing Rows

To add a row of data displayed in the Data Table:
1 Select a row from the Data Table display where you want to add a row. Click Additional options to open the Data Table Options dialog box.

2 Click Add row. The default is to add the row up. To add a row down, select Insertion option and then click Add down. In addition, you can select the Insertion option of Date to designate a specific date. (If a date is not specified, the added row will contain a date that is chronologically in order with respect to the initial row.)

When you add rows, the Data Table display is immediately updated.
To remove a row of data from the Data Table:
1 Select one or more rows in the Data Table display that you want to remove. Click Additional options to open the Data Table Options dialog box.

2 Click Remove row(s). The default is to remove the selected rows. In addition, to remove selected rows, select Removal options and then select other options for row removal from the Remove rows list box. You can specify a Start and End date or you can click the Non-uniform range setting option to designate a range.

When you remove rows, the Data Table display is updated immediately.

## Adding and Removing Columns

To add a column of data displayed in the Data Table:
1 Select a column from the Data Table display where you want to add a column. Click Additional options to open the Data Table Options dialog box.

2 Click Add column. The default is to add the column to the left of the selected column.

Note For time series objects, you cannot add a column to the left of the Date/Times column; there is no restriction for double data.

To add a column to the right, select Insertion option and then click Add right. In addition, you can use the Insertion option of New Column Name to designate a specific column name. (If a New Column Name is not specified, an added column will contain a column name of series1, series2, and so on.)

When you add columns, the Data Table display is updated immediately.
To remove a column of data displayed in the Data Table:
1 Select one or more columns in the Data Table display that you want to remove. Click Additional options to open the Data Table Options dialog box.

2 Click Remove column(s). The default is to remove the selected rows. In addition, to remove selected columns, select Removal options and then select columns for removal from the Remove columns list box.

When you remove columns, the Data Table display is updated immediately.

## Viewing and Modifying the Properties for a FINTS Object

The FINTS Object Properties pane in FTSTool lets you modify financial time series (fints) object properties. This area becomes active whenever the Data Table displays a financial time series object.

To modify the properties for a fints object:
1 After you create a fints object, double-click the object name in the MATLAB Workspace Variables list box to open the Data Table and display the fints object properties.

2 Click to modify the Description, Frequency, or Series Names fields.
The Frequency drop-down list supports the following conversion functions:

| Function | New Frequency |
| :--- | :--- |
| toannual | Annual |
| todaily | Daily |
| tomonthy | Monthly |
| toquarterly | Quarterly |
| tosemi | Semiannually |
| toweekly | Weekly |

3 Click Update properties to save the changes. This action also updates the associated workspace variable.

## Using FTSTool with Other Time Series GUls

FTSTool works with Datafeed Toolbox and Database Toolbox software to load data. In addition, FTSTool interoperates with chartfts to display an interactive plot and ftsgui to perform further time series data analysis.

The workflow for using FTSTool with chartfts is:
1 After loading data from either Datafeed Toolbox or Database Toolbox software or an internal file, select a variable from the MATLAB Workspace Variables list box.

2 Click the Plot tab, click Type, and then select Interactive Chart.
3 Click Plot. The interactive plot is displayed in chartfts. You can then use chartfts menu items for further display options.

For more information on chartfts, select Help > Graphics Help.
The workflow for using FTSTool with the Financial Time Series GUI (ftsgui) is:

1 After loading data from either Datafeed Toolbox or Database Toolbox software or an internal file, select a variable from the MATLAB Workspace Variables list box.

2 Select Tools > FTSGUI to open the Financial Time Series GUI window.
3 Select a variable from the MATLAB Workspace Variables list box. Click the Plot tab and then select one of the following from the Type drop-down list: Line Plot, High-Low Plot, or Candlestick Plot.

4 Click Plot. The plot is displayed in a MATLAB graphic window. In addition, the Financial Time Series GUI window displays an entry for the plotted fints object. You can then use the menu items in the Financial Time Series GUI window to perform further analysis.

For more information on ftsgui, select Help > Help on Financial Time Series GUI.

[^2]
# Financial Time Series Graphical User Interface 

- "Introduction" on page 12-2
- "Using the Financial Time Series GUI" on page 12-7


## Introduction

Use the financial time series graphical user interface (GUI) to analyze your time series data and display the results graphically without resorting to the command line. The GUI lets you visualize the data and the results at the same time.
"Using the Financial Time Series GUI" on page 12-7 discusses how to use this GUI.

## Main Window

Start the financial time series GUI with the command
ftsgui
The Financial Time Series GUI window opens.


The title bar acts as an active time series object indicator (indicates the currently active financial time series object). For example, if you load the file disney.mat and want to use the time series data in the file dis, the title bar on the main GUI would read as shown.

| - Financial Time Series GUI, Active: Walt Disney Company (DIS) |  | - - ${ }^{\text {a }}$ |
| :---: | :---: | :---: |
| File Data Analysis Graphs Window H | Help |  |
| Started: Main FTS GUI Window <br> Loaded file: J: \disney.mat <br> Displaying: Wall Disney Company (DIS) <br> Displaying: TOQUARTERLY: Walt Disney Company (DIS) <br> Displaying: Wall Disney Company (DIS) - No VOLUME |  | $\Delta$ |

The menu bar consists of six menu items: File, Data, Analysis, Graphs, Window, and Help. Under the menu bar is a status box that displays the steps you are doing.

## File Menu



The File menu contains the commands for input and output. You can read and save (Load, Save, and Save As) MATLAB MAT-files, ASCII (text) data files, as well as import (Import) Excel XLS files. MATLAB software does not support the export of Excel XLS files at this time.

The File menu also contains the printing suite (Page Setup, Print Preview, and Print). Lastly, from this menu you can close the GUI itself (Close FTS GUI) and quit MATLAB (Exit MATLAB).

## Data Menu



The Data menu provides a collection of data manipulation functions and data conversion functions.

| Resample data |  |
| :--- | :--- |
| ConvertData Frequency To | To Daily |
|  | To Weekly |
|  | To Monthly |
|  | To Quarterly |
|  | To Semi-annual |
|  | To Annual |

To use any of the functions here, make sure that the correct financial time series object is displayed in the title bar of the main GUI window.

Analysis Menu


## The Analysis menu provides

- A set of exponentiation and logarithmic functions.
- Statistical tools (Basic Statistics), which calculate and display the minimum, maximum, average (mean), standard deviation, and variance of the current (active) time series object; these basic statistics numbers are displayed in a dialog window.
- Data difference (Difference) and periodic average (Periodic Average) calculations. Data difference generates a vector of data that is the difference between the first data point and the second, the second and the third, and so on. The periodic average function calculates the average per defined length period, for example, averages of every five days.
- Technical analysis functions. See Chapter 14, "Technical Analysis" for a list of the provided technical analysis functions.

As with the Data menu, to use any of the Analysis menu functions, make sure that the correct financial time series object is displayed in the title bar of the main GUI window.

## Graphs Menu



The Graphs menu contains functions that graphically display the current (active) financial time series object. You can also start up the interactive charting function (chartfts) from this menu.

## Window Menu

| - Financial T | isney Company (DIS) $\quad$ - 回区 $^{\text {a }}$ |
| :---: | :---: |
| File Data Analysis Graphs | Window Help |
| Started: Main FTS GUI Window Loaded file: J: \disney.mat Displaying: Walt Disney Company (DIS) Displaying: TOQUARTERLY: Walt Disney ( | 2 TOQUARTERLY: Walt Disney Company (DIS) 3 Walt Disney Company (DIS) <br> 4 Walt Disney Company (DIS) - No VOLUME |
|  | 1 Financial Time Series GUI, Active: Walt Disney Company (DIS) @ MATLAB Command Window |

The Window menu lists open windows under the current MATLAB session.

## Help Menu

| - Financial Time Series GUI, Active: Walt Disney Company (DIS) |  | - $\square^{\text {a }}$ |
| :---: | :---: | :---: |
| File Data Analysis Graphs Window | Help |  |
| Started: Main FTS GUI WindowLoaded file: J:\disney.matDisplaying: Walt Disney Company (DIS)Displaying: TOQUARTERLY: Walt Disney Company (DISDisplaying: Walt Disney Company (DIS) - No VOLUME | Help on Financial Time Series GUI | - |
|  | Help on Financial Time Series Toolbox |  |
|  | Help on MATLAB |  |
|  | MATLAB Demos | $\nabla$ |
|  | About Financial Time Series GUI |  |
|  | About MATLAB |  |

The Help menu provides a standard set of Help menu links.

## Using the Financial Time Series GUI

In this section...<br>"Getting Started" on page 12-7<br>"Data Menu" on page 12-9<br>"Analysis Menu" on page 12-13<br>"Graphs Menu" on page 12-15<br>"Saving Time Series Data" on page 12-19

## Getting Started

To use the Financial Time Series GUI, first start the financial time series GUI with the command ftsgui. Then load (or import) the time series data.

For example, if your data is in a MATLAB MAT-file, select Load from the File menu.



For illustration purposes, choose the file ftsdata.mat from the dialog presented.

If you don't see the MAT-file, look in the directory matlabroot $\backslash$ toolbox \finance $\backslash$ findemos, where matlabroot is the MATLAB root directory (the directory where MATLAB is installed).

Note Data loaded through the Financial Time Series GUI is not available in the MATLAB workspace. You can access this data only through the GUI itself, not with any MATLAB command-line functions.

Each financial time series object inside the MAT-file is presented as a line plot in a separate window. The status window is updated accordingly.


Whirlpool (WHR) is the last plot displayed, as indicated on the title bar of the main window.

## Data Menu

The Data menu provides functions that manipulate time series data.


Here are some example tasks that illustrate the use of the functions on this menu.

## Fill Missing Data

First, look at filling missing data. The Fill Missing Data item uses the toolbox function fillts. With the data loaded from the file ftsdata, you have three time series: IBM Corp. (IBM), Walt Disney Co. (DIS), and Whirlpool (WHR). Click on the window that shows the time series data for Walt Disney Co. (DIS).


To view any missing data in this time series data set, zoom into the plot using the Zoom tool (the magnifying glass icon with the plus sign) from the toolbar and select a region.


The gaps represent the missing data in the series. To fill these gaps, select Data > Fill Missing Data. This selection automatically fills the gaps and generates a new plot that displays the filled time series data.


You cannot see the filled gaps when you display the entire data set. However, when you zoom into the plot, you see that the gaps have been eliminated.
Note that the title bar has changed; the title has been prefixed with the word Filled to reflect the filled time series data.

## Frequency Conversion

The Data menu also provides access to frequency conversion functions.

| Resample data |  |
| :--- | :--- |
| Convert Data Frequency To $\quad$ To Daily |  |
|  | To Weekly |
|  | To Monthly |
|  | To Quarterly |
|  | To Semi-annual |
|  | To Annual |

This example changes the DIS time series data frequency from daily to monthly. Close the Filled Walt Disney Company (DIS) window, and click the Walt Disney Company (DIS) window to make it active (current) again. Then, from the Data menu, select Convert Data Frequency To and To Monthly.

A new figure window displays the result of this conversion.


The title reflects that the data displayed had its frequency changed to monthly.

## Analysis Menu

The Analysis menu provides functions that analyze time series data, including the technical analysis functions. (See Chapter 14, "Technical Analysis" for a complete list of the technical analysis functions and several usage examples.)

For example, you can use the Analysis menu to calculate the natural logarithm (log) of the data contained within the data set ftsdata.mat. This data file provides time series data for IBM (IBM), Walt Disney (DIS), and Whirlpool (WHR). Click the window displaying the data for IBM Corporation (IBM) to make it active (current). Then select the Analysis menu, followed by $\log (\ldots$ ). The result appears in its own window.


Close the above window and click again on the IBM data window to make it active (current).

Note Before proceeding with any time series analysis, make certain that the title bar confirms that the active data series is the correct one.

From the Analysis menu on the main window, select Technical Analysis and MACD. The result, again, is displayed in its own window.


Other analysis functions work similarly.

## Graphs Menu

The Graphs menu displays time series data using the provided graphics functions. Included in the Graphs menu are several types of bar charts (bar, barh and bar3, bar3h), line plot (plot), candle plot (candle), and High-Low plot (highlow). The Graphs menu also provides access to the interactive charting function, chartfts.

## Candle Plot

For example, you can display the candle plot of a set of time series data and start up the interactive chart on the same data set.

Load the ftsdata.mat data set, and click the window that displays the Whirlpool (WHR) time series data to make it active (current). From the main window, select the Graphs menu and then Candle Plot.


The result is shown below.


This does not look much like a candle plot because there are too many data points in the data set. All the candles are too compressed for effective
viewing. However, when you zoom into a region of this plot, the candles become apparent.


## Interactive Chart

To create an interactive chart (chartfts) on the Whirlpool data, click the window that displays the Whirlpool (WHR) data to make it active (current). Then, go to the Graphs menu and select Interactive Chart.


The chart that results is shown below.


You can use this interactive chart as if you had invoked it with the chartfts command from the MATLAB command line. For a tutorial on the use of chartfts, see "Visualizing Financial Time Series Objects" on page 9-18.

## Saving Time Series Data

The Save and Save As items on the main window File menu let you save the time series data that results from your analyses and computations. These items save all time series data that has been loaded or processed during the current session, even if the window displaying the results of a computation has previously been dismissed.

Note The Save and Save As items on the File menu of the individual plot windows will not save the time series data, but will save the actual plot.

You can save your time series data in two ways:

- Into the latest MAT-file loaded (Save)
- Into a MAT-file chosen (or named) from the window (Save As)

To illustrate this, start by loading the data file testftsdata.mat (located in matlabroot/toolbox/finance/findemos). Then, convert the Disney (DIS) data from daily (the original frequency) to monthly data. Next, run the MACD analysis on the Whirlpool (WHR) data. You now have a set of five open figure windows.


## Saving into the Original File (Save)

To save the data back into the original file (testftsdata.mat), select Save from the File menu.

A confirmation window appears. It confirms that the data has been saved in the latest MAT-file loaded (testftsdata.mat in this example).


## Saving into a New File (Save As)

To save the data in a different file, choose Save As from the File menu.
The dialog box that appears lets you choose an existing MAT-file from a list or type in the name of a new MAT-file you want to create.


After you click the Save button, another confirmation window appears.


This confirmation window indicates that the data has been saved in a new file named myftstestdata.mat.

## Trading Date Utilities

- "Trading Calendars Graphical User Interface" on page 13-2
- "UICalendar Graphical User Interface" on page 13-4


## Trading Calendars Graphical User Interface

Use the createholidays function to open the Trading Calendars graphical user interface.


The createholidays function supports http://www.FinancialCalendar.com trading calendars. This function can be used from the command line or from the Trading Calendars graphical user interface. For more information on using the command line to programmatically generate the market specific holidays.m files without displaying the interface, see createholidays.

To use the Trading Calendars graphical user interface:
1 From the command line, type the following command to open the Trading Calendars graphical user interface.
createholidays

2 Click Choose data file to select the data file.
3 Click Choose codes file to select the codes file.
4 Click Choose info file to select the info file.
5 Click Choose directory for writing holiday files to select the output directory.

6 Select Include weekends to include weekends in the holiday list and click Prompt for target directory to be prompted for the file location for each holidays.m file that is created.

7 Click Create holiday files to convert FinancialCalendar.com financial center holiday data into market-specific holidays.m files.

The market-specific holidays.m files can be used in place of the standard holidays.m that ships with Financial Toolbox software.

## UICalendar Graphical User Interface

## In this section... <br> "Using UICalendar in Standalone Mode" on page 13-4 <br> "Using UICalendar with an Application" on page 13-5

## Using UICalendar in Standalone Mode

You can use the UICalendar graphical user interface in standalone mode to look up any date. To use the standalone mode:

1 Type the following command to open the UICalendar GUI:
uicalendar
The UICalendar interface is displayed:


2 Click the date and year controls to locate any date.

## Using UICalendar with an Application

You can use the UICalendar graphical user interface with an application to look up any date. To use the UICalendar graphical interface with an application, use the following command:

```
uicalendar('PARAM1', VALUE1, 'PARAM2', VALUE2', ...)
```

For more information, see uicalendar.

## Example of Using UICalendar with an Application

The UICalendar example creates a function that displays a graphical user interface that lets you select a date from the UICalendar graphical user interface and fill in a text field with that date.

1 Create a figure.

```
function uicalendarGUIExample
f = figure('Name', 'uicalendarGUIExample');
```

2 Add a text control field.

```
dateTextHandle = uicontrol(f, 'Style', 'Text', ...
'String', 'Date:', ...
'Horizontalalignment', 'left', ...
Position', [100 200 50 20]);
```

3 Add a uicontrol editable text field to display the selected date.

```
dateEditBoxHandle = uicontrol(f, 'Style', 'Edit', ...
'Position', [140 200 100 20], ...
'BackgroundColor', 'w');
```

4 Create a push button that startups the UICalendar.

```
calendarButtonHandle = uicontrol(f, 'Style', 'PushButton', ...
'String', 'Select a single date', ...
'Position', [275 200 200 20], ...
'callback', @pushbutton_cb);
```

5 To startup UICalendar, create a nested function (callback function) for the push button.

```
function pushbutton_cb(hcbo, eventStruct)
% Create a UICALENDAR with the following properties:
% 1) Highlight weekend dates.
% 2) Only allow a single date to be selected at a time.
% 3) Send the selected date to the edit box uicontrol.
uicalendar('Weekend', [1 0 0 0 0 0 1], ...
'SelectionType', 1, ...
'DestinationUI', dateEditBoxHandle);
end
end
```

6 Run the function uicalendarGUIExample to display the application interface:


7 Click Select a single date to display the UICalendar graphical user interface:


8 Select a date and click OK to display the date in the text field:


## Technical Analysis

- "Introduction" on page 14-2
- "Examples" on page 14-4


## Introduction

Technical analysis (or charting) is used by some investment managers to help manage portfolios. Technical analysis relies heavily on the availability of historical data. Investment managers calculate different indicators from available data and plot them as charts. Observations of price, direction, and volume on the charts assist managers in making decisions on their investment portfolios.

The technical analysis functions in this toolbox are tools to help analyze your investments. The functions in themselves will not make any suggestions or perform any qualitative analysis of your investment.

## Technical Analysis: Oscillators

| Function | Type |
| :--- | :--- |
| adosc | Accumulation/distribution oscillator |
| chaikosc | Chaikin oscillator |
| macd | Moving Average |
|  | Convergence/Divergence |
| stochosc | Stochastic oscillator |
| tsaccel | Acceleration |
| tsmom | Momentum |

## Technical Analysis: Stochastics

| Function | Type |
| :--- | :--- |
| chaikvolat | Chaikin volatility |
| fpctkd | Fast stochastics |
| spctkd | Slow stochastics |
| willpctr | Williams \%R |

## Technical Analysis: Indexes

| Function | Type |
| :--- | :--- |
| negvolidx | Negative volume index |
| posvolidx | Positive volume index |
| rsindex | Relative strength index |

## Technical Analysis: Indicators

| Function | Type |
| :--- | :--- |
| adline | Accumulation/distribution line |
| bollinger | Bollinger band |
| hhigh | Highest high |
| llow | Lowest low |
| medprice | Median price |
| onbalvol | On balance volume |
| prcroc | Price rate of change |
| pvtrend | Price-volume trend |
| typprice | Typical price |
| volroc | Volume rate of change |
| wclose | Weighted close |
| willad | Williams accumulation/distribution |

## Examples

```
In this section...
"Overview" on page 14-4
"Moving Average Convergence/Divergence (MACD)" on page 14-4
"Williams %R" on page 14-6
"Relative Strength Index (RSI)" on page 14-7
"On-Balance Volume (OBV)" on page 14-8
```


## Overview

To illustrate some the technical analysis functions, this section uses the IBM stock price data contained in the supplied file ibm9599.dat. First create a financial time series object from the data using ascii2fts:

```
ibm = ascii2fts('ibm9599.dat', 1, 3, 2);
```

The time series data contains the open, close, high, and low prices, as well as the volume traded on each day. The time series dates start on January 3, 1995, and end on April 1, 1999, with some values missing for weekday holidays; weekend dates are not included.

## Moving Average Convergence/Divergence (MACD)

Moving Average Convergence/Divergence (MACD) is an oscillator function used by technical analysts to spot overbought and oversold conditions. Look at the portion of the time series covering the 3 -month period between October 1, 1995 and December 31, 1995. At the same time fill any missing values due to holidays within the time period specified:

```
part_ibm = fillts(ibm('10/01/95::12/31/95'));
```

Now calculate the MACD, which when plotted produces two lines; the first line is the MACD line itself and the second is the nine-period moving average line:

```
macd_ibm = macd(part_ibm);
```

Note When you call macd without giving it a second input argument to specify a particular data series name, it searches for a closing price series named Close (in all combinations of letter cases).

Plot the MACD lines and the High-Low plot of the IBM stock prices in two separate plots in one window.

```
subplot(2, 1, 1);
plot(macd_ibm);
title('MACD of IBM Close Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
subplot(2, 1, 2);
highlow(part_ibm);
title('IBM Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy')
```

The following figure shows the result.


## Williams \%R

Williams \%R is an indicator that measures overbought and oversold levels. The function willpctr is from the stochastics category. All the technical analysis functions can accept a different name for a required data series. If, for example, a function needs the high, low, and closing price series but your time series object does not have the data series names exactly as High, Low, and Close, you can specify the correct names as follows.

```
wpr = willpctr(tsobj, 14, 'HighName', 'Hi', 'LowName', 'Lo',...
'CloseName', 'Closing')
```

The function willpctr now assumes that your high price series is named Hi , low price series is named Lo, and closing price series is named Closing.

Since the time series object part_ibm has its data series names identical to the required names, name adjustments are not needed. The input argument to the function is only the name of the time series object itself.

Calculate and plot the Williams \%R indicator for IBM stock along with the price range using these commands:

```
wpctr_ibm = willpctr(part_ibm);
subplot(2, 1, 1);
plot(wpctr_ibm);
title('Williams %R of IBM stock, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
hold on;
plot(wpctr_ibm.dates, -80*ones(1, length(wpctr_ibm)),...
'color', [0.5 0 0], 'linewidth', 2)
plot(wpctr_ibm.dates, -20*ones(1, length(wpctr_ibm)),...
'color', [0 0.5 0], 'linewidth', 2)
subplot(2, 1, 2);
highlow(part_ibm);
title('IBM Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
```

The next figure shows the results. The top plot has the Williams \%R line plus two lines at $-20 \%$ and $-80 \%$. The bottom plot is the High-Low plot of the IBM stock price for the corresponding time period.


## Relative Strength Index (RSI)

The Relative Strength Index (RSI) is a momentum indicator that measures an equity's price relative to itself and its past performance. The function name is rsindex.

The rsindex function needs a series that contains the closing price of a stock. The default period length for the RSI calculation is 14 periods. This length can be changed by providing a second input argument to the function. Similar to the previous commands, if your closing price series is not named Close, you can provide the correct name.

Calculate and plot the RSI for IBM stock along with the price range using these commands:

```
rsi_ibm = rsindex(part_ibm);
subplot(2, 1, 1);
plot(rsi_ibm);
title('RSI of IBM stock, 10/01/95-12/31/95');
```

```
datetick('x', 'mm/dd/yy');
hold on;
plot(rsi_ibm.dates, 30*ones(1, length(wpctr_ibm)),...
'color', [0.5 0 0], 'linewidth', 2)
plot(rsi_ibm.dates, 70*ones(1, length(wpctr_ibm)),...
'color',[0 0.5 0], 'linewidth', 2)
subplot(2, 1, 2);
highlow(part_ibm);
title('IBM Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
```

The next figure shows the result.


## On-Balance Volume (OBV)

On-Balance Volume (OBV) relates volume to price change. The function onbalvol requires you to have the closing price (Close) series as well as the volume traded (Volume) series.

Calculate and plot the OBV for IBM stock along with the price range using these commands:

```
obv_ibm = onbalvol(part_ibm);
subplot(2, 1, 1);
plot(obv_ibm);
title('On-Balance Volume of IBM Stock, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
subplot(2, 1, 2);
highlow(part_ibm);
title('IBM Stock Prices, 10/01/95-12/31/95');
datetick('x', 'mm/dd/yy');
```

The next figure shows the result.


14 Technical Analysis

## Function Reference

Dates (p. 15-2)
Currency and Price (p. 15-6)
Financial Data Charts (p. 15-6)
Cash Flows (p. 15-7)
Fixed-Income Securities (p. 15-9)
Portfolio Optimization Objects (p. 15-12)

Portfolio Analysis (p. 15-15)

Financial Statistics (p. 15-18)

Derivatives (p. 15-20)
Credit Risk Utilities (p. 15-21)
GARCH Processes (p. 15-22)
Financial Time Series Object and File Construction (p. 15-23)

Financial Time Series Arithmetic (p. 15-23)

Financial Time Series Math (p. 15-24)

Financial Time Series Descriptive Statistics (p. 15-24)

Work with dates
Work with currency and price data
Create charts
Work with cash flows
Work with fixed-income securities
Create and manage portfolios using portfolio objects

Analyze and measure performance for portfolios

Perform statistical analysis of financial data

Price and analyze derivatives
Measure and analyze credit risk
Introduce GARCH analysis
Functions for creating financial time series

Arithmetic in financial time series

Mathematical calculations in financial time series

Statistics in financial time series

| Financial Time Series Utility (p. 15-25) | Utility work with financial time series |
| :---: | :---: |
| Financial Time Series Data Transformation (p. 15-26) | Data transformations of financial time series |
| Financial Time Series Indicator (p. 15-27) | Work with indicators for financial time series |
| Financial Time Series GUI (p. 15-28) | Work with Financial Time Series GUI |
| Financial Time Series Tool (p. 15-28) | Work with Financial Time Series Tool |

## Dates

Current Time and Date (p. 15-2)
Date and Time Components (p. 15-2) Compute dates and times
Date Conversion (p. 15-3)
Financial Dates (p. 15-4)
Coupon Bond Dates (p. 15-5)

Work with current date and time

Convert dates
Compute financial dates
Compute coupon bond dates

## Current Time and Date

| now | Current date and time |
| :--- | :--- |
| today | Current date |

today
Current date

## Date and Time Components

| datefind | Indices of date numbers in matrix |
| :--- | :--- |
| datevec | Date components |
| day | Day of month |

eomdate
eomday
hour
lweekdate
minute
month
months
nweekdate
second
weekday
weeknum
year
yeardays

## Date Conversion

| date2time | Time and frequency from dates |
| :--- | :--- |
| datedisp | Display date entries |
| datenum | Create date number |
| datestr | Create date string |
| dec2thirtytwo | Decimal to thirty-second quotation |
| m2xdate | MATLAB serial date number to |
|  | Excel serial date number |
| thirtytwo2dec | Thirty-second quotation to decimal |
| time2date | Dates from time and frequency |

Last date of month
Last day of month
Hour of date or time
Date of last occurrence of weekday in month

Minute of date or time
Month of date
Number of whole months between dates

Date of specific occurrence of weekday in month
Seconds of date or time
Day of week
Week in a year
Year of date
Number of days in year

uicalendar<br>x2mdate

Graphical calendar
Excel serial date number to MATLAB serial date number

## Financial Dates

| busdate | Next or previous business day |
| :--- | :--- |
| busdays | Business days in serial date format |
| createholidays | Create trading calendars |
| datemnth | Date of day in future or past month |
| datewrkdy | Date of future or past workday |
| days360 | Days between dates based on <br> 360 -day year |
| days360e | Days between dates based on <br> 360-day year (European) |
| days360isda | Days between dates based on <br>  <br>  <br> 360-day year (International |
| days360psa | Swap Dealer Association (ISDA) <br> compliant) |
| days365 | Days between dates based on <br> 360-day year (Public Securities <br> Association (PSA) compliant) |
| daysact | Days between dates based on <br>  <br> daysadd |
| 365-day year |  |
| daysdif | Actual number of days between <br> dates |
| fbusdate | Date away from starting date for any <br> day-count basis |
|  | Days between dates for any <br> day-count basis |
|  | First business date of month |

holidays
isbusday
lbusdate
nyseclosures
thirdwednesday
wrkdydif
yearfrac

## Coupon Bond Dates

accrfrac
cfamounts
cfdates
cfport
cftimes
cpncount
cpndaten
cpndatenq
cpndatep
cpndatepq

Holidays and nontrading days
True for dates that are business days
Last business date of month
New York Stock Exchange closures from 1885 to 2050

Find third Wednesday of month
Number of working days between dates

Fraction of year between dates

Fraction of coupon period before settlement

Cash flow and time mapping for bond portfolio
Cash flow dates for fixed-income security
Portfolio form of cash flow amounts
Time factors corresponding to bond cash flow dates

Coupon payments remaining until maturity

Next coupon date for fixed-income security

Next quasi coupon date for fixed income security

Previous coupon date for fixed-income security
Previous quasi coupon date for fixed income security
cpndaysn
cpndaysp
cpnpersz

## Currency and Price

cur2frac<br>cur2str<br>dec2thirtytwo<br>frac2cur<br>thirtytwo2dec

Number of days to next coupon date
Number of days since previous coupon date

Number of days in coupon period

Decimal currency values to fractional values

Bank-formatted text
Decimal to thirty-second quotation
Fractional currency value to decimal value
Thirty-second quotation to decimal

## Financial Data Charts

| bar, barh | Bar chart |
| :--- | :--- |
| bar3, bar3h | 3-D bar chart |
| bolling | Bollinger band chart |
| candle | Candlestick chart |
| candle (fts) | Time series candle plot |
| chartfts | Interactive display |
| dateaxis | Convert serial-date axis labels to <br> calendar-date axis labels |
| highlow | High, low, open, close chart |
| highlow (fts) | Time series High-Low plot |

kagi<br>linebreak<br>movavg<br>plot<br>pointfig<br>priceandvol<br>renko<br>volarea

Kagi chart
Line break chart
Leading and lagging moving averages chart

Plot data series
Point and figure chart
Price and volume chart
Renko chart
Price and volume chart

## Cash Flows

Annuities (p. 15-7)
Amortization and Depreciation (p. 15-8)

Present Value (p. 15-8)
Future Value (p. 15-8)
Payment Calculations (p. 15-8)
Rates of Return (p. 15-9)
Cash Flow Sensitivities (p. 15-9)

## Annuities

annurate
annuterm

Work with annuities
Work with amortization and depreciation
Work with present values
Work with future values
Work with payment calculations
Work with rates of return
Work with cash flow sensitivities

Periodic interest rate of annuity
Number of periods to obtain value

## Amortization and Depreciation

amortize<br>depfixdb<br>depgendb<br>deprdv<br>depsoyd<br>depstln

Amortization schedule
Fixed declining-balance depreciation schedule

General declining-balance depreciation schedule

Remaining depreciable value
Sum of years' digits depreciation
Straight-line depreciation schedule

Present value with fixed periodic payments

Present value of varying cash flow

Future value of discounted security
Future value with fixed periodic payments

Future value of varying cash flow

## Payment Calculations

| payadv | Periodic payment given number of <br> advance payments |
| :--- | :--- |
| payodd | Payment of loan or annuity with odd <br> first period |

payper
payuni

## Rates of Return

## effrr

elpm
irr
mirr
nomrr
taxedrr
xirr

## Cash Flow Sensitivities

cfconv
cfdur

Periodic payment of loan or annuity
Uniform payment equal to varying cash flow

Effective rate of return
Compute expected lower partial moments for normal asset returns

Internal rate of return
Modified internal rate of return
Nominal rate of return
After-tax rate of return
Internal rate of return for nonperiodic cash flow

Cash flow convexity
Cash-flow duration and modified duration

## Fixed-Income Securities

Accrued Interest (p. 15-10)
Prices (p. 15-10)
Term Structure of Interest Rates (p. 15-10)

Yields (p. 15-11)

Work with accrued interest
Work with prices
Work with term structure of interest rates

Work with yields

| Spreads (p. 15-11) | Work with spreads |
| :--- | :--- |
| Interest Rate Sensitivities (p. 15-11) | Work with interest rate sensitivities |

## Accrued Interest

acrubond
acrudisc

## Prices

bndprice<br>prdisc<br>prmat<br>prtbill

## Term Structure of Interest Rates

| disc2zero | Zero curve given discount curve |
| :--- | :--- |
| fwd2zero | Zero curve given forward curve |
| prbyzero | Price bonds in portfolio by set of zero <br> curves |
| pyld2zero | Zero curve given par yield curve |
| tbl2bond | Treasury bond parameters given <br> Treasury bill parameters |
| tr2bonds | Term-structure parameters given <br> Treasury bond parameters |
| zbtprice | Zero curve bootstrapping from <br> coupon bond data given price |

zbtyield
zero2disc
zero2fwd
zero2pyld

## Yields

beytbill
bndyield
discrate
ylddisc
yldmat
yldtbill

## Spreads

bndspread

## Interest Rate Sensitivities

bndconvp
bndconvy
bnddurp
bnddury
bndkrdur

Static spread over spot curve
Zero curve bootstrapping from coupon bond data given yield Discount curve given zero curve Forward curve given zero curve Par yield curve given zero curve

Bond equivalent yield for Treasury bill

Yield to maturity for fixed income security

Bank discount rate of money market security

Yield of discounted security
Yield with interest at maturity
Yield of Treasury bill

## Portfolio Optimization Objects

| Portfolio Objects (p. 15-12) | Construct portfolio object |
| :--- | :--- |
| Get Methods (p. 15-12) | Obtain portfolio object information |
| Set Methods (p. 15-13) | Set portfolio object information |
| Add Methods (p. 15-14) | Add portfolio object information |
| Preprocessing Methods (p. 15-14) | Preprocess portfolio object <br> information |
| Efficient Portfolio Estimation | Efficient portfolio estimation <br> methods for portfolio object |
| Methods (p. 15-14) | Efficient frontier methods for <br> pertfolio object |
| Utility Methods (p. 15-15) | Utility methods for portfolio object |

## Portfolio Objects

AbstractPortfolio<br>Portfolio

## Get Methods

getAssetMoments (Portfolio)<br>getBounds (Portfolio)<br>getBudget (Portfolio)<br>getCosts (Portfolio)


#### Abstract

portfolio object for portfolio optimization and analysis

Portfolio object for mean-variance portfolio optimization and analysis


Obtain mean and covariance of asset returns from portfolio object

Obtain bounds for portfolio weights from portfolio object

Obtain budget constraint bounds from portfolio object

Obtain buy and sell transaction costs from portfolio object

```
getEquality (Portfolio)
getGroupRatio (Portfolio)
getGroups (Portfolio)
getInequality (Portfolio)
```


## Set Methods

| setAssetList (Portfolio) | Set up list of identifiers for assets |
| :--- | :--- |
| setAssetMoments (Portfolio) | Set moments (mean and covariance) <br> of asset returns |
| setBounds (Portfolio) | Set up bounds for portfolio weights |
| setBudget (Portfolio) | Set up budget constraints |
| setCosts (Portfolio) | Set up proportional transaction costs |
| setDefaultConstraints (Portfolio) | Set up portfolio constraints with <br> nonnegative weights that must sum <br> to 1 |
| setEquality (Portfolio) | Set up linear equality constraints for <br> portfolio weights |
| setGroupRatio (Portfolio) | Set up group ratio constraints for <br> portfolio weights |
| setGroups (Portfolio) | Set up group constraints for portfolio <br> weights |
| setInequality (Portfolio) | Set up linear inequality constraints <br> for portfolio weights |
| setInitPort (Portfolio) | Set up initial or current portfolio <br> Set hidden properties in portfolio <br> object |
| setOptions (Portfolio) |  |

$\begin{array}{ll}\text { setSolver (Portfolio) } & \begin{array}{l}\text { Choose main solver and specify } \\ \text { associated solver options for portfolio } \\ \text { optimization }\end{array} \\ \text { setTurnover (Portfolio) } & \begin{array}{l}\text { Set up maximum portfolio turnover } \\ \text { constraint }\end{array}\end{array}$

## Add Methods

addEquality (Portfolio)<br>addGroupRatio (Portfolio)<br>addGroups (Portfolio)<br>addInequality (Portfolio)

Add linear equality constraints for portfolio weights to existing constraints

Add group ratio constraints for portfolio weights to existing group ratio constraints

Add group constraints for portfolio weights to existing group constraints
Add linear inequality constraints for portfolio weights to existing constraints

## Preprocessing Methods

estimateAssetMoments (Portfolio)

Estimate mean and covariance of asset returns from data

## Efficient Portfolio Estimation Methods

estimateFrontier (Portfolio)
estimateFrontierByReturn (Portfolio)

Estimate specified number of optimal portfolios over entire efficient frontier

Estimate optimal portfolios with targeted portfolio returns
> estimateFrontierByRisk (Portfolio)
> estimateFrontierLimits (Portfolio)

Estimate optimal portfolios with targeted portfolio risks

Estimate optimal portfolios at endpoints of efficient frontier

## Efficient Frontier Methods

estimatePortMoments (Portfolio)
estimatePortReturn (Portfolio)
estimatePortRisk (Portfolio)
plotFrontier (Portfolio)
Estimate moments of portfolio returns

Estimate mean of portfolio returns (portfolio return)

Estimate standard deviation of portfolio returns (portfolio risk)

Plot efficient frontier

## Utility Methods

checkFeasibility (Portfolio)
estimateBounds (Portfolio)
Check feasibility of input portfolios against a portfolio object

Estimate global lower and upper bounds for set of portfolios

## Portfolio Analysis

Basic Portfolio Optimization (p. 15-16)

Performance Metrics (p. 15-16)

Portfolio Utilities (p. 15-17)

Perform portfolio analysis

Calculate portfolio performance metrics

Work with portfolio statistics

## Basic Portfolio Optimization

| frontcon | Mean-variance efficient frontier |
| :--- | :--- |
| frontier | Rolling efficient frontier |
| pcalims | Linear inequalities for individual <br> asset allocation |
| pcgcomp | Linear inequalities for asset group <br> comparison constraints |
| pcglims | Linear inequalities for asset group <br> minimum and maximum allocation |
| pcpval | Linear inequalities for fixing total <br> portfolio value |
| portalloc | Optimal capital allocation to efficient <br> frontier portfolios |
| portcons | Portfolio constraints |
| portopt | Portfolios on constrained efficient <br> frontier |
| portror | Portfolio expected rate of return |
| selectreturn | Portfolio configurations from 3-D <br> efficient frontier |
| targetreturn | Portfolio weight accuracy |

## Performance Metrics

emaxdrawdown<br>inforatio<br>lpm<br>maxdrawdown

Compute expected maximum drawdown for Brownian motion Calculate information ratio for one or more assets

Compute sample lower partial moments of data

Compute maximum drawdown for one or more price series

portalpha<br>sharpe

Compute risk-adjusted alphas and returns for one or more assets

Compute Sharpe ratio for one or more assets

## Portfolio Utilities

| abs2active | Convert constraints from absolute to <br> active format |
| :--- | :--- |
| active2abs | Convert constraints from active to <br> absolute format |
| arith2geom | Arithmetic to geometric moments of <br> asset returns |
| corr2cov | Convert standard deviation and <br> correlation to covariance |
| cov2corr | Convert covariance to standard <br> deviation and correlation coefficient |
| ewstats | Expected return and covariance from <br> return time series |
| geom2arith | Geometric to arithmetic moments of <br> asset returns |
| holdings2weights | Portfolio holdings into weights <br> periodicreturns |
| portrand | return prices returns from total |
| partsim | Randomized portfolio risks, returns, <br> and weights |
| portstats | Monte Carlo simulation of correlated <br> asset returns |
| portvar | Portfolio expected return and risk <br> portvrisk |
| Variance for portfolio of assets |  |

ret2tick
ret2tick (fts)
tick2ret
tick2ret (fts)
totalreturnprice
weights2holdings

Convert return series to price series
Convert return series to price series for time series object

Convert price series to return series
Convert price series to return series for time series object

Total return price time series
Portfolio values and weights into holdings

## Financial Statistics

Expectation Conditional
Maximization (p. 15-18)
Multivariate Normal Regression (p. 15-19)

Expectation Conditional Maximization - Multivariate
Normal Regression (p. 15-19)
Expectation Conditional
Maximization - Least-Squares Regression (p. 15-20)
Seemingly Unrelated Regression (p. 15-20)

Work with expectation conditional maximization
Work with multivariate normal regression
Work with expectation conditional maximization and multivariate normal regression

Work with least-squares regression

Work with unrelated regression

## Expectation Conditional Maximization

ecmnfish

ecmnhess

| ecmninit | Initial mean and covariance |
| :--- | :--- |
| ecmnmle | Mean and covariance of incomplete <br> multivariate normal data |
| ecmnobj | Multivariate normal negative <br> log-likelihood function |
| ecmnstd | Standard errors for mean and <br> covariance of incomplete data |

## Multivariate Normal Regression

| mvnrfish | Fisher information matrix for <br> multivariate normal or least-squares <br> regression |
| :--- | :--- |
| mvnrmle | Multivariate normal regression <br> (ignore missing data) |
| mvnrobj | Log-likelihood function for <br> multivariate normal regression <br> without missing data |
| mvnrstd | Evaluate standard errors for <br> multivariate normal regression <br> model |

## Expectation Conditional Maximization - Multivariate Normal Regression

ecmmvnrfish<br>ecmmvnrmle

Fisher information matrix for multivariate normal regression model

Multivariate normal regression with missing data

| ecmmvnrobj | Log-likelihood function for <br> multivariate normal regression <br> with missing data |
| :--- | :--- |
| ecmmvnrstd | Evaluate standard errors for <br> multivariate normal regression <br> model |

## Expectation Conditional Maximization -Least-Squares Regression

| ecmlsrmle | Least-squares regression with <br> missing data |
| :--- | :--- |
| ecmlsrobj | Log-likelihood function for <br> least-squares regression with <br> missing data |

## Seemingly Unrelated Regression

convert2sur

Convert multivariate normal regression model to seemingly unrelated regression (SUR) model

## Derivatives

Option Valuation and Sensitivity
(p. 15-21)

Work with option valuation and sensitivity

## Option Valuation and Sensitivity

| binprice | Binomial put and call pricing |
| :---: | :---: |
| blkimpv | Implied volatility for futures options from Black's model |
| blkprice | Black's model for pricing futures options |
| blsdelta | Black-Scholes sensitivity to underlying price change |
| blsgamma | Black-Scholes sensitivity to underlying delta change |
| blsimpv | Black-Scholes implied volatility |
| blslambda | Black-Scholes elasticity |
| blsprice | Black-Scholes put and call option pricing |
| blsrho | Black-Scholes sensitivity to interest rate change |
| blstheta | Black-Scholes sensitivity to time-until-maturity change |
| blsvega | Black-Scholes sensitivity to underlying price volatility |
| opprofit | Option profit |

## Credit Risk Utilities

Estimation of Transition
Probabilities (p. 15-22)

Estimate transition probabilities

## Estimation of Transition Probabilities

transprob<br>transprobbytotals

Estimation of transition probabilities from credit ratings data

Estimation of transition probabilities from preprocessed credit ratings data

## GARCH Processes

Univariate GARCH Processes (p. 15-22)

## Univariate GARCH Processes

| ugarch | Univariate GARCH $(\mathrm{P}, \mathrm{Q})$ parameter <br> estimation with Gaussian <br> innovations |
| :--- | :--- |
| ugarchllf | Log-likelihood objective function of <br> univariate GARCH $(\mathrm{P}, \mathrm{Q})$ processes <br> with Gaussian innovations |
| ugarchpred | Forecast conditional variance of <br> univariate GARCH $(\mathrm{P}, \mathrm{Q})$ processes |
| ugarchsim | Simulate univariate GARCH(P,Q) <br> process with Gaussian innovations |

## Financial Time Series Object and File Construction

ascii2fts<br>fints<br>fts2ascii<br>fts2mat<br>merge

## Financial Time Series Arithmetic

## end

horzcat
length
minus
mrdivide
mtimes
plus
power
rdivide
size
subsasgn
subsref
times

Create financial time series object from ASCII data file

Construct financial time series object
Write elements of time-series data into ASCII file

Convert to matrix
Merge multiple financial time series objects

Last date entry
Concatenate financial time series objects horizontally
Get number of dates (rows)
Financial time series subtraction
Financial time series matrix division
Financial time series matrix multiplication

Financial time series addition
Financial time series power
Financial time series division
Number of dates and data series
Content assignment
Subscripted reference
Financial time series multiplication

| uminus | Unary minus of financial time series <br> object |
| :--- | :--- |
| uplus | Unary plus of financial time series <br> object |
| vertcat | Concatenate financial time series <br> objects vertically |

## Financial Time Series Math

cumsum
$\exp$
hist
$\log$
$\log 10$
$\log 2$
$\max$
mean
min
std

Cumulative sum
Exponential values
Histogram
Natural logarithm
Common logarithm
Base 2 logarithm
Maximum value
Arithmetic average
Minimum value
Standard deviation

## Financial Time Series Descriptive Statistics

| corrcoef | Correlation coefficients |
| :--- | :--- |
| cov | Covariance matrix |
| isempty | True for empty financial time series <br> objects |
| nancov | Covariance ignoring NaNs |

nanmax
nanmean
nanmedian
nanmin
nanstd
nansum
nanvar
var

Maximum ignoring NaNs
Mean ignoring NaNs
Median ignoring NaNs
Minimum ignoring NaNs
Standard deviation ignoring NaNs
Sum ignoring NaNs
Variance ignoring NaNs
Variance

## Financial Time Series Utility

| chfield | Change data series name |
| :---: | :---: |
| eq (fts) | Multiple financial times series object equality |
| extfield | Data series extraction |
| fetch | Data from financial time series object |
| fieldnames | Get names of fields |
| freqnum | Convert string frequency indicator to numeric frequency indicator |
| freqstr | Convert numeric frequency indicator to string representation |
| ftsbound | Start and end dates |
| ftsinfo | Financial time series object information |
| ftsuniq | Determine uniqueness |
| getfield | Content of specific field |
| getnameidx | Find name in list |


| iscompatible | Structural equality |
| :--- | :--- |
| isequal | Multiple object equality |
| isfield | Check whether string is field name |
| issorted | Check whether dates and times are <br> monotonically increasing |
| rmfield | Remove data series |
| setfield | Set content of specific field |
| sortfts | Sort financial time series |

## Financial Time Series Data Transformation

boxcox
convert2sur

convertto
diff
fillts
filter
lagts
leadts
peravg
resamplets
smoothts
toannual
todaily
todecimal

Box-Cox transformation
Convert multivariate normal regression model to seemingly unrelated regression (SUR) model

Convert to specified frequency
Differencing
Fill missing values in time series
Linear filtering
Lag time series object
Lead time series object
Periodic average of FINTS object
Downsample data
Smooth data
Convert to annual
Convert to daily
Fractional to decimal conversion
tomonthly
toquarterly
toquoted
tosemi
toweekly
tsmovavg

## Financial Time Series Indicator

adline
adosc
bollinger
chaikosc
chaikvolat
fpctkd
hhigh
llow
macd
medprice
negvolidx
onbalvol
posvolidx
prcroc
pvtrend
rsindex
spctkd

Convert to monthly
Convert to quarterly
Decimal to fractional conversion
Convert to semiannual
Convert to weekly
Moving average

Accumulation/Distribution line
Accumulation/Distribution oscillator
Time series Bollinger band
Chaikin oscillator
Chaikin volatility
Fast stochastics
Highest high
Lowest low
Moving Average Convergence/Divergence (MACD)
Median price
Negative volume index
On-Balance Volume (OBV)
Positive volume index
Price rate of change
Price and Volume Trend (PVT)
Relative Strength Index (RSI)
Slow stochastics
stochosc
tsaccel
tsmom
typprice
volroc
wclose
willad
willpctr

Financial Time Series GUI
ftsgui

Financial Time Series Tool
ftstool

Stochastic oscillator
Acceleration between times
Momentum between times
Typical price
Volume rate of change
Weighted close
Williams Accumulation/Distribution line

Williams \%R

Financial time series GUI

Financial time series tool

## Class Reference

AbstractPortfolio<br>Portfolio

Abstract portfolio object for portfolio optimization and analysis

Portfolio object for mean-variance portfolio optimization and analysis

Functions - Alphabetical
List

## abs2active

# Purpose Convert constraints from absolute to active format 

Syntax ActiveConSet = abs2active (AbsConSet, Index)

Description

## Input <br> Arguments

## Output

 Arguments
## Definitions

ActiveConSet = abs2active(AbsConSet, Index) transforms a constraint matrix to an equivalent matrix expressed in active weight format (relative to the index).

AbsConSet
Portfolio linear inequality constraint matrix expressed in absolute weight format. AbsConSet is formatted as [A b] such that $A^{*} w<=b$, where $A$ is a number of constraints (NCONSTRAINTS) by number of assets (NASSETS) weight coefficient matrix, and $b$ and w are column vectors of length NASSETS. The value w represents a vector of absolute asset weights whose elements sum to the total portfolio value. See the output ConSet from portcons for additional details about constraint matrices.

## Index

NASSETS-by- 1 vector of index portfolio weights. The sum of the index weights must equal the total portfolio value (for example, a standard portfolio optimization imposes a sum-to-one budget constraint).

ActiveConSet
The transformed portfolio linear inequality constraint matrix expressed in active weight format, also of the form [A b] such that $A^{*} w<=b$. The value $w$ represents a vector of active asset weights (relative to the index portfolio) whose elements sum to zero.
abs2active transforms a constraint matrix to an equivalent matrix expressed in active weight format (relative to the index). The transformation equation is

$$
A w_{\text {absolute }}=A\left(w_{\text {active }}+w_{\text {index }}\right) \leq b_{\text {absolute }} .
$$

Therefore

$$
A w_{\text {active }} \leq b_{\text {absolute }}-A w_{\text {index }}=b_{\text {active }}
$$

The initial constraint matrix consists of NCONSTRAINTS portfolio linear inequality constraints expressed in absolute weight format. The index portfolio vector contains NASSETS assets.

## Examples

Set up constraints for a portfolio optimization for portfolio wo with constraints in the form $A^{*} w<=b$, where $w$ is absolute portfolio weights. (Absolute weights do not depend on the tracking portfolio.) Use abs2active to convert constraints in terms of absolute weights into constraints in terms of active portfolio weights, defined relative to the tracking portfolio w0. Assume three assets with the following mean and covariance of asset returns:

```
m = [ 0.14; 0.10; 0.05 ];
C = [ 0.29^2 0.4*0.29*0.17 0.1*0.29*0.08; 0.4*0.29*0.17 0.17^2 0.3*0.17*0.08; ...
0.1*0.29*0.08 0.3*0.17*0.08 0.08^2 ];
```

Absolute portfolio constraints are the typical ones (weights sum to 1 and fall from 0 through 1 ), create the $A$ and $b$ matrices using portcons:

```
AbsCons = portcons('PortValue',1,3,'AssetLims', [0; 0; 0], [1; 1; 1;]);
```

The efficient frontier is:

```
portopt(m, C, [], [], AbsCons);
```

The tracking portfolio w0 is:

```
wO = [ 0.1; 0.55; 0.35 ];
```

Use abs2active to compute the constraints for active portfolio weights:

```
ActCons = abs2active(AbsCons, w0)
```

This returns:

| ActCons $=$ |  |  |  |
| ---: | ---: | ---: | ---: |
|  |  |  |  |
| 1.0000 | 1.0000 | 1.0000 | 0 |
| -1.0000 | -1.0000 | -1.0000 | 0 |
| 1.0000 | 0 | 0 | 0.9000 |
| 0 | 1.0000 | 0 | 0.4500 |
| 0 | 0 | 1.0000 | 0.6500 |
| -1.0000 | 0 | 0 | 0.1000 |
| 0 | -1.0000 | 0 | 0.5500 |
| 0 | 0 | -1.0000 | 0.3500 |

The efficient frontier demonstrates expected returns and risk relative to the tracking portfolio w0:

```
portopt(m, C, [], [], ActCons);
```

The returns:


See Also
active2abs | pcalims | pcgcomp | pcglims | pcpval | portcons

## AbstractPortfolio

## Purpose <br> Abstract portfolio object for portfolio optimization and analysis

Construction There is no constructor for the abstract class. To construct a portfolio

Description

## Properties

The portfolio object implements mean-variance portfolio optimization and is derived from the abstract class AbstractPortfolio. object, see the Portfolio class.

Name
Name for instance of the portfolio object ([] or [string]).
Attributes:
SetAccess public
GetAccess public

NumAssets
Number of assets in universe ([] or [integer scalar]).
Attributes:
SetAccess public
GetAccess public

## AssetList

Names or symbols of assets in universe ([] or [vector cell of strings]).

Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

InitPort

Initial portfolio ([] or vector).
Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

## AInequality

Linear inequality constraint matrix ([] or [matrix ]).
Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

## bInequality

Linear inequality constraint vector ([ ] or [vector]).
Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

## AEquality

Linear equality constraint matrix ([] or [matrix]).
Attributes:

```
SetAccess public
```

GetAccess public
bEquality
Linear equality constraint vector ([] or [vector ]).
Attributes:

## AbstractPortfolio

SetAccess ..... public
GetAccess ..... public
LowerBoundLower-bound constraint ([] or [vector]).Attributes:
SetAccess ..... public
GetAccess ..... public
UpperBound
Upper-bound constraint ([] or [vector]).
Attributes:
SetAccess ..... public
GetAccess ..... public
LowerBudget
Lower-bound budget constraint ([ ] or [scalar]).
Attributes:
SetAccess ..... public
GetAccess ..... public
UpperBudget
Upper-bound budget constraint ([] or [scalar]).
Attributes:
SetAccess ..... public
GetAccess public

## GroupMatrix

Group membership matrix ([] or [matrix ]).
Attributes:
SetAccess public

GetAccess public

## LowerGroup

Lower-bound group constraint ([] or [vector]).
Attributes:

SetAccess public
GetAccess public

## UpperGroup

Upper-bound group constraint ([] or [vector]).
Attributes:
SetAccess public
GetAccess public

## GroupA

Group A weights to be bounded by weights in group B ([] or [matrix]).

## Attributes:

SetAccess public

GetAccess public
GroupB
Group B weights ([ ] or [matrix]).

## AbstractPortfolio

## Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

LowerRatio
Minimum ratio of allocations between groups A and B ([] or [vector]).

## Attributes:

SetAccess public
GetAccess public
UpperRatio
Maximum ratio of allocations between groups A and B ([] or [vector]).

Attributes:
SetAccess public
GetAccess public

## Methods

| addEquality | Add linear equality constraints <br> for portfolio weights to existing <br> constraints |
| :--- | :--- |
| addGroupRatio | Add group ratio constraints for <br> portfolio weights to existing group <br> ratio constraints |
| addGroups | Add group constraints for <br> portfolio weights to existing <br> group constraints |

## AbstractPortfolio

addInequality<br>checkFeasibility<br>estimateBounds<br>estimateFrontier<br>estimateFrontierByReturn<br>estimateFrontierByRisk<br>estimateFrontierLimits<br>estimatePortReturn<br>estimatePortRisk<br>getBounds<br>getBudget<br>getEquality<br>getGroupRatio

Add linear inequality constraints for portfolio weights to existing constraints

Check feasibility of input portfolios against a portfolio object

Estimate global lower and upper bounds for set of portfolios

Estimate specified number of optimal portfolios over entire efficient frontier

Estimate optimal portfolios with targeted portfolio returns

Estimate optimal portfolios with targeted portfolio risks

Estimate optimal portfolios at endpoints of efficient frontier

Estimate mean of portfolio returns (portfolio return)

Estimate standard deviation of portfolio returns (portfolio risk)

Obtain bounds for portfolio weights from portfolio object
Obtain budget constraint bounds from portfolio object

Obtain equality constraint arrays from portfolio object

Obtain group ratio constraint arrays from portfolio object

## AbstractPortfolio

| getGroups | Obtain group constraint arrays from portfolio object |
| :---: | :---: |
| getInequality | Obtain inequality constraint arrays from portfolio object |
| plotFrontier | Plot efficient frontier |
| setAssetList | Set up list of identifiers for assets |
| setBounds | Set up bounds for portfolio weights |
| setBudget | Set up budget constraints |
| setDefaultConstraints | Set up portfolio constraints with nonnegative weights that must sum to 1 |
| setEquality | Set up linear equality constraints for portfolio weights |
| setGroupRatio | Set up group ratio constraints for portfolio weights |
| setGroups | Set up group constraints for portfolio weights |
| setInequality | Set up linear inequality constraints for portfolio weights |
| setInitPort | Set up initial or current portfolio |
| setOptions | Set hidden properties in portfolio object |
| setSolver | Choose main solver and specify associated solver options for portfolio optimization |


| Instance | The AbstractPortfolio class has one subclass, Portfolio, that |
| :--- | :--- |
| Hierarchy | inherits properties and methods from theAbstractPortfolio class. |

## AbstractPortfolio

## Attributes

Copy
Semantics
Alternatives You can perform portfolio optimization using a collection of special-purpose functions in Financial Toolbox software. For more information, see "Portfolio Optimization Functions" on page 3-3.

See Also Portfolio<br>How To - Class Attributes<br>- Property Attributes

## accrfrac

Purpose Fraction of coupon period before settlement

```
Syntax Fraction = accrfrac(Settle, Maturity)
Fraction = accrfrac(Settle, Maturity, Period, Basis,
EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate,
StartDate)
```


## Description

## Input <br> Arguments

Fraction = accrfrac(Settle, Maturity) returns the fraction of the coupon period before settlement.

Fraction = accrfrac(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the fraction of the coupon period before settlement with optional inputs.

Use accrfrac for computing accrued interest.

Settle
Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

Maturity
Maturity date. A vector of serial date numbers or date strings.
Period
Coupons per year of the bond. A vector of integers. Values are $0,1,2,3,4,6$, and 12 .

Default: 2

## Basis

Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual $/ 360$
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0

## EndMonthRule

End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. $1=$ set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

## IssueDate

Issue date for a bond.
FirstCouponDate

## accrfrac

First actual coupon date.

## LastCouponDate

Last actual coupon date.

## StartDate

Future implementation.

## Output Arguments

Fraction
The cash flow matrix of a portfolio of bonds. Each row represents the cash flow vector of a single bond. Each element in a column represents a specific cash flow for that bond.

## Examples Find the accrued interest for given bond data:

```
Settle = '14-Mar-1997';
Maturity = ['30-Nov-2000'
        '31-Dec-2000'
        '31-Jan-2001'];
Period = 2;
Basis = 0;
EndMonthRule = 1;
Fraction = accrfrac(Settle, Maturity, Period, Basis,...
EndMonthRule)
```

This returns:

```
Fraction =
```

0.5714
0.4033
0.2320

## See Also

cfdates | cfamounts | cpncount | cpndaten | cpndatenq | cpndatep | cpndatepq | cpndaysn | cpndaysp | cpnpersz

## Purpose

Accrued interest of security with periodic interest payments
Synfax AccruInterest = acrubond(IssueDate, Settle, FirstCouponDate, Face, CouponRate, Period, Basis)

## Arguments

IssueDate Enter as serial date number or date string.
Settle
Enter as serial date number or date string.
FirstCouponDate
Face
CouponRat
Period

Basis

Enter as serial date number or date string.
Redemption (par, face) value.
Enter as decimal fraction.
(Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), $3,4,6$, and 12.
(Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)


## acrubond

- $10=$ actual $/ 365$ (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

## Description

AccruInterest = acrubond(IssueDate, Settle, FirstCouponDate, Face, CouponRate, Period, Basis) returns the accrued interest for a security with periodic interest payments. This function computes the accrued interest for securities with standard, short, and long first coupon periods.

Note cfamounts or accrfrac is recommended when calculating accrued interest beyond the first period.

```
Examples
```

```
AccruInterest = acrubond('31-jan-1983', '1-mar-1993', ...
```

AccruInterest = acrubond('31-jan-1983', '1-mar-1993', ...
'31-jul-1983', 100, 0.1, 2, 0)
'31-jul-1983', 100, 0.1, 2, 0)
AccruInterest =
AccruInterest =
0.8011

```
    0.8011
```

See Also accrfrac | acrudisc | bndprice | bndyield | cfamounts | datenum

```
Purpose
Accrued interest of discount security paying at maturity
Syntax AccruInterest = acrudisc(Settle, Maturity, Face, Discount, Period,
Basis)
```


## Arguments

| Settle | Enter as serial date number or date string. <br> Settle must be earlier than Maturity. |
| :--- | :--- |
| Maturity | Enter as serial date number or date string. |
| Face | Redemption (par, face) value. |
| Discount | Discount rate of the security. Enter as decimal <br> fraction. |
| Period | (Optional) Coupons per year of the bond. A vector <br> of integers. Allowed values are 0, 1, 2 (default), 3, <br> 4,6, and 12. |
| Basis | (Optional) Day-count basis of the instrument. A <br> vector of integers. |

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

## Description

## Examples

References

See Also

AccruInterest = acrudisc(Settle, Maturity, Face, Discount, Period, Basis) returns the accrued interest of a discount security paid at maturity.

```
AccruInterest = acrudisc('05/01/1992', '07/15/1992', ...
    100, 0.1, 2, 0)
AccruInterest =
    2.0604 (or $2.06)
```

Mayle, Standard Securities Calculation Methods, Volumes I-II, 3rd edition. Formula D.
acrubond | prdisc | prmat | ylddisc | yldmat

# Purpose Convert constraints from active to absolute format <br> Syntax AbsConSet = active2abs(ActiveConSet, Index) 

## Arguments

| ActiveConSet | Portfolio linear inequality constraint matrix expressed in active weight format. ActiveConSet is formatted as [A b] such that $A^{*} \mathrm{w}$ <= b , where A is a number of constraints (NCONSTRAINTS) by number of assets (NASSETS) weight coefficient matrix, and $b$ and $w$ are column vectors of length NASSETS. The value w represents a vector of active asset weights (relative to the index portfolio) whose elements sum to 0 . |
| :---: | :---: |
|  | See the output ConSet from portcons for additional details about constraint matrices. |
| Index | NASSETS-by- 1 vector of index portfolio weights. The sum of the index weights must equal the total portfolio value (for example, a standard portfolio optimization imposes a sum-to-one budget constraint). |

Description AbsConSet = active2abs(ActiveConSet, Index) transforms a constraint matrix to an equivalent matrix expressed in absolute weight format. The transformation equation is

$$
A w_{\text {active }}=A\left(w_{\text {absolute }}-w_{\text {index }}\right) \leq b_{\text {active }} .
$$

Therefore

## active2abs

$$
A w_{\text {absolute }} \leq b_{\text {active }}+A w_{\text {index }}=b_{\text {absolute }} \text {. }
$$

The initial constraint matrix consists of NCONSTRAINTS portfolio linear inequality constraints expressed in active weight format (relative to the index portfolio). The index portfolio vector contains NASSETS assets.

AbsConSet is the transformed portfolio linear inequality constraint matrix expressed in absolute weight format, also of the form [A b] such that $A * W$ <= $b$. The value $w$ represents a vector of active asset weights (relative to the index portfolio) whose elements sum to the total portfolio value.

[^3]
## Superclasses AbstractPortfolio

Purpose

Syntax
Description

Tips

Arguments

Add linear equality constraints for portfolio weights to existing constraints
obj = addEquality(obj, AEquality, bEquality)
obj = addEquality (obj, AEquality, bEquality) to add linear equality constraints for portfolio weights to existing constraints.

Given a linear equality constraint matrix AEquality and vector bEquality, every weight in a portfolio Port must satisfy:

```
AEquality * Port = bEquality
```

An results if AEquality is empty and bEquality is nonempty, or if AEquality is nonempty and bEquality is empty.

This method "stacks" additional linear equality constraints onto any existing linear equality constraints that already exist in the input portfolio object. If no constraints already exist, this method is the same as setEquality.

- Use dot notation to add the linear equality constraints for portfolio weights:

```
obj = obj.addEquality(AEquality, bEquality)
```

- To remove linear equality constraints from a portfolio object:

```
obj = obj.setEquality([ ], [ ])
```

obj
A portfolio object [Portfolio].
AEquality
Matrix to form linear equality constraints [matrix].

## Portfolio.addEquality

## bEquality

Vector to form linear equality constraints [vector].

## Output Arguments obj

 Updated portfolio object [Portfolio].
## Attributes

## Examples

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

To add another linear equality constraint to ensure that the last 3 assets constitute $50 \%$ of a portfolio, use addEquality to build up linear equality constraints:

```
p = Portfolio;
A = [ 1 1 1 0 0 ]; % first equality constraint
b = 0.5;
p = p.setEquality(A, b);
A = [ 0 0 1 1 1 1 ]; % second equality constraint
b = 0.5;
p = p.addEquality(A, b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);
5
\begin{tabular}{lllll}
1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1
\end{tabular}
```

$$
\begin{aligned}
& 0.5000 \\
& 0.5000
\end{aligned}
$$

See Also setEquality | Portfolio

## Portfolio.addGroupRatio

Superclasses AbstractPortfolio

Purpose

Syntax

Description

Add group ratio constraints for portfolio weights to existing group ratio constraints
obj = addGroupRatio(obj, GroupA, GroupB, LowerRatio) obj = addGroupRatio(obj, GroupA, GroupB, LowerRatio, UpperRatio)
obj = addGroupRatio(obj, GroupA, GroupB, LowerRatio) to add group ratio constraints for the portfolio weights to existing group ratio constraints with just a lower bound on the ratio between groups.
obj = addGroupRatio(obj, GroupA, GroupB, LowerRatio, UpperRatio) to add group ratio constraints for the portfolio weights to existing group ratio constraints with an additional option for UpperRatio.

Given base and comparison group matrices GroupA and GroupB and, either LowerRatio, or UpperRatio bounds, group ratio constraints require any portfolio in Port to satisfy:

```
(GroupB * Port) .* LowerRatio <= GroupA * Port <= (GroupB * Port) .* UpperRatio
```


## Caution

This collection of constraints usually require that portfolio weights be nonnegative and that the products GroupA * Port and GroupB * Port are always nonnegative. Although negative portfolio weights and non-Boolean group ratio matrices are supported, use with caution.

## Tips

- Use dot notation to add group ratio constraints for the portfolio weights to existing group ratio constraints:

```
obj = obj.addGroupRatio(GroupA, GroupB, LowerRatio, UpperRatio)
```


## Portfolio.addGroupRatio

- To remove group ratio constraints from a portfolio object, enter empty arrays for the corresponding arrays.


## Input <br> obj

Arguments
A portfolio object [Portfolio].
GroupA
Matrix that forms base groups for comparison [matrix].

## GroupB

Matrix that forms comparison groups [matrix].

Note The group matrices GroupA and GroupB are usually indicators of membership in groups, which means that their elements are usually either 0 or 1 . Because of this interpretation, the GroupA and GroupB matrices can be logical or numerical arrays.

## LowerGroup

Lower-bound for ratio of GroupB groups to GroupA groups [vector].

Note If input is scalar, LowerGroup undergoes scalar expansion to be conformable with the group matrices.

## UpperRatio

(Optional) Upper-bound for ratio of GroupB groups to GroupA groups [vector].

## Portfolio.addGroupRatio

> Note If input is scalar, UpperRatio undergoes scalar expansion to be conformable with the group matrices.

## Output Arguments

## Attributes

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

## Examples

obj
Updated portfolio object [Portfolio].


To add another group ratio constraint to ensure that the weight in odd-numbered assets constitute at least $20 \%$ of the weight in nonfinancial assets of a portfolio, use addGroupRatio to build up group ratio constraints by creating another group matrix for the second group constraint:

```
p = Portfolio;
GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % non-financial companies
p = p.setGroupRatio(GA, GB, [], 0.5);
GA = [ true false true false true false ]; % odd-numbered companies
GB = [ false false false true true true ]; % non-financial companies
p = p.addGroupRatio(GA, GB, 0.2);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.LowerRatio);
```

| disp(p.UpperRatio) ; |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 |  |  |  |  |  |
| 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 |
| - Inf |  |  |  |  |  |
| 0.2000 |  |  |  |  |  |
| 0.5000 |  |  |  |  |  |
| Inf |  |  |  |  |  |

See Also<br>setGroupRatio | Portfolio

## Portfolio.addGroups

## Superclasses AbstractPortfolio

Purpose
Add group constraints for portfolio weights to existing group constraints
Syntax
Description

Tips

Input
Arguments
obj = addGroups(obj, GroupMatrix, LowerGroup)
obj = addGroups(obj, GroupMatrix, LowerGroup, UpperGroup)
obj = addGroups(obj, GroupMatrix, LowerGroup) to add group constraints for portfolio weights to existing group constraints subject to a lower bound on groups.
obj = addGroups(obj, GroupMatrix, LowerGroup, UpperGroup) to add the group constraints for portfolio weights to existing group constraints with an additional option for UpperGroup.
Given GroupMatrix and either LowerGroup or UpperGroup, a portfolio Port must satisfy:

```
LowerGroup <= GroupMatrix * Port <= UpperGroup
```

- Use dot notation to add group constraints for portfolio weights:
obj = obj.addGroups(GroupMatrix, LowerGroup, UpperGroup)
- To remove group constraints from a portfolio object, enter empty arrays for the corresponding arrays.
obj
A portfolio object [Portfolio].


## GroupMatrix

Group constraint matrix [matrix].

> Note The group matrix GroupMatrix is usually an indicator of membership in groups, which means that its elements are usually either 0 or 1 . Because of this interpretation, GroupMatrix can be a logical or numerical matrix.

## LowerGroup

Lower bound for group constraints [vector].

Note If input is scalar, LowerGroup undergoes scalar expansion to be conformable with GroupMatrix.

## UpperGroup

(Optional) Upper bound for group constraints [vector].

Note If input is scalar, UpperGroup undergoes scalar expansion to be conformable with GroupMatrix.

## Output obj

Arguments
Updated portfolio object [Portfolio].

## Attributes

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

## Portfolio.addGroups



## Superclasses AbstractPortfolio

Purpose

Syntax
Description

Tips

Input Arguments

Add linear inequality constraints for portfolio weights to existing constraints

```
obj = addInequality(obj, AInequality, bInequality)
```

obj = addInequality(obj, AInequality, bInequality) to add linear inequality constraints for portfolio weights to existing constraints.

Given linear inequality constraint matrix AInequality and vector bInequality, every weight in a portfolio Port must satisfy:

```
AInequality * Port <= bInequality
```

- Use dot notation to add linear inequality constraints for portfolio weights to existing constraints:

```
obj = obj.addInequality(AInequality, bInequality)
```

- To remove linear inequality constraints for portfolio weights from a portfolio object:

```
obj = obj.setInequality([ ], [ ])
```

obj
A portfolio object [Portfolio].
AEquality
Matrix to form linear inequality constraints [matrix].
bEquality
Vector to form linear inequality constraints [vector].

## Portfolio.addInequality

> Note An error results if AInequality is empty and bInequality is nonempty, or if AInequality is nonempty and bInequality is empty.

## Output Arguments

obj
Updated portfolio object [Portfolio].

## Attributes

## Examples

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

To add another linear inequality constraint to ensure that the last three assets constitute at least $50 \%$ of a portfolio, use addInequality to build up linear inequality constraints by creating another system of inequalities:

```
p = Portfolio;
A = [ 1 1 1 0 0 ]; % first inequality constraint
b = 0.5;
p = p.setInequality(A, b);
A = [ 0 0 -1 -1 -1 ]; % second inequality constraint
b = -0.5;
p = p.addInequality(A, b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);
```

5

| 1 | 1 | 1 | 0 | 0 |
| :--- | ---: | ---: | ---: | ---: |
| 0 | 0 | -1 | -1 | -1 |
| 0.5000 |  |  |  |  |
|  |  |  |  |  |
| -0.5000 |  |  |  |  |

See Also<br>setInequality | Portfolio

## Purpose Accumulation/Distribution line

```
Syntax
adln = adline(highp, lowp, closep, tvolume)
adln = adline([highp lowp closep tvolume])
adlnts = adline(tsobj)
adlnts = adline(tsobj, ParameterName, ParameterValue, ...)
```


## Arguments

| highp | High price (vector) |
| :--- | :--- |
| lowp | Low price (vector) |
| closep | Closing price (vector) |
| tvolume | Volume traded (vector) |
| tsobj | Time series object |

## Description

adln = adline(highp, lowp, closep, tvolume) computes the Accumulation/Distribution line for a set of stock price and volume traded data. The prices required for this function are the high (highp), low (lowp), and closing (closep) prices.
adln = adline([highp lowp closep tvolume]) accepts a four-column matrix as input. The first column contains the high prices, the second contains the low prices, the third contains the closing prices, and the fourth contains the volume traded.
adlnts = adline(tsobj) computes the Williams
Accumulation/Distribution line for a set of stock price data contained in the financial time series object tsobj. The object must contain the high, low, and closing prices plus the volume traded. The function assumes that the series are named High, Low, Close, and

Volume. All are required. adlnts is a financial time series object with the same dates as tsobj but with a single series named ADLine.
adlnts = adline(tsobj, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- HighName: high prices series name
- LowName: low prices series name
- CloseName: closing prices series name
- VolumeName: volume traded series name

Parameter values are the strings that represent the valid parameter names.

Examples Compute the Accumulation/Distribution line for Disney stock and plot the results:
load disney.mat
dis_ADLine = adline(dis)
plot(dis_ADLine)
title('Accumulation/Distribution Line for Disney')


## References

See Also

Achelis, Steven B., Technical Analysis from A to Z, Second Edition, McGraw-Hill, 1995, pp. 56-58.

## Purpose Accumulation/Distribution oscillator

```
Syntax ado = adosc(highp, lowp, openp, closep)
ado = adosc([highp lowp openp closep])
adots = adosc(tsobj)
adots = adosc(tsojb, ParameterName, ParameterValue, ...)
```


## Arguments

| highp | High price (vector) |
| :--- | :--- |
| lowp | Low price (vector) |
| openp | Opening price (vector) |
| closep | Closing price (vector) |
| tsobj | Time series object |

## Description

ado $=$ adosc(highp, lowp, openp, closep) returns a vector, ado, that represents the Accumulation/Distribution (A/D) oscillator. The A/D oscillator is calculated based on the high, low, opening, and closing prices of each period. Each period is treated individually.
ado = adosc([highp lowp openp closep]) accepts a four-column matrix as input. The order of the columns must be high, low, opening, and closing prices.
adots $=$ adosc(tsobj) calculates the Accumulation/Distribution (A/D) oscillator, adots, for the set of stock price data contained in the financial time series object tsobj. The object must contain the high, low, opening, and closing prices. The function assumes that the series are named High, Low, Open, and Close. All are required. adots is a financial time series object with similar dates to tsobj and only one series named ADOsc.
adots $=$ adosc(tsobj, ParameterName, ParameterValue, ...) accepts parameter name-parameter value pairs as input. These pairs
specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- HighName: high prices series name
- LowName: low prices series name
- OpenName: opening prices series name
- CloseName: closing prices series name

Parameter values are the strings that represents the valid parameter names.

## Examples

Compute the Accumulation/Distribution oscillator for Disney stock and plot the results:
load disney.mat
dis_ADOsc = adosc(dis)
plot(dis_ADOsc)
title('A/D Oscillator for Disney')


See Also adline | willad

## Purpose Amortization schedule

Syntax [Principal, Interest, Balance, Payment] = amortize(Rate, NumPeriods, PresentValue, FutureValue, Due)

## Arguments

$$
\begin{array}{ll}
\text { Rate } & \text { Interest rate per period, as a decimal fraction. } \\
\text { NumPeriods } & \text { Number of payment periods. } \\
\text { PresentValue } & \text { Present value of the loan. } \\
\text { Futurevalue } & \begin{array}{l}
\text { (Optional) Future value of the loan. Default }=0 . \\
\text { Due }
\end{array} \\
& \begin{array}{l}
\text { (Optional) When payments are due: } 0=\text { end of } \\
\text { period (default), or } 1=\text { beginning of period. }
\end{array}
\end{array}
$$

[Principal, Interest, Balance, Payment] = amortize(Rate, NumPeriods, PresentValue, FutureValue, Due) returns the principal and interest payments of a loan, the remaining balance of the original loan amount, and the periodic payment.

| Principal | Principal paid in each period. A <br> 1-by-NumPeriods vector. |
| :--- | :--- |
| Interest | Interest paid in each period. A 1-by-NumPeriods <br> vector. |
| Balance | Remaining balance of the loan in each payment <br> period. A 1-by-NumPeriods vector. |
| Payment | Payment per period. A scalar. |

Examples Compute an amortization schedule for a conventional 30-year, fixed-rate mortgage with fixed monthly payments. Assume a fixed rate of $12 \%$ APR and an initial loan amount of $\$ 100,000$.

```
Rate = 0.12/12; % 12 percent APR = 1 percent per month
NumPeriods = 30*12; % 30 years = 360 months
PresentValue = 100000;
[Principal, Interest, Balance, Payment] = amortize(Rate, ...
NumPeriods, PresentValue);
```

The output argument Payment contains the fixed monthly payment.

```
format bank
```


## Payment

```
Payment =
```

1028.61

Finally, summarize the amortization schedule graphically by plotting the current outstanding loan balance, the cumulative principal, and the interest payments over the life of the mortgage. In particular, note that total interest paid over the life of the mortgage exceeds $\$ 270,000$, far in excess of the original loan amount.

```
plot(Balance,'b'), hold('on')
plot(cumsum(Principal),'--k')
plot(cumsum(Interest),':r')
xlabel('Payment Month')
ylabel('Dollars')
grid('on')
title('Outstanding Balance, Cumulative Principal & Interest')
legend('Outstanding Balance', 'Cumulative Principal', ...
'Cumulative Interest')
```



The solid blue line represents the declining principal over the 30 -year period. The dotted red line indicates the increasing cumulative interest payments. Finally, the dashed black line represents the cumulative principal payments, reaching $\$ 100,000$ after 30 years.

See Also
annurate | annuterm | payadv | payodd | payper

\author{
Purpose Periodic interest rate of annuity <br> ```
Syntax Rate = annurate(NumPeriods, Payment, PresentValue, FutureValue, <br> Due)

```
}

\section*{Arguments}
\begin{tabular}{ll} 
NumPeriods & Number of payment periods. \\
Payment & Payment per period. \\
PresentValue & Present value of the loan or annuity. \\
FutureValue & \begin{tabular}{l} 
(Optional) Future value of the loan or annuity. \\
Default \(=0\).
\end{tabular} \\
Due & \begin{tabular}{l} 
(Optional) When payments are due: \(0=\) end of \\
period (default), or \(1=\) beginning of period.
\end{tabular}
\end{tabular}

\section*{Description Rate = annurate(NumPeriods, Payment, PresentValue, FutureValue, Due) returns the periodic interest rate paid on a loan or annuity.}

\section*{Examples}

Find the periodic interest rate of a four-year, \(\$ 5000\) loan with a \(\$ 130\) monthly payment made at the end of each month.
```

Rate = annurate(4*12, 130, 5000, 0, 0)
Rate =
0.0094

```
(Rate multiplied by 12 gives an annual interest rate of \(11.32 \%\) on the loan.)

See Also amortize | annuterm | bndyield | irr

Purpose Number of periods to obtain value
```

Syntax NumPeriods = annuterm(Rate, Payment, PresentValue, FutureValue,
Due)

```

\section*{Arguments}
\[
\begin{array}{ll}
\text { Rate } & \text { Interest rate per period, as a decimal fraction. } \\
\text { Payment } & \text { Payment per period. } \\
\text { PresentValue } & \text { Present value. } \\
\text { FutureValue } & \text { (Optional) Future value. Default }=0 . \\
\text { Due } & \begin{array}{l}
\text { (Optional) When payments are due: } 0=\text { end of } \\
\text { period (default), or } 1=\text { beginning of period. }
\end{array} \\
&
\end{array}
\]

\section*{Description NumPeriods = annuterm(Rate, Payment, PresentValue,} FutureValue, Due) calculates the number of periods needed to obtain a future value. To calculate the number of periods needed to pay off a loan, enter the payment or the present value as a negative value.

A savings account has a starting balance of \(\$ 1500 . \$ 200\) is added at the end of each month and the account pays \(9 \%\) interest, compounded monthly. How many years will it take to save \(\$ 5,000\) ?
```

NumPeriods = annuterm(0.09/12, 200, 1500, 5000, 0)
NumPeriods =
15.68 months or 1.31 years.

```

See Also annurate | amortize | fvfix | pvfix

\section*{Purpose}

Arithmetic to geometric moments of asset returns

\section*{Syntax}
```

[mg, Cg] = arith2geom(ma, Ca);
[mg, Cg] = arith2geom(ma, Ca, t);

```

\section*{Arguments}
\[
\begin{array}{ll}
\text { ma } & \text { Arithmetic mean of asset-return data (n-vector). } \\
\mathrm{Ca} & \begin{array}{l}
\text { Arithmetic covariance of asset-return data (n-by-n } \\
\text { symmetric, positive-semidefinite matrix. }
\end{array} \\
\mathrm{t} & \begin{array}{l}
\text { (Optional) Target period of geometric moments in } \\
\text { terms of periodicity of arithmetic moments with } \\
\text { default value } 1 \text { (scalar). }
\end{array}
\end{array}
\]

Description
arith2geom transforms moments associated with a simple Brownian motion into equivalent continuously-compounded moments associated with a geometric Brownian motion with a possible change in periodicity.
[mg, Cg] = arith2geom(ma, Ca, t) returns mg, continuously-compounded or "geometric" mean of asset returns over the target period ( n -vector), and Cg , which is a continuously-compounded or "geometric" covariance of asset returns over the target period (n-by-n matrix).
Arithmetic returns over period \(t_{A}\) are modeled as multivariate normal random variables with moments
\[
E[\mathrm{X}]=\mathrm{m}_{A}
\]
and
\[
\operatorname{cov}(\mathrm{X})=\mathrm{C}_{A}
\]

Geometric returns over period \(t_{G}\) are modeled as multivariate lognormal random variables with moments
\[
\begin{aligned}
& E[\mathrm{Y}]=1+\mathrm{m}_{G} \\
& \operatorname{cov}(\mathrm{Y})=\mathrm{C}_{G}
\end{aligned}
\]

Given \(t=t_{G} / t_{A}\), the transformation from geometric to arithmetic moments is
\[
\begin{aligned}
& 1+\mathrm{m}_{G_{i}}=\exp \left(t \mathrm{~m}_{A_{i}}+\frac{1}{2} t \mathrm{C}_{A_{i i}}\right) \\
& \mathrm{C}_{G_{i j}}=\left(1+\mathrm{m}_{G_{i}}\right)\left(1+\mathrm{m}_{G_{\mathrm{j}}}\right)\left(\exp \left(t \mathrm{C}_{A i j}\right)-1\right)
\end{aligned}
\]

For \(i, j=1, \ldots, n\).

Note If \(t=1\), then \(\mathbf{Y}=\exp (\mathbf{X})\).

This function has no restriction on the input mean ma but requires the input covariance Ca to be a symmetric positive-semidefinite matrix.

The functions arith2geom and geom2arith are complementary so that, given \(m, C\), and \(t\), the sequence
```

[mg, Cg] = arith2geom(m, C, t);
[ma, Ca] = geom2arith(mg, Cg, 1/t);

```
yields \(\mathrm{ma}=\mathrm{m}\) and \(\mathrm{Ca}=\mathrm{C}\).

\section*{Examples}

Example 1. Given arithmetic mean \(m\) and covariance C of monthly total returns, obtain annual geometric mean mg and covariance Cg . In this case, the output period (1 year) is 12 times the input period ( 1 month) so that \(\mathrm{t}=12\) with
```

[mg, Cg] = arith2geom(m, C, 12);

```

Example 2. Given annual arithmetic mean \(m\) and covariance \(C\) of asset returns, obtain monthly geometric mean mg and covariance Cg . In this case, the output period ( 1 month) is \(1 / 12\) times the input period ( 1 year) so that \(\mathrm{t}=1 / 12\) with
```

[mg, Cg] = arith2geom(m, C, 1/12);

```

Example 3. Given arithmetic means m and standard deviations s of daily total returns (derived from 260 business days per year), obtain annualized continuously-compounded mean mg and standard deviations sg with
```

[mg, Cg] = arith2geom(m, diag(s .^2), 260);
sg = sqrt(diag(Cg));

```

Example 4. Given arithmetic mean \(m\) and covariance C of monthly total returns, obtain quarterly continuously-compounded return moments. In this case, the output is 3 of the input periods so that \(t=3\) with
```

[mg, Cg] = arith2geom(m, C, 3);

```

Example 5. Given arithmetic mean \(m\) and covariance C of 1254 observations of daily total returns over a 5 -year period, obtain annualized continuously-compounded return moments. Since the periodicity of the arithmetic data is based on 1254 observations for a 5 -year period, a 1-year period for geometric returns implies a target period of \(t=1254 / 5\) so that
[mg, Cg] = arith2geom(m, C, 1254/5);

\section*{See Also}
geom2arith

Purpose Create financial time series object from ASCII data file
```

Syntax tsobj = ascii2fts(filename, descrow, colheadrow, skiprows)
tsobj = ascii2fts(filename, timedata, descrow, colheadrow,
skiprows)

```

Arguments
\begin{tabular}{ll} 
filename & \begin{tabular}{l} 
ASCII data file \\
descrow \\
(Optional) Row number in the data file that contains \\
the description to be used for the description field of \\
the financial time series object
\end{tabular} \\
colheadrow & \begin{tabular}{l} 
(Optional) Row number that has the column \\
headers/names
\end{tabular} \\
skiprows & \begin{tabular}{l} 
(Optional) Scalar or vector of row numbers to be \\
skipped in the data file
\end{tabular} \\
timedata & \begin{tabular}{l} 
Set to ' \(T\) ' if time-of-day data is present in the ASCII \\
data file or to 'NT' if no time-of-day data is present.
\end{tabular}
\end{tabular}

\section*{Description}
tsobj = ascii2fts(filename, descrow, colheadrow, skiprows) creates a financial time series object tsobj from the ASCII file named filename. This form of the function can only read a data file without time-of-day information and create a financial time series object without time information. If time information is present in the ASCII file, an error message appears.

The general format of the text data file is
- Can contain header text lines.
- Can contain column header information. The column header information must immediately precede the data series columns unless skiprows is specified.
- Leftmost column must be the date column.
- Dates must be in a valid date string format:
- 'ddmmmyy' or 'ddmmmyyyy'
- 'mm/dd/yy' or 'mm/dd/yyyy'
- 'dd-mmm-yy' or 'dd-mmm-yyyy'
- 'mmm.dd,yy' or 'mmm.dd,yyyy'
- Each column must be separated either by spaces or a tab.
tsobj = ascii2fts(filename, timedata, descrow, colheadrow, skiprows) creates a financial time series object containing time-of-day data. Set timedata to ' \(T\) ' to create a financial time series object containing time-of-day data.

\section*{Examples}

Example 1. If your data file contains no description or column header rows,
\begin{tabular}{lllllll}
\(1 / 3 / 95\) & 36.75 & 36.9063 & 36.6563 & 36.875 & 1167900 & \\
\(1 / 4 / 95\) & 37 & 37.2813 & 36.625 & 37.1563 & 1994700 & \(\ldots\)
\end{tabular}
you can create a financial time series object from it with the simplest form of the ascii2fts function:
```

myinc = ascii2fts('my_inc.dat');
myinc =
desc: my_inc.dat
freq: Unknown (0)

```


Example 2: If your data file contains description and column header information with the data series immediately following the column header row,
\begin{tabular}{llllllll} 
International & Business Machines & Corporation (IBM) & \\
Daily prices & \((1 / 3 / 95\) & to & \(4 / 5 / 99)\) & & & \\
DATE & OPEN & HIGH & LOW & CLOSE & VOLUME & \\
\(1 / 3 / 95\) & 36.75 & 36.9063 & 36.6563 & 36.875 & 1167900 & \\
\(1 / 4 / 95\) & 37 & 37.2813 & 36.625 & 37.1563 & 1994700 & \(\ldots\).
\end{tabular}
you must specify the row numbers containing the description and column headers:
```

ibm = ascii2fts('ibm9599.dat', 1, 3);
ibm =
desc: International Business Machines Corporation (IBM)
freq: Unknown (0)
'dates: (2)' 'OPEN: (2)' 'HIGH: (2)' 'LOW: (2)' ...
'03-Jan-1995' [ 36.7500] [ 36.9063] [ 36.6563]
04-Jan-1995' [ 37] [ 37.2813] [ 36.6250]

```

Example 3: If your data file contains rows between the column headers and the data series, for example,
\begin{tabular}{lllllll} 
Staples, Inc. (SPLS) & & & & \\
Daily prices & & & & \\
DATE OPEN & HIGH & LOW & CLOSE & VOLUME & \\
Starting date: & \(04 / 08 / 1996\) & & & & \\
Snding date: & \(04 / 07 / 1999\) & & & & \\
4/8/96 19.50 & 19.75 & 19.25 & 19.375 & 548500 & \\
\(4 / 9 / 96\) & 19.75 & 20.125 & 19.375 & 20 & 1135900 & \(\ldots\)
\end{tabular}
you need to indicate to ascii2fts the rows in the file that must be skipped. Assume that you have called the data file containing the Staples data above staples.dat. The command
```

spls = ascii2fts('staples.dat', 1, 3, [4 5]);

```
indicates that the fourth and fifth rows in the file should be skipped in creating the financial time series object:
```

spls =
desc: Staples, Inc. (SPLS)
freq: Unknown (0)
'dates: (2)' 'OPEN: (2)' 'HIGH: (2)' 'LOW: (2)'
'08-Apr-1996' [ 19.5000] [ 19.7500] [19.2500]
'09-Apr-1996' [ 19.7500] [ 20.1250] [19.3750]

```

Example 4: Create a financial time series object containing time-of-day information.

First create a data file with time information:
```

dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
'02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
serial_dates_times = [datenum(dates), datenum(times)];
data = round(10*rand(6,2));
stat = fts2ascii('myfts_file2.txt',serial_dates_times,data, ...
{'dates';'times';'Data1';'Data2'},'My FTS with Time');

```

Now read the data file back and create a financial time series object:
```

MyFts = ascii2fts('myfts_file2.txt','t',1,2,1)
MyFts =
desc: My FTS with Time
freq: Unknown (0)
'dates: (6)' 'times: (6)' 'Data1: (6)' 'Data2: (6)'
'01-Jan-2001' '11:00' [ 9] [ 4]

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline ' " ' & '12:00' & [ & 7] & [ & \(9]\) \\
\hline '02-Jan-2001' & '11:00' & [ & 2] & [ & \(1]\) \\
\hline ' " & '12:00' & [ & 4] & [ & \(4]\) \\
\hline '03-Jan-2001' & '11:00' & [ & 9] & [ & \(8]\) \\
\hline ' " ' & '12:00' & [ & 9] & [ & \(0]\) \\
\hline
\end{tabular}

See Also fints | fts2ascii
\begin{tabular}{|c|c|}
\hline Purpose & Bar chart \\
\hline \multirow[t]{6}{*}{Syntax} & bar(tsobj) \\
\hline & bar(tsobj, width) \\
\hline & bar(..., 'style') \\
\hline & hbar \(=\operatorname{bar}(\ldots\). \\
\hline & barh (...) \\
\hline & hbarh \(=\operatorname{barh}(\ldots)\) \\
\hline
\end{tabular}

\section*{Arguments}

\section*{Description bar, barh draw vertical and horizontal bar charts. \\ bar(tsobj) draws the columns of data series of the object tsobj. The number of data series dictates the number of vertical bars per group. Each group is the data for one particular date. \\ bar(tsobj, width) specifies the width of the bars. \\ bar (..., 'style') changes the style of the bar chart. \\ hbar \(=\operatorname{bar}(\ldots) \quad\) returns a vector of bar handles. \\ Use the MATLAB command shading faceted to put edges on the bars. Use shading flat to turn edges off.}

\section*{Examples}

Create bar charts for Disney stock showing high, low, opening, and closing prices.

\section*{bar, barh}

```

load disney
barh(q_dis)
title('Horizontal Bar Chart of Disney Prices')

```

See Also bar3, bar3h | candle | highlow

\section*{bar3, bar3h}

Purpose 3-D bar chart
```

Syntax
bar3(tsobj)
bar3(tsobj, width)
bar3(..., 'style')
nbar3 = bar3(...)
bar3h(...)
hbar3h = bar3h(...)

```

\section*{Arguments}

\section*{Description}
tsobj Financial time series object.
width Width of the bars and separation of bars within a group. (Default \(=0.8\).) If width is 1 , the bars within a group touch one another. Values \(>1\) produce overlapping bars.
style 'detached' (default), 'grouped', or 'stacked'.
bar3, bar3h draw three-dimensional vertical and horizontal bar charts.
bar3(tsobj) draws the columns of data series of the object tsobj. The number of data series dictates the number of vertical bars per group. Each group is the data for one particular date.
bar3(tsobj, width) specifies the width of the bars.
bar3(..., 'style') changes the style of the bar chart.
hbar3 = bar3(...) returns a vector of bar handles.
Use the MATLAB command shading faceted to put edges on the bars. Use shading flat to turn edges off.

\section*{Examples}

Create three-dimensional bar charts for Disney stock showing high, low, opening, and closing prices.

```

load disney
bar3(q_dis, 'stacked')
title('Three-Dimensional Bar Chart of Disney Prices')

```


\section*{bar3, bar3h}
load disney
bar3(q_dis, 'stacked')
title('Three-Dimensional Bar Chart of Disney Prices (Stacked)')
See Also bar, barh | candle | highlow
\begin{tabular}{ll} 
Purpose & Bond equivalent yield for Treasury bill \\
Syntax & Yield \(=\) beytbill(Settle, Maturity, Discount)
\end{tabular}

\section*{Arguments}

Settle Enter as serial date numbers or date strings. Settle must be earlier than Maturity.

Maturity Enter as serial date numbers or date strings.
Discount Discount rate of the Treasury bill. Enter as decimal fraction.

\section*{Description}

\section*{Examples}

Yield = beytbill(Settle, Maturity, Discount) returns the bond equivalent yield for a Treasury bill.

The settlement date of a Treasury bill is February 11, 2000, the maturity date is August 7, 2000, and the discount rate is \(5.77 \%\). The bond equivalent yield is
```

Yield = beytbill('2/11/2000', '8/7/2000', 0.0577)
Yield =
0.0602

```

See Also

\section*{binprice}

Purpose
Binomial put and call pricing

\section*{Syntax}
[AssetPrice, OptionValue] = binprice(Price, Strike, Rate, Time, Increment, Volatility, Flag, DividendRate, Dividend, ExDiv)

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline Price & Underlying asset price. A scalar. \\
\hline Strike & Option exercise price. A scalar. \\
\hline Rate & Risk-free interest rate. A scalar. Enter as a decimal fraction. \\
\hline Time & Option's time until maturity in years. A scalar. \\
\hline Increment & Time increment. A scalar. Increment is adjusted so that the length of each interval is consistent with the maturity time of the option. (Increment is adjusted so that Time divided by Increment equals an integer number of increments.) \\
\hline Volatility & Asset's volatility. A scalar. \\
\hline Flag & Specifies whether the option is a call (Flag = 1) or a put (Flag = 0). A scalar. \\
\hline DividendRate & (Optional) The dividend rate, as a decimal fraction. A scalar. Default \(=0\). If you enter a value for DividendRate, set Dividend and ExDiv \(=0\) or do not enter them. If you enter values for Dividend and ExDiv, set DividendRate \(=0\). \\
\hline
\end{tabular}
\begin{tabular}{ll} 
Dividend & \begin{tabular}{l} 
(Optional) The dividend payment at an \\
ex-dividend date, ExDiv. A row vector. For \\
each dividend payment, there must be a \\
corresponding ex-dividend date. Default \(=0\). If \\
you enter values for Dividend and ExDiv, set \\
DividendRate \(=0\).
\end{tabular} \\
ExDiv & \begin{tabular}{l} 
(Optional) Ex-dividend date, specified in \\
number of periods. A row vector. Default \(=0\).
\end{tabular}
\end{tabular}

\section*{Description}
[AssetPrice, OptionValue] = binprice(Price, Strike, Rate, Time, Increment, Volatility, Flag, DividendRate, Dividend, ExDiv) prices an option using the Cox-Ross-Rubinstein binomial pricing model.

Examples
Consider a put option with an exercise price of \(\$ 50\) that matures in 5 months. The current asset price is \(\$ 52\), the risk-free interest rate is \(10 \%\), and the volatility is \(40 \%\). There is one dividend payment of \(\$ 2.06\) in \(3-1 / 2\) months. To specify the input argument ExDiv in terms of number of periods, divide the ex-dividend date, specified in years, by the time Increment.
```

ExDiv = ( 3.5/12) / (1/12) = 3.5
[Price, Option] = binprice(52, 50, 0.1, 5/12, 1/12, 0.4, 0, 0, 2.06, 3.5)

```
returns the asset price and option value at each node of the binary tree.
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|l|}{Price \(=\)} \\
\hline 52.0000 & 58.1367 & 65.0226 & 72.7494 & 79.3515 & 89.0642 \\
\hline 0 & 46.5642 & 52.0336 & 58.1706 & 62.9882 & 70.6980 \\
\hline 0 & 0 & 41.7231 & 46.5981 & 49.9992 & 56.1192 \\
\hline 0 & 0 & 0 & 37.4120 & 39.6887 & 44.5467 \\
\hline 0 & 0 & 0 & 0 & 31.5044 & 35.3606 \\
\hline 0 & 0 & 0 & 0 & 0 & 28.0688 \\
\hline \multicolumn{6}{|l|}{Option \(=\)} \\
\hline
\end{tabular}

\section*{binprice}
\begin{tabular}{rrrrrr}
4.4404 & 2.1627 & 0.6361 & 0 & 0 & 0 \\
0 & 6.8611 & 3.7715 & 1.3018 & 0 & 0 \\
0 & 0 & 10.1591 & 6.3785 & 2.6645 & 0 \\
0 & 0 & 0 & 14.2245 & 10.3113 & 5.4533 \\
0 & 0 & 0 & 0 & 18.4956 & 14.6394 \\
0 & 0 & 0 & 0 & 0 & 21.9312
\end{tabular}

\section*{References Cox, J., S. Ross, and M. Rubenstein, "Option Pricing: A Simplified Approach", Journal of Financial Economics 7, Sept. 1979, pp. 229-263. \\ Hull, John C., Options, Futures, and Other Derivative Securities, 2nd edition, Chapter 14.}

See Also blkprice | blsprice

\section*{Purpose}

Implied volatility for futures options from Black's model
Syntax
Volatility = blsimpv(Price, Strike, Rate, Time, Value, Limit, ... Tolerance, Class)

\section*{Arguments}
\(\left.\begin{array}{ll}\text { Price } & \begin{array}{l}\text { Current price of the underlying asset (a futures } \\ \text { contract). }\end{array} \\ \text { Strike } & \begin{array}{l}\text { Exercise price of the futures option. }\end{array} \\ \text { Rate } & \begin{array}{l}\text { Annualized, continuously compounded risk-free rate } \\ \text { of return over the life of the option, expressed as a } \\ \text { positive decimal number. }\end{array} \\ \text { Time } & \begin{array}{l}\text { Time to expiration of the option, expressed in years. }\end{array} \\ \text { Value } & \begin{array}{l}\text { Price of a European futures option from which the } \\ \text { implied volatility of the underlying asset is derived. }\end{array} \\ \text { Limit } & \begin{array}{l}\text { (Optional) Positive scalar representing the upper } \\ \text { bound of the implied volatility search interval. If } \\ \text { Limit is empty or unspecified, the default = 10, or }\end{array} \\ \text { Tolerance } & \begin{array}{l}\text { 1000\% per annum. } \\ \text { (Optional) Implied volatility termination tolerance. }\end{array} \\ \text { Class } & \begin{array}{l}\text { A positive scalar. Default = 1e-6. } \\ \text { (Optional) Option class (call or put) indicating the }\end{array} \\ & \begin{array}{l}\text { option type from which the implied volatility is } \\ \text { derived. May be either a logical indicator or a cell } \\ \text { array of characters. To specify call options, set }\end{array} \\ \text { Class = true or Class = \{'call'\}; to specify put } \\ \text { options, set Class = false or Class = \{ 'put '\}. If }\end{array}\right\}\)

Description

\section*{Examples}

\section*{References}

Volatility = blkimpv(Price, Strike, Rate, Time, CallPrice, MaxIterations, Tolerance) computes the implied volatility of a futures price from the market value of European futures options using Black's model.

Volatility is the implied volatility of the underlying asset derived from European futures option prices, expressed as a decimal number. If no solution is found, blkimpv returns NaN.

Any input argument may be a scalar, vector, or matrix. When a value is a scalar, that value is used to compute the implied volatility of all the options. If more than one input is a vector or matrix, the dimensions of all nonscalar inputs must be identical.

Rate and Time must be expressed in consistent units of time.

Consider a European call futures option that expires in four months, trading at \(\$ 1.1166\), with an exercise price of \(\$ 20\). Assume that the current underlying futures price is also \(\$ 20\) and that the risk-free rate is \(9 \%\) per annum. Furthermore, assume that you are interested in implied volatilities no greater than 0.5 (50\% per annum). Under these conditions, the following commands all return an implied volatility of 0.25 , or \(25 \%\) per annum:
```

Volatility = blkimpv(20, 20, 0.09, 4/12, 1.1166, 0.5)
Volatility = blkimpv(20, 20, 0.09, 4/12, 1.1166, 0.5, [],
{'Call'})
Volatility = blkimpv(20, 20, 0.09, 4/12, 1.1166, 0.5, [], true)

```

Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003, pp. 287-288.
Black, Fischer, "The Pricing of Commodity Contracts," Journal of Financial Economics, March 3, 1976, pp. 167-79.

\footnotetext{
See Also
blkprice | blsimpv | blsprice
}

\section*{Purpose}

Black's model for pricing futures options

\section*{Syntax}

\section*{Arguments}

\section*{Description}

Examples positive decimal number. futures option prices. all non-scalar inputs must be identical. time.
\begin{tabular}{ll} 
Price & \begin{tabular}{l} 
Current price of the underlying asset (a futures \\
contract).
\end{tabular} \\
Strike & \begin{tabular}{l} 
Strike or exercise price of the futures option.
\end{tabular} \\
Rate & \begin{tabular}{l} 
Annualized, continuously compounded, risk-free rate \\
of return over the life of the option, expressed as a \\
positive decimal number.
\end{tabular} \\
Time & \begin{tabular}{l} 
Time until expiration of the option, expressed in \\
years. Must be greater than 0.
\end{tabular}
\end{tabular}

Volatility Annualized futures price volatility, expressed as a
[Call, Put] = blkprice(ForwardPrice, Strike, Rate, Time, Volatility) uses Black's model to compute European put and call

Any input argument may be a scalar, vector, or matrix. When a value is a scalar, that value is used to compute the implied volatility from all options. If more than one input is a vector or matrix, the dimensions of

Rate, Time, and Volatility must be expressed in consistent units of

Consider European futures options with exercise prices of \(\$ 20\) that expire in four months. Assume that the current underlying futures price is also \(\$ 20\) with a volatility of \(25 \%\) per annum. The risk-free rate is \(9 \%\) per annum. Using this data

\section*{blkprice}
[Call, Put] = blkprice(20, 20, 0.09, 4/12, 0.25)
returns equal call and put prices of \(\$ 1.1166\).
References Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003, pp. 287-288.

Black, Fischer, "The Pricing of Commodity Contracts," Journal of Financial Economics, March 3, 1976, pp. 167-179.

See Also binprice | blsprice

\section*{Purpose}

Black-Scholes sensitivity to underlying price change

\section*{Syntax}
[CallDelta, PutDelta] = blsdelta(Price, Strike, Rate, Time, Volatility, Yield)

\section*{Arguments}
\begin{tabular}{ll} 
Price & Current price of the underlying asset. \\
Strike & \begin{tabular}{l} 
Exercise price of the option.
\end{tabular} \\
Rate & \begin{tabular}{l} 
Annualized, continuously compounded risk-free rate \\
of return over the life of the option, expressed as a \\
positive decimal number.
\end{tabular} \\
Time & \begin{tabular}{l} 
Time to expiration of the option, expressed in years.
\end{tabular} \\
Volatility & \begin{tabular}{l} 
Annualized asset price volatility (annualized \\
standard deviation of the continuously compounded \\
asset return), expressed as a positive decimal
\end{tabular} \\
Yield & \begin{tabular}{l} 
number. \\
(Optional) Annualized, continuously compounded \\
yield of the underlying asset over the life of the \\
option, expressed as a decimal number. (Default \\
\(=\)
\end{tabular} \\
& \begin{tabular}{l} 
indices, Yield could represent the dividend yield.
\end{tabular} \\
& \begin{tabular}{l} 
For currency options, Yield could be the foreign \\
risk-free interest rate.
\end{tabular}
\end{tabular}

Description
[CallDelta, PutDelta] = blsdelta(Price, Strike, Rate, Time, Volatility, Yield) returns delta, the sensitivity in option value to change in the underlying asset price. Delta is also known as the hedge ratio.

\section*{blsdelta}

Note blsdelta can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:
```

Yield = Rate

```

When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
```

Yield = ForeignRate

```
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.

\section*{Examples}
```

[CallDelta, PutDelta] = blsdelta(50, 50, 0.1, 0.25, 0.3, 0)
CallDelta =
0.5955
PutDelta =
-0.4045

```

\section*{References}

Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003.

See Also
blsgamma | blslambda | blsprice | blsrho | blstheta | blsvega

\section*{Purpose}

Black-Scholes sensitivity to underlying delta change
Syntax Gamma = blsgamma(Price, Strike, Rate, Time, Volatility, Yield)

\section*{Arguments}
\begin{tabular}{ll} 
Price & Current price of the underlying asset. \\
Strike & \begin{tabular}{l} 
Exercise price of the option. \\
Rate
\end{tabular} \\
Time & \begin{tabular}{l} 
Annualized, continuously compounded risk-free rate \\
of return over the life of the option, expressed as a \\
positive decimal number.
\end{tabular} \\
Volatility & \begin{tabular}{l} 
Time to expiration of the option, expressed in years. \\
Annualized asset price volatility (annualized \\
standard deviation of the continuously compounded \\
asset return), expressed as a positive decimal \\
number.
\end{tabular} \\
Yield & \begin{tabular}{l} 
(Optional) Annualized, continuously compounded \\
yield of the underlying asset over the life of the \\
option, expressed as a decimal number. (Default \\
\(=0\).\() For example, for options written on stock\)
\end{tabular} \\
indices, Yield could represent the dividend yield.
\end{tabular}

\section*{Description}

Gamma = blsgamma(Price, Strike, Rate, Time, Volatility, Yield) returns gamma, the sensitivity of delta to change in the underlying asset price.

\section*{blsgamma}

Note blsgamma can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:
```

Yield = Rate

```

When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
```

Yield = ForeignRate

```
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.

\author{
Examples \\ Gamma \(=\) blsgamma( \(50,50,0.12,0.25,0.3,0)\) \\ Gamma = \\ 0.0512 \\ References \(\begin{aligned} & \text { Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, } \\ & \text { 5th edition, 2003. }\end{aligned}\) \\ See Also blsdelta | blslambda | blsprice | blsrho | blstheta | blsvega
}

\section*{Purpose Black-Scholes implied volatility}

Syntax Volatility = blsimpv(Price, Strike, Rate, Time, Value, Limit, ... Yield, Tolerance, Class)

\section*{Arguments}
\begin{tabular}{ll} 
Price & Current price of the underlying asset. \\
Strike & \begin{tabular}{l} 
Exercise price of the option.
\end{tabular} \\
Rate & \begin{tabular}{l} 
Annualized, continuously compounded risk-free rate \\
of return over the life of the option, expressed as a \\
positive decimal number.
\end{tabular} \\
Time & \begin{tabular}{l} 
Time to expiration of the option, expressed in years.
\end{tabular} \\
Value & \begin{tabular}{l} 
Price of a European option from which the implied \\
volatility of the underlying asset is derived.
\end{tabular} \\
Limit & \begin{tabular}{l} 
(Optional) Positive scalar representing the upper \\
bound of the implied volatility search interval. If \\
Limit is empty or unspecified, the default = 10, or
\end{tabular} \\
Yield & \begin{tabular}{l}
\(1000 \%\) per annum. \\
(Optional) Annualized, continuously compounded \\
yield of the underlying asset over the life of the \\
option, expressed as a decimal number. (Default \\
\(=0\).\() For example, for options written on stock\) \\
indices, Yield could represent the dividend yield.
\end{tabular} \\
& \begin{tabular}{l} 
For currency options, Yield could be the foreign \\
risk-free interest rate.
\end{tabular}
\end{tabular}

\section*{blsimpv}

Tolerance (Optional) Implied volatility termination tolerance. A positive scalar. Default \(=1 \mathrm{e}-6\).
Class
(Optional) Option class (call or put) indicating the option type from which the implied volatility is derived. May be either a logical indicator or a cell array of characters. To specify call options, set Class = true or Class \(=\) \{'call' \(\}\); to specify put options, set Class \(=\) false or Class \(=\{\) 'put' \(\}\). If Class is empty or unspecified, the default is a call option.

\section*{Description}

\section*{Examples}

Volatility = blsimpv(Price, Strike, Rate, Time, Value, Limit, Yield, Tolerance, Class) using a Black-Scholes model computes the implied volatility of an underlying asset from the market value of European call and put options.

Volatility is the implied volatility of the underlying asset derived from European option prices, expressed as a decimal number. If no solution is found, blsimpv returns NaN.

Any input argument may be a scalar, vector, or matrix. When a value is a scalar, that value is used to price all the options. If more than one input is a vector or matrix, the dimensions of all non-scalar inputs must be identical.

Rate, Time, and Yield must be expressed in consistent units of time.

Consider a European call option trading at \(\$ 10\) with an exercise price of \(\$ 95\) and three months until expiration. Assume that the underlying stock pays no dividend and trades at \(\$ 100\). The risk-free rate is \(7.5 \%\) per annum. Furthermore, assume that you are interested in implied volatilities no greater than 0.5 ( \(50 \%\) per annum).
Under these conditions, the following statements all compute an implied volatility of 0.3130 , or \(31.30 \%\) per annum.
```

Volatility = blsimpv(100, 95, 0.075, 0.25, 10, 0.5)

```

\section*{blsimpv}
```

Volatility = blsimpv(100, 95, 0.075, 0.25, 10, 0.5, 0, [], {'Call'})
Volatility = blsimpv(100, 95, 0.075, 0.25, 10, 0.5, 0, [], true)

```

\author{
References Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003. \\ Luenberger, David G., Investment Science, Oxford University Press, 1998.
}

See Also blsdelta | blsgamma | blslambda | blsprice | blsrho | blstheta

\section*{blslambda}

Purpose
Syntax

Arguments

\section*{Description}

Black-Scholes elasticity
[CallEl, PutEl] = blslambda(Price, Strike, Rate, Time, Volatility, Yield)
\begin{tabular}{ll} 
Price & Current price of the underlying asset. \\
Strike & \begin{tabular}{l} 
Exercise price of the option. \\
Rate
\end{tabular} \\
Time & \begin{tabular}{l} 
Annualized, continuously compounded risk-free rate \\
of return over the life of the option, expressed as a \\
positive decimal number.
\end{tabular} \\
Volatility & \begin{tabular}{l} 
Time to expiration of the option, expressed in years. \\
Annualized asset price volatility (annualized \\
standard deviation of the continuously compounded \\
asset return), expressed as a positive decimal \\
number.
\end{tabular} \\
Yield & \begin{tabular}{l} 
(Optional) Annualized, continuously compounded \\
yield of the underlying asset over the life of the \\
option, expressed as a decimal number. (Default \\
=0.) For example, for options written on stock
\end{tabular} \\
& \begin{tabular}{l} 
indices, Yield could represent the dividend yield. \\
For currency options, Yield could be the foreign \\
risk-free interest rate.
\end{tabular}
\end{tabular}

Price Current price of the underlying asset.
Strike Exercise price of the option.
Rate Annualized, continuously compounded risk-free rate of return over the life of the option, expressed as a positive decimal number.
Time Time to expiration of the option, expressed in years.
Volatility Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), expressed as a positive decimal number.
(Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, expressed as a decimal number. (Default \(=0\).) For example, for options written on stock indices, Yield could represent the dividend yield. risk-free interest rate.
[CallEl, PutEl] = blslambda(Price, Strike, Rate, Time, Volatility, yield) returns the elasticity of an option. CallEl is the call option elasticity or leverage factor, and PutEl is the put option elasticity or leverage factor. Elasticity (the leverage of an option position) measures the percent change in an option price per one percent change in the underlying asset price.

\section*{blslambda}
```

Note blslambda can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:

```
```

Yield = Rate

```
```

Yield = Rate

```

When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.
```

Examples [CallEl, PutEl] = blslambda(50, 50, 0.12, 0.25, 0.3)
CallEl =
8.1274
PutEl =
-8.6466

```
References Daigler, Advanced Options Trading, Chapter 4.
See Also blsdelta | blsgamma | blsprice | blsrho | blstheta | blsvega

\section*{blsprice}

Purpose
Black-Scholes put and call option pricing
Syntax
[Call, Put] = blsprice(Price, Strike, Rate, Time, Volatility, Yield)

\section*{Arguments}

Price Current price of the underlying asset.
Strike Exercise price of the option.
Rate Annualized, continuously compounded risk-free rate of return over the life of the option, expressed as a positive decimal number.

Time Time to expiration of the option, expressed in years.
Volatility Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), expressed as a positive decimal number.
Yield (Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, expressed as a decimal number. (Default \(=0\).) For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate.

\section*{Description}
[Call, Put] = blsprice(Price, Strike, Rate, Time, Volatility, Yield) computes European put and call option prices using a Black-Scholes model.

Any input argument may be a scalar, vector, or matrix. When a value is a scalar, that value is used to price all the options. If more than one input is a vector or matrix, the dimensions of all non-scalar inputs must be identical.

\section*{blsprice}

Rate, Time, Volatility, and Yield must be expressed in consistent units of time.

Note blsprice can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:
Yield = Rate

When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.

\section*{Examples}

\section*{References}

See Also

Consider European stock options that expire in three months with an exercise price of \(\$ 95\). Assume that the underlying stock pays no dividend, trades at \(\$ 100\), and has a volatility of \(50 \%\) per annum. The risk-free rate is \(10 \%\) per annum. Using this data
```

[Call, Put] = blsprice(100, 95, 0.1, 0.25, 0.5)

```
returns call and put prices of \(\$ 13.70\) and \(\$ 6.35\), respectively.
Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003.

Luenberger, David G., Investment Science, Oxford University Press, 1998.
blkprice | blsdelta | blsgamma | blsimpv | blslambda | blsrho | blstheta | blsvega

\section*{blsho}

Purpose
Black-Scholes sensitivity to interest rate change
Syntax
[CallRho, PutRho]= blsrho(Price, Strike, Rate, Time, Volatility, Yield)

\section*{Arguments}
\begin{tabular}{ll} 
Price & Current price of the underlying asset. \\
Strike & \begin{tabular}{l} 
Exercise price of the option.
\end{tabular} \\
Rate & \begin{tabular}{l} 
Annualized, continuously compounded risk-free rate \\
of return over the life of the option, expressed as a \\
positive decimal number.
\end{tabular} \\
Time & \begin{tabular}{l} 
Time to expiration of the option, expressed in years.
\end{tabular} \\
Volatility & \begin{tabular}{l} 
Annualized asset price volatility (annualized \\
standard deviation of the continuously compounded \\
asset return), expressed as a positive decimal
\end{tabular} \\
Yield & \begin{tabular}{l} 
number. \\
(Optional) Annualized, continuously compounded \\
yield of the underlying asset over the life of the \\
option, expressed as a decimal number. (Default \\
= 0.) For example, for options written on stock \\
indices, Yield could represent the dividend yield.
\end{tabular} \\
& \begin{tabular}{l} 
For currency options, Yield could be the foreign \\
risk-free interest rate.
\end{tabular}
\end{tabular}

Description
[CallRho, PutRho]= blsrho(Price, Strike, Rate, Time, Volatility, Yield) returns the call option rho CallRho, and the put option rho PutRho. Rho is the rate of change in value of derivative securities with respect to interest rates.

Note blsrho can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:
```

Yield = Rate

```

When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.

\section*{Examples}

\section*{References}

See Also
```

[CallRho, PutRho] = blsrho(50, 50, 0.12, 0.25, 0.3, 0)
CallRho =
6.6686
PutRho =
-5.4619

```

Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003.
blsdelta | blsgamma | blslambda | blsprice | blstheta | blsvega

\section*{blstheta}

Purpose
Black-Scholes sensitivity to time-until-maturity change
Syntax [CallTheta, PutTheta] = blstheta(Price, Strike, Rate, Time, Volatility, Yield)

\section*{Arguments}
\begin{tabular}{ll} 
Price & Current price of the underlying asset. \\
Strike & \begin{tabular}{l} 
Exercise price of the option.
\end{tabular} \\
Rate & \begin{tabular}{l} 
Annualized, continuously compounded risk-free rate \\
of return over the life of the option, expressed as a \\
positive decimal number.
\end{tabular} \\
Time & \begin{tabular}{l} 
Time to expiration of the option, expressed in years.
\end{tabular} \\
Volatility & \begin{tabular}{l} 
Annualized asset price volatility (annualized \\
standard deviation of the continuously compounded \\
asset return), expressed as a positive decimal \\
number.
\end{tabular} \\
Yield & \begin{tabular}{l} 
(Optional) Annualized, continuously compounded \\
yield of the underlying asset over the life of the \\
option, expressed as a decimal number. (Default
\end{tabular} \\
\(=0\).\() For example, for options written on stock\) \\
indices, Yield could represent the dividend yield.
\end{tabular}

\section*{Description}
[CallTheta, PutTheta] = blstheta(Price, Strike, Rate, Time, Volatility, Yield) returns the call option theta CallTheta, and the put option theta PutTheta. Theta is the sensitivity in option value with respect to time.

Note blstheta can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:
Yield = Rate

When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.

\section*{Examples}

\section*{References}

See Also
```

[CallTheta, PutTheta] = blstheta(50, 50, 0.12, 0.25, 0.3, 0)

```
CallTheta \(=\)
    -8.9630

\section*{PutTheta =}
-3. 1404
Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003.
blsdelta | blsgamma | blslambda | blsprice | blsrho | blsvega

\section*{blsvega}

\section*{Purpose}

Black-Scholes sensitivity to underlying price volatility
Syntax Vega = blsvega(Price, Strike, Rate, Time, Volatility, Yield)

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline Price & Current price of the underlying asset. \\
\hline Strike & Exercise price of the option. \\
\hline Rate & Annualized, continuously compounded risk-free rate of return over the life of the option, expressed as a positive decimal number. \\
\hline Time & Time to expiration of the option, expressed in years. \\
\hline Volatility & Annualized asset price volatility (annualized standard deviation of the continuously compounded asset return), expressed as a positive decimal number. \\
\hline Yield & (Optional) Annualized, continuously compounded yield of the underlying asset over the life of the option, expressed as a decimal number. (Default \(=0\).) For example, for options written on stock indices, Yield could represent the dividend yield. For currency options, Yield could be the foreign risk-free interest rate. \\
\hline
\end{tabular}

\section*{Description}

Vega = blsvega(Price, Strike, Rate, Time, Volatility, Yield) returns vega, the rate of change of the option value with respect to the volatility of the underlying asset.

\section*{blsvega}

Note blsvega can handle other types of underlies like Futures and Currencies. When pricing Futures (Black model), enter the input argument Yield as:
Yield = Rate

When pricing currencies (Garman-Kohlhagen model), enter the input argument Yield as:
Yield = ForeignRate
where ForeignRate is the continuously compounded, annualized risk free interest rate in the foreign country.
```

Examples Vega = blsvega(50, 50, 0.12, 0.25, 0.3, 0)
Vega =
9.6035
References Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall,
See Also blsdelta | blsgamma | blslambda | blsprice | blsrho | blstheta

```

Purpose
Bond convexity given price

\author{
Syntax
}

\section*{Description}

\section*{Input \\ Arguments} Settle, Maturity) Face) 'ParameterValue ...) are case-insensitive.

\section*{Price}
[YearConvexity, PerConvexity] = bndconvp(Price, CouponRate,
[YearConvexity, PerConvexity] = bndconvp(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate,
[YearConvexity, PerConvexity] = bndconvp(Price, CouponRate, Settle, Maturity, 'ParameterName',
[YearConvexity, PerConvexity] = bndconvp(Price, CouponRate, Settle, Maturity) computes the convexity of NUMBONDS fixed income securities given a clean price for each bond.
[YearConvexity, PerConvexity] = bndconvp(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)
[YearConvexity, PerConvexity] = bndconvp(Price,CouponRate, Settle, Maturity, 'ParameterName','ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. ParameterName' is the name of the parameter inside single quotes. Parametervalue is the value corresponding to 'ParameterName'. Specify parameter/value pairs in any order. Names

Clean price (excludes accrued interest).

\section*{CouponRate}

Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.

\section*{Settle}

\section*{bndconvp}

Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

\section*{Maturity}

Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

\section*{Default: 2}

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)

\section*{bndconvp}
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\) actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0
EndMonthRule
End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1
IssueDate
Issue date for a bond.

\section*{FirstCouponDate}

Irregular or normal first coupon date.

\section*{LastCouponDate}

Irregular or normal last coupon date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

Face
Face or par value.

\section*{bndconvp}

Default: 100

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.

\section*{CompoundingFrequency}

Compounding frequency for yield calculation. By default, SIA bases ( \(0-7\) ) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day count to compute discount factors. If you use ISMA day counts and BUS/252, the specified bases are used.

\section*{Output Arguments}

\section*{YearConvexity}

NUMBONDS-by-1 vector for the yearly (annualized) convexity.

\section*{PerConvexity}

NUMBONDS-by-1 vector for the periodic convexity reported on a semiannual bond basis (in accordance with SIA convention).

\section*{Definitions}
bndconvp determines the convexity for a bond whether the first or last coupon periods in the coupon structure are short or long (that is, whether the coupon structure is synchronized to maturity). This function also determines the convexity of a zero coupon bond.

All specified arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Use an empty matrix ([]) as a placeholder for an optional argument. Fill in unspecified entries input vectors with NaNs. Dates can be serial date numbers or date strings.

\section*{bndconvp}
Examples Find the convexity of three bonds given their prices:
```

Price = [106; 100; 98];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;
[YearConvexity, PerConvexity] = bndconvp(Price,...
CouponRate,Settle, Maturity, Period, Basis)

```
This returns:
```

YearConvexity =

```
    21.4447
    21.0363
    20.8951
PerConvexity =
    85.7788
    84.1454
    83.5803
References Krgin, Dragomir, Handbook of Global Fixed Income Calculations, JohnWiley \& Sons, 2002.Mayle, Jan, "Standard Securities Calculations Methods: Fixed IncomeSecurities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994.Stigum, Marcia, and Franklin Robinson, Money Market and BondCalculations, McGraw-Hill, 1996.
See Also cfconv | bndconvy | bnddurp | bnddury | cfdur
How To - "Yield Conventions" on page 2-31

\section*{Purpose}

Bond convexity given yield

\section*{Description}

\section*{Input Arguments}
[YearConvexity, PerConvexity] = bndconvy(Yield, CouponRate, Settle, Maturity)
[YearConvexity, PerConvexity] = bndconvy(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)
[YearConvexity, PerConvexity] = bndconvy(Yield, CouponRate, Settle, Maturity, 'ParameterName', 'ParameterValue ...)
[YearConvexity, PerConvexity] = bndconvy(Yield, CouponRate, Settle, Maturity) computes the convexity of NUMBONDS fixed income securities given the yield to maturity for each bond.
[YearConvexity, PerConvexity] = bndconvy(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)
[YearConvexity, PerConvexity] = bndconvy(Yield, CouponRate, Settle, Maturity, 'ParameterName','ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. Parametervalue is the value corresponding to 'ParameterName'. Specify parameter/value pairs in any order. Names are case-insensitive.

\section*{Yield}

Yield to maturity on a semiannual basis.

\section*{CouponRate}

Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
```

Settle

```

\section*{bndconvy}

Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

\section*{Maturity}

Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

Period
Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

Default: 2

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0

\section*{EndMonthRule}

End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1

\section*{IssueDate}

Issue date for a bond.

\section*{FirstCouponDate}

Irregular or normal first coupon date.

\section*{LastCouponDate}

Irregular or normal last coupon date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Face}

Face or par value.

\section*{bndconvy}

Default: 100

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.
CompoundingFrequency
Compounding frequency for yield calculation. By default, SIA bases ( \(0-7\) ) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day count to compute discount factors. If you use ISMA day counts and BUS/252, the specified bases are used.

\section*{Output Arguments}

YearConvexity
NUMBONDS-by-1 vector for the yearly (annualized) convexity. PerConvexity

NUMBONDS-by-1 vector for the periodic convexity reported on a semiannual bond basis (in accordance with SIA convention).

\section*{Definitions}
bndconvy determines the convexity for a bond whether the first or last coupon periods in the coupon structure are short or long (that is, whether the coupon structure is synchronized to maturity). This function also determines the convexity of a zero coupon bond.

All specified arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Use an empty matrix ([]) as a placeholder for an optional argument. Fill in unspecified entries input vectors with NaNs. Dates can be serial date numbers or date strings.

\section*{Examples}
```

Yield = [0.04; 0.055; 0.06];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;
[YearConvexity, PerConvexity]=bndconvy(Yield, CouponRate,...
Settle, Maturity, Period, Basis)

```

This returns:
```

YearConvexity =

```
    21.4825
    21.0358
    20.8885
PerConvexity =
    85.9298
    84.1434
    83.5541
\begin{tabular}{ll} 
References & \begin{tabular}{l} 
Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John \\
Wiley \& Sons, 2002.
\end{tabular} \\
& \begin{tabular}{l} 
Mayle, Jan, "Standard Securities Calculations Methods: Fixed Income \\
Securities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994.
\end{tabular} \\
& \begin{tabular}{l} 
Stigum, Marcia, and Franklin Robinson, Money Market and Bond \\
Calculations, McGraw-Hill, 1996.
\end{tabular} \\
See Also & cfconv | bndconvp | bnddurp | bnddury | cfdur \\
How To & . "Yield Conventions" on page 2-31
\end{tabular}

Purpose
Syntax

\section*{Description}

\section*{Input \\ Arguments}

Bond duration given price
[ModDuration, YearDuration, PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity) [ModDuration, YearDuration, PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)
[ModDuration, YearDuration, PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity, 'ParameterName', 'ParameterValue ...)
[ModDuration, YearDuration, PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity) computes the convexity of NUMBONDS fixed income securities given a clean price for each bond.
[ModDuration, YearDuration, PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)
[ModDuration, YearDuration, PerDuration] = bnddurp(Price, CouponRate, Settle, Maturity, 'ParameterName', 'ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. Parametervalue is the value corresponding to 'ParameterName'. Specify parameter/value pairs in any order. Names are case-insensitive.

Price
Clean price (excludes accrued interest).

\section*{CouponRate}

Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.

Settle

Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity. Maturity

Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

Default: 2

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)

\section*{bnddurp}
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\) actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0
EndMonthRule
End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1
IssueDate
Issue date for a bond.

\section*{FirstCouponDate}

Irregular or normal first coupon date.

\section*{LastCouponDate}

Irregular or normal last coupon date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

Face
Face or par value.

Default: 100

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.

\section*{CompoundingFrequency}

Compounding frequency for yield calculation. By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day count to compute discount factors. If you use ISMA day counts and BUS/252, the specified bases are used.

\section*{Output Arguments}

\section*{Definitions}

ModDuration
NUMBONDS-by- 1 vector for the modified duration in years, reported on a semiannual bond basis (in accordance with SIA convention).

\section*{YearDuration}

NUMBONDS-by-1 vector for the Macaulay duration in years.

\section*{PerDuration}

NUMBONDS-by-1 vector for the periodic Macaulay duration reported on a semiannual bond basis (in accordance with SIA convention).
bnddurp determines the Macaulay and modified duration for a bond whether the first or last coupon periods in the coupon structure are short or long (that is, whether the coupon structure is synchronized to maturity). This function also determines the Macaulay and modified duration for a zero coupon bond.

All specified arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Use an

\section*{bnddurp}
empty matrix ([]) as a placeholder for an optional argument. Fill in unspecified entries input vectors with NaNs. Dates can be serial date numbers or date strings.

\section*{Examples Find the duration of three bonds given their prices:}
```

Price = [106; 100; 98];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;
[ModDuration, YearDuration, PerDuration] = bnddurp(Price,...
CouponRate, Settle, Maturity, Period, Basis)

```

This returns:
ModDuration \(=\)
4.2400
4.1925
4.1759

YearDuration \(=\)
4.3275
4.3077
4.3007

PerDuration =
8.6549
8.6154
8.6014
References Krgin, Dragomir, Handbook of Global Fixed Income Calculations, JohnWiley \& Sons, 2002.Mayle, Jan, "Standard Securities Calculations Methods: Fixed IncomeSecurities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994.Stigum, Marcia, and Franklin Robinson, Money Market and BondCalculations, McGraw-Hill, 1996.
See Also bndconvy | bndconvp | bnddury | bndkrdur
How To - "Yield Conventions" on page 2-31

\section*{bnddury}
\begin{tabular}{|c|c|}
\hline Purpose & Bond duration given yield \\
\hline Syntax & \begin{tabular}{l}
[ModDuration, YearDuration, PerDuration] = bnddury(Yield, CouponRate, Settle, Maturity) \\
[ModDuration, YearDuration, PerDuration] = bnddury(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) \\
[ModDuration, YearDuration, PerDuration] = bnddury(Yield, CouponRate, Settle, Maturity, 'ParameterName', 'ParameterValue ...)
\end{tabular} \\
\hline Description & \begin{tabular}{l}
[ModDuration, YearDuration, PerDuration] = bnddury(Yield, CouponRate, Settle, Maturity) computes the Macaulay and modified duration of NUMBONDS fixed income securities given yield to maturity for each bond. \\
[ModDuration, YearDuration, PerDuration] = bnddury(Yield, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) \\
[ModDuration, YearDuration, PerDuration] = bnddury(Yield, CouponRate, Settle, Maturity, \\
'ParameterName','ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. \\
'ParameterName' is the name of the parameter inside single quotes. ParameterValue is the value corresponding to 'ParameterName'. \\
Specify parameter/value pairs in any order. Names are case-insensitive.
\end{tabular} \\
\hline Input Arguments & \begin{tabular}{l}
Yield \\
Yield to maturity on a semiannual basis. CouponRate
\end{tabular} \\
\hline
\end{tabular}

Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.

\section*{Settle}

Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

Maturity
Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

Period
Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

\section*{Default: 2}

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- \(2=\) actual \(/ 360\)
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)

\section*{bnddury}
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\mathrm{actual} / 365\) (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0

\section*{EndMonthRule}

End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1
IssueDate
Issue date for a bond.
FirstCouponDate
Irregular or normal first coupon date.
LastCouponDate
Irregular or normal last coupon date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Face}
(Optional) Face or par value.
Default: 100

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.

\section*{CompoundingFrequency}

Compounding frequency for yield calculation. By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day count to compute discount factors. If you use ISMA day counts and BUS/252, the specified bases are used.

\section*{Output Arguments}

\section*{Definitions}

\section*{ModDuration}

NUMBONDS-by-1 vector for the modified duration in years, reported on a semiannual bond basis (in accordance with SIA convention).

\section*{YearDuration}

NUMBONDS-by-1 vector for the Macaulay duration in years.
PerDuration
NUMBONDS-by-1 vector for the periodic Macaulay duration reported on a semiannual bond basis (in accordance with SIA convention).
bnddurp determines the duration for a bond whether the first or last coupon periods in the coupon structure are short or long (that is, whether the coupon structure is synchronized to maturity). This function also determines the Macaulay and modified duration for a zero coupon bond.

\section*{bnddury}

All specified arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalar arguments. Use an empty matrix ([]) as a placeholder for an optional argument. Fill in unspecified entries input vectors with NaNs. Dates can be serial date numbers or date strings.

\section*{Examples Find the duration of a bond at three different yield values:}
```

Yield = [0.04; 0.055; 0.06];
CouponRate = 0.055;
Settle = '02-Aug-1999';
Maturity = '15-Jun-2004';
Period = 2;
Basis = 0;
[ModDuration,YearDuration,PerDuration]=bnddury(Yield,...
CouponRate, Settle, Maturity, Period, Basis)

```

This returns:
ModDuration \(=\)
4.2444
4.1924
4.1751

YearDuration =
4.3292
4.3077
4.3004

PerDuration =
8.6585
8.6154
8.6007
References Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John Wiley \& Sons, 2002.
Mayle, Jan, "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994.
Stigum, Marcia, and Franklin Robinson, Money Market and Bond Calculations, McGraw-Hill, 1996.
See Also bndconvp | bndconvy | bnddurp | bndkrdur
How To - "Yield Conventions" on page 2-31

Purpose Bond key rate duration given zero curve
```

Syntax KRDUR = bndkrdur(ZeroData, CouponRate,
Settle, Maturity)
KRDUR = bndkrdur(ZeroData, CouponRate, Settle,
Maturity, 'Parameter1', Value1, 'Parameter2',
Value2, ...)

```

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline ZeroData & Zero curve represented as a numRates-by-2 matrix where the first column is a MATLAB date number and the second column is accompanying zero rates. \\
\hline CouponRate & numBonds-by- 1 vector of coupon rates in decimal form. \\
\hline Settle & Scalar MATLAB date number for the settlement date for all the bonds and the zero data. Settle must be the same settlement date for all the bonds and the zero curve. \\
\hline Maturity & numBonds-by-1 vector of maturity dates. \\
\hline Period & (Optional) Coupons per year of the bond. A vector of integers. Acceptable values are 0, 1, 2 (default), 3, 4, 6, and 12. \\
\hline InterpMethod & (Optional) Interpolation method used to obtain points from the zero curve. Acceptable values are: \\
\hline & \begin{tabular}{l}
- 'linear' (default) \\
- 'cubic'
\end{tabular} \\
\hline & - 'pchip' \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline ShiftValue & (Optional) Scalar value that zero curve is shifted up and down to compute duration. Default is . 01 ( 100 basis points). \\
\hline KeyRates & (Optional) Rates to perform the duration calculation, specified as a time to maturity. By default, KeyRates is set to each of the zero dates. \\
\hline CurveCompounding & (Optional) Compounding frequency of the curve. Default is semiannual. \\
\hline CurveBasis & (Optional) Basis of the curve, where the choices are identical to Basis below. Default is 0 (actual/actual). \\
\hline \multirow[t]{14}{*}{Basis} & (Optional) Day-count basis of the bond instrument. A vector of integers: \\
\hline & - \(0=\) actual/actual (default) \\
\hline & - \(1=30 / 360\) (SIA) \\
\hline & - 2 = actual/360 \\
\hline & - \(3=\) actual/365 \\
\hline & - \(4=30 / 360\) (PSA) \\
\hline & - \(5=30 / 360\) (ISDA) \\
\hline & - \(6=30 / 360\) (European) \\
\hline & - \(7=\) actual/365 (Japanese) \\
\hline & - \(8=\mathrm{actual} /\) actual (ISMA) \\
\hline & - 9 = actual/360 (ISMA) \\
\hline & - \(10=\mathrm{actual} / 365\) (ISMA) \\
\hline & - \(11=30 / 360 \mathrm{E}\) (ISMA) \\
\hline & - \(12=\mathrm{actual} / 365\) (ISDA) \\
\hline
\end{tabular}

\section*{bndkrdur}
\begin{tabular}{ll} 
& - \(13=\) BUS/252 \\
& For more information, seebasis on page \\
Glossary-1. \\
EndMonthRule \\
& \begin{tabular}{l} 
Optional) End-of-month rule. This rule applies \\
only when Mar \\
for a month having is an end-of-month date \\
values are:
\end{tabular} \\
& - \(0=\) ignore rule, meaning that a bond's \\
coupon payment date is always the same \\
& numerical day of the month.
\end{tabular}
\begin{tabular}{ll} 
StartDate & \begin{tabular}{l} 
(Optional) Date when a bond actually starts \\
(the date from which a bond cash flow \\
is considered). To make an instrument \\
forward-starting, specify this date as a future \\
date. If you do not specify StartDate, the \\
effective start date is the Settle date.
\end{tabular} \\
Face & \begin{tabular}{l} 
(Optional) Face or par value. Default \(=100\). \\
Face has no impact on key rate duration.
\end{tabular}
\end{tabular}

Note You must enter the optional arguments as parameter/value pairs.

\section*{Description}
```

KRDUR = bndkrdur(ZeroData, CouponRate, Settle, Maturity)
KRDUR = bndkrdur(ZeroData, CouponRate, Settle, Maturity,
'Parameter1', Value1, 'Parameter2', Value2, ...)

```

The output argument KRDUR is a numBonds-by-numRates matrix of key rate durations.
bndkrdur computes the key rate durations for one or more bonds given a zero curve and a set of key rates. By default, the key rates are each of the zero curve rates. For each key rate, the duration is computed by shifting the zero curve up and down by a specified amount (ShiftValue) at that particular key rate, computing the present value of the bond in each case with the new zero curves, and then evaluating the following:
\[
k r d u r_{i}=\frac{\left(P V_{\text {down }}-P V_{u p}\right)}{(P V \times \text { ShiftValue } \times 2)}
\]

\section*{bndkrdur}

Note The shift to the curve is computed by shifting the particular key rate by the ShiftValue and then interpolating the values of the curve in the interval between the previous and next key rates. For the first key rate, any curve values before the date are equal to the ShiftValue; likewise, for the last key rate, any curve values after the date are equal to the ShiftValue.

\section*{Examples}

\section*{References}

See Also

Find the key rate duration of a bond for key rate times of \(2,5,10\), and 30 years.
```

ZeroRates = [0.0476 .0466 .0465 .0468 . 0473 .0478 ...
.0493 .0539 .0572 .0553 .0530]';
ZeroDates = daysadd('31-Dec-1998',[30 360 360*2 360*3 360*5 ...
360*7 360*10 360*15 360*20 360*25 360*30],1);
ZeroData = [ZeroDates ZeroRates];
krdur = bndkrdur(ZeroData,.0525,'12/31/1998',...
'11/15/2028','KeyRates',[2 5 10 30])
krdur =
0.2986 0.8791 4.1354 9.5811

```

Golub, B.W. and L.M. Tilman, Risk Management: Approaches for Fixed Income Markets Wiley, 2000.

Tuckman, B. Fixed Income Securities: Tools for Today's Markets Wiley, 2002.
bndconvp | bndconvy | bnddurp | bnddury

Purpose
Syntax
```

```
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,
```

```
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,
Maturity)
Maturity)
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,
Maturity, Period, Basis, EndMonthRule, IssueDate,
Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face)
FirstCouponDate, LastCouponDate, StartDate, Face)
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,
Maturity, 'ParameterName', 'ParameterValue ...)
```

```
Maturity, 'ParameterName', 'ParameterValue ...)
```

```

\section*{Description}

\section*{Input Arguments}

Price fixed income security from yield to maturity
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle, Maturity), given bonds with SIA date parameters and semiannual yields to maturity, returns the clean prices and accrued interest due.
```

[Price, AccruedInt] = bndprice(Yield, CouponRate,
Settle, Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face) given bonds
with SIA date parameters and semiannual yields to maturity and
optional inputs, returns the clean prices and accrued interest due.

```
[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,
Maturity, 'ParameterName', 'ParameterValue ...) accepts
optional inputs as one or more comma-separated parameter/value pairs.
'ParameterName' is the name of the parameter inside single quotes.
ParameterValue is the value corresponding to 'ParameterName'.
Specify parameter/value pairs in any order. Names are case-insensitive.

\section*{Yield}

Bond yield to maturity is on a semiannual basis for basis values 0 through 7 and an annual basis for basis values 8 through 12.

\section*{CouponRate}

Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
```

Settle

```

\section*{bndprice}

Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

Maturity
Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter value pairs. You cannot mix ordered syntax with parameter value pairs.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

\section*{Default: 2}

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0
EndMonthRule
End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1
IssueDate
Issue date for a bond.

Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

\section*{FirstCouponDate}

Irregular or normal first coupon date.
Default: If you do not specify a FirstCouponDate, the effective start is the Settle date.

\section*{LastCouponDate}

Irregular or normal last coupon date.

\section*{bndprice}

Default: If you do not specify a LastCouponDate, the effective date is the Settle date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

Default: If you do not specify StartDate, the effective start date is the Settle date.

\section*{Face}

Face or par value.
Default: 100

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.
CompoundingFrequency
Compounding frequency for yield calculation. By default, SIA bases ( \(0-7\) ) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

Default: SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day count to compute discount factors. If you use ISMA day counts and BUS/252, the specified bases are used.

Default: SIA bases use the actual/actual day count to compute discount factors.

\section*{LastCouponInterest}

Compounding convention for computing the yield of a bond in the last coupon period. This is based on only the last coupon and the face value to be repaid. Acceptable values are simple or compound.

Default: compound

\section*{Output}

Arguments

\section*{Definitions}

Examples Price a treasury bond at three different yield values:
```

Yield = [0.04; 0.05; 0.06];
CouponRate = 0.05;
Settle = '20-Jan-1997';
Maturity = '15-Jun-2002';
Period = 2;
Basis = 0;

```

\section*{bndprice}
```

[Price, AccruedInt] = bndprice(Yield, CouponRate, Settle,...
Maturity, Period, Basis)
This returns:
Price =
104.8106
99.9951
95.4384
AccruedInt =
0.4945
0.4945
0.4945

```

Price a Treasury bond at two different yield values that include parameter/value pairs for CompoundingFrequency, DiscountBasis, and LastCouponPeriodInterest:
```

bndprice(.04,0.08,'5/25/2004','4/21/2005','Period',1,'Basis', 8, ...
'LastCouponInterest','simple')

```

This returns:
```

ans =

```
    103.4743

Algorithms
For SIA conventions, the following formula defines bond price and yield:
\[
P V=\frac{C F}{\left(1+\frac{z}{f}\right)^{T F}},
\]
where:
\begin{tabular}{ll}
\(P V=\) & Present value of a cash flow. \\
\(C F=\) & The cash flow amount. \\
\(z=\) & \begin{tabular}{l} 
The risk-adjusted annualized rate or yield corresponding \\
to a given cash flow. The yield is quoted on a semiannual \\
basis.
\end{tabular} \\
\(f=\) & \begin{tabular}{l} 
The frequency of quotes for the yield.
\end{tabular} \\
\(T F=\) & \begin{tabular}{l} 
Time factor for a given cash flow. Time is measured \\
in semiannual periods from the settlement date to the \\
cash flow date. In computing time factors, use SIA \\
actual/actual day count conventions for all time factor \\
calculations.
\end{tabular}
\end{tabular}

For ISMA conventions, the frequency of annual coupon payments determines bond price and yield.

\author{
References Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John Wiley \& Sons, 2002. \\ Mayle, Jan, "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994. \\ Stigum, Marcia, and Franklin Robinson, Money Market and Bond Calculations, McGraw-Hill, 1996. \\ See Also bndyield | cfamounts \\ Tutorials . "Pricing Functions" on page 2-31
}

\section*{bndspread}

Purpose
Syntax

\section*{Description}

\section*{Input Arguments}

Spread = bndspread(SpotInfo, Price, Coupon, Settle, Maturity) computes the static spread (Z-spread) to benchmark in basis points.
Spread = bndspread(SpotInfo, Price, Coupon, Settle, MaturityPeriod, Basis, EndMonthRule, IssueDate, FirstCouponDateLastCouponDate, StartDate, Face) computes the
static spread (Z-spread) to benchmark in basis points including optional FirstCouponDateLastCouponDate, StartDate, Face) computes the
static spread (Z-spread) to benchmark in basis points including optional inputs.

Spread = bndspread(SpotInfo, Price, Coupon, Settle, Maturity,'ParameterName', 'ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. ParameterValue is the value corresponding to 'ParameterName'. Specify parameter/value pairs in any order. Names are case-insensitive.
Static spread over spot curve
Spread = bndspread(SpotInfo, Price, Coupon, Settle, Maturity) Spread = bndspread(SpotInfo, Price, Coupon, Settle, Maturity Period, Basis, EndMonthRule, IssueDate, FirstCouponDate LastCouponDate, StartDate, Face) Spread = bndspread(SpotInfo, Price, Coupon, Settle, Maturity, 'ParameterName', 'ParameterValue ...)

\section*{SpotInfo}

Two-column matrix: [SpotDates ZeroRates]. Zero rates correspond to maturities on the spot dates, continuously compounded. Choose evenly spaced rates close together to obtain the best results. For example, using the 3 -month deposit rates:
```

SpotInfo = ...
[datenum('2-Jan-2004') , 0.03840;
datenum('2-Jan-2005') , 0.04512;
datenum('2-Jan-2006') , 0.05086];

```

Price
Price for every \(\$ 100\) notional amount of bonds whose spreads are computed.

\section*{Coupon}

Annual coupon rate of bonds whose spreads are computed.

\section*{Settle}

Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

\section*{Maturity}

Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

Default: 2

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- 3 = actual \(/ 365\)
- \(4=30 / 360\) (PSA)

\section*{bndspread}
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- \(13=\mathrm{BUS} / 252\)

For more information, seebasis on page Glossary-1.
Default: 0

\section*{EndMonthRule}

End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

\section*{Default: 1}

\section*{IssueDate}

Issue date for a bond.

\section*{FirstCouponDate}

Irregular or normal first coupon date.

\section*{LastCouponDate}

Irregular or normal last coupon date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

Face
Face or par value.
Default: 100

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.

\section*{CompoundingFrequency}

Compounding frequency for yield calculation. By default, SIA bases ( \(0-7\) ) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day count to compute discount factors. If you use ISMA day counts and BUS/252, the specified bases are used.

\section*{Output \\ Arguments}

\section*{Examples Compute a Federal National Mortgage Association (FNMA) \(43 / 8\) spread over a Treasury spot curve:}
```

RefMaturity = [datenum('02/27/2003');
datenum('05/29/2003');
datenum('10/31/2004');

```

\section*{bndspread}
```

            datenum('11/15/2007');
            datenum('11/15/2012');
            datenum('02/15/2031')];
    RefCpn = [0;
0;
2.125;
3;
4;
5.375] / 100;
RefPrices = [99.6964;
99.3572;
100.3662;
99.4511;
99.4299;
106.5756];
RefBonds = [RefPrices, RefMaturity, RefCpn];
Settle = datenum('26-Nov-2002');
[ZeroRates, CurveDates] = zbtprice(RefBonds(:, 2:end), ...
RefPrices, Settle)
% FNMA 4 3/8 maturing 10/06 at 4.30 pm Tuesday
Price = 105.484;
Coupon = 0.04375;
Maturity = datenum('15-0ct-2006');
% All optional inputs are supposed to be accounted by default,
% except the accrued interest under 30/360 (SIA), so:
Period = 2;
Basis = 1;
SpotInfo = [CurveDates, ZeroRates];
% Compute static spread over treasury curve, taking into account
% the shape of curve as derived by bootstrapping method embedded
% within bndspread.

```
```

SpreadInBP = bndspread(SpotInfo, Price, Coupon, Settle, ...
Maturity, Period, Basis)

```

This returns:
\[
\begin{array}{r}
\text { ZeroRates }= \\
0.0121 \\
0.0127 \\
0.0194 \\
0.0317 \\
0.0423 \\
0.0550
\end{array}
\]
CurveDates =

731639
731730
732251
733361
735188
741854

SpreadInBP =
18.7582

Plot the results:
```

plot(CurveDates, ZeroRates*100, 'b', CurveDates, ...
ZeroRates*100+SpreadInBP/100, 'r--')
legend({'Treasury'; 'FNMA 4 3/8'})
xlabel('Curve Dates')
ylabel('Spot Rate [%]')
grid;

```

\section*{bndspread}


References Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John Wiley \& Sons, 2002.

Mayle, Jan, "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994.
Stigum, Marcia, and Franklin Robinson, Money Market and Bond Calculations, McGraw-Hill, 1996.

See Also
bndyield | bndprice

Purpose
Yield to maturity for fixed income security
Syntax
Description

\section*{Input Arguments}
```

Yield = bndyield(Price, CouponRate, Settle, Maturity)
Yield = bndyield(Price, CouponRate, Settle, Maturity,
Period, Basis, EndMonthRule, IssueDate, FirstCouponDate,
LastCouponDate, StartDate, Face)
Yield = bndyield(Price, CouponRate, Settle, Maturity,
'ParameterName', 'ParameterValue ...)

``` accrued interest), returns the bond equivalent yields to maturity.
Yield = bndyield(Price, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, the bond equivalent yields to maturity.

Yield = bndyield(Price, CouponRate, Settle, Maturity, pairs in any order. Names are case-insensitive.

\section*{Price}

Yield = bndyield(Price, CouponRate, Settle, Maturity), given NUMBONDS bonds with SIA date parameters and clean prices (excludes LastCouponDate, StartDate, Face) bonds with SIA date parameters and clean prices (excludes accrued interest) and optional inputs, returns 'ParameterName', 'ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. ParameterValue is the value corresponding to 'ParameterName'. Specify parameter/value

Clean price of the bond (current price without accrued interest).

\section*{CouponRate}

Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.

Settle

\section*{bndyield}

Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

Maturity
Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

Period
Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

Default: 2

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0
EndMonthRule
End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1
IssueDate
Issue date for a bond.

Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

\section*{FirstCouponDate}

Irregular or normal first coupon date.
Default: If you do not specify a FirstCouponDate, the effective start is the Settle date.

\section*{LastCouponDate}

Irregular or normal last coupon date.

\section*{bndyield}

Default: If you do not specify a LastCouponDate, the effective date is the Settle date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

Default: If you do not specify StartDate, the effective start date is the Settle date.

\section*{Face}

Face or par value.
Default: 100

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.
CompoundingFrequency
Compounding frequency for yield calculation. By default, SIA bases ( \(0-7\) ) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

Default: SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day count to compute discount factors. If you use ISMA day counts and BUS/252, the specified bases are used.

Default: SIA bases use the actual/actual day count to compute discount factors.

\section*{LastCouponInterest}

Compounding convention for computing the yield of a bond in the last coupon period. This computation is based on only the last coupon and the face value to be repaid. Acceptable values are simple or compound.

Default: compound

\section*{Output}

Arguments

\section*{Definitions}

Examples

Yield
NUMBONDS-by-1 vector of the yield to maturity with semiannual compounding.

All nonscalar or empty matrix input arguments must be either NUMBONDS-by-1 or 1 -by-NUMBONDS conforming vectors. Fill in unspecified entries input vectors with NaNs. Dates can be serial date numbers or date strings.

Compute the yield of a Treasury bond at three different price values:
```

Price = [95; 100; 105];
CouponRate = 0.05;
Settle = '20-Jan-1997';
Maturity = '15-Jun-2002';
Period = 2;
Basis = 0;
Yield = bndyield(Price, CouponRate, Settle,...
Maturity, Period, Basis)

```

This returns:
```

Yield =

```

\section*{bndyield}
0.0610
0.0500
0.0396

Algorithms
\[
P V=\frac{C F}{\left(1+\frac{z}{f}\right)^{T F}},
\]
where:
\begin{tabular}{ll}
\(P V=\) & Present value of a cash flow. \\
\(C F=\) & The cash flow amount. \\
\(z=\) & \begin{tabular}{l} 
The risk-adjusted annualized rate or yield corresponding \\
to a given cash flow. The yield is quoted on a semiannual \\
basis.
\end{tabular} \\
\(f=\) & \begin{tabular}{l} 
The frequency of quotes for the yield.
\end{tabular} \\
\(T F=\) & \begin{tabular}{l} 
Time factor for a given cash flow. Time is measured \\
in semiannual periods from the settlement date to the \\
cash flow date. In computing time factors, use SIA \\
actual/actual day count conventions for all time factor \\
calculations.
\end{tabular}
\end{tabular}

For ISMA conventions, the frequency of annual coupon payments determines bond price and yield.

\author{
References Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John Wiley \& Sons, 2002. \\ Mayle, Jan, "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994. \\ Stigum, Marcia, and Franklin Robinson, Money Market and Bond Calculations, McGraw-Hill, 1996.
}
\(\begin{array}{ll}\text { See Also } & \text { bndprice | cfamounts } \\ \text { How To } & \text { - "Yield Functions" on page 2-32 }\end{array}\)

Purpose Bollinger band chart
```

Syntax
bolling(Asset, Samples, Alpha)
[Movavgv, UpperBand, LowerBand] = bolling(Asset, Samples, Alpha,
Width)

```

\section*{Arguments}
\begin{tabular}{ll} 
Asset & \begin{tabular}{l} 
Vector of asset data. \\
Samples \\
Number of samples to use in computing the moving \\
average.
\end{tabular} \\
Alpha & \begin{tabular}{l} 
(Optional) Exponent used to compute the element \\
weights of the moving average. Default = (simple \\
moving average).
\end{tabular} \\
Width & \begin{tabular}{l} 
(Optional) Number of standard deviations to include \\
in the envelope. A multiplicative factor specifying \\
how tight the bands should be around the simple \\
moving average. Default \(=2\).
\end{tabular}
\end{tabular}

\section*{Description}
bolling(Asset, Samples, Alpha, Width) plots Bollinger bands for given Asset data. This form of the function does not return any data.
[Movavgv, UpperBand, LowerBand] = bolling(Asset, Samples, Alpha, Width) returns Movavgv with the moving average of the Asset data, UpperBand with the upper band data, and LowerBand with the lower band data. This form of the function does not plot any data.

Note The standard deviations are normalized by \(\mathrm{N}-1\), where \(\mathrm{N}=\) the sequence length.

\section*{Examples If Asset is a column vector of closing stock prices}
```

bolling(Asset, 20, 1)

```
plots linear 20-day moving average Bollinger bands based on the stock prices.
```

[Movavgv, UpperBand, LowerBand] = bolling(Asset, 20, 1)

```
returns Movavgv, UpperBand, and LowerBand as vectors containing the moving average, upper band, and lower band data, without plotting the data.

See Also candle | dateaxis | highlow | movavg | pointfig

Purpose
Time series Bollinger band

\section*{Syntax}
```

[mid, uppr, lowr] = bollinger(data, wsize, wts, nstd)
[midfts, upprfts, lowrfts] = bollinger(tsobj, wsize, wts, nstd)

```

\section*{Arguments}
\begin{tabular}{ll} 
data & Data vector. \\
wsize & (Optional) Window size. Default = 20. \\
wts & \begin{tabular}{l} 
(Optional) Weight factor. Determines the type of \\
moving average used. Default = (box). \(1=\) linear.
\end{tabular} \\
nstd & \begin{tabular}{l} 
(Optional) Number of standard deviations for upper \\
and lower bands. Default \(=2\).
\end{tabular} \\
tsobj & Financial time series object.
\end{tabular}

\section*{Description [mid, uppr, lowr] = bollinger(data, wsize, wts, nstd)} calculates the middle (mid), upper (uppr), and lower (lowr) bands that make up the Bollinger bands from the vector data.
mid is the vector that represents the middle band, a simple moving average with a window size of wsize. uppr and lowr are vectors that represent the upper and lower bands. uppr is a vector representing the upper band that is +nstd times. lowr is a vector representing the lower band that is -nstd times.
[midfts, upprfts, lowrfts] = bollinger(tsobj, wsize, wts, nstd) calculates the middle, upper, and lower bands that make up the Bollinger bands from a financial time series object tsobj.
midfts is a financial time series object that represents the middle band for all series in tsobj. Both upprfts and lowrfts are financial time series objects that represent the upper and lower bands of all series, which are +nstd times and -nstd times moving standard deviations away from the middle band.

Examples

\section*{References}

See Also

Compute the Bollinger bands for Disney stock closing prices and plot the results:
```

load disney.mat
[dis_Mid,dis_Uppr,dis_Lowr]= bollinger(dis);
dis_CloseBolling = [dis_Mid.CLOSE, dis_Uppr.CLOSE,...
dis_Lowr.CLOSE];
plot(dis_CloseBolling)
title('Bollinger Bands for Disney Closing Prices')

```


Achelis, Steven B., Technical Analysis from A to Z, Second Edition, McGraw-Hill, 1995, pp. 72-74.

Purpose Box-Cox transformation

\author{
Syntax [transdat, lambda] = boxcox(data) \\ [transfts, lambdas] = boxcox(tsobj) \\ transdat = boxcox(lambda, data) \\ transfts = boxcox(lambda, tsobj)
}

\section*{Arguments}
\begin{tabular}{ll} 
data & Data vector. Must be positive. \\
tsobj & Financial time series object.
\end{tabular}

Description boxcox transforms nonnormally distributed data to a set of data that has approximately normal distribution. The Box-Cox transformation is a family of power transformations.

If \(\lambda\) is not \(=0\), then
\[
\operatorname{data}(\lambda)=\frac{d a t a^{\lambda}-1}{\lambda}
\]

If \(\lambda\) is \(=0\), then
\[
\operatorname{data}(\lambda)=\log (d a t a)
\]

The logarithm is the natural logarithm (log base e). The algorithm calls for finding the \(\lambda\) value that maximizes the Log-Likelihood Function (LLF). The search is conducted using fminsearch.
[transdat, lambda] = boxcox(data) transforms the data vector data using the Box-Cox transformation method into transdat. It also estimates the transformation parameter \(\lambda\).
[transfts, lambda] = boxcox(tsojb) transforms the financial time series object tsobj using the Box-Cox transformation method into transfts. It also estimates the transformation parameter \(\lambda\).

If the input data is a vector, lambda is a scalar. If the input is a financial time series object, lambda is a structure with fields similar to the components of the object; for example, if the object contains series names Open and Close, lambda has fields lambda.Open and lambda.Close.
transdat = boxcox(lambda, data) and transfts = boxcox(lambda, tsobj) transform the data using a certain specified \(\lambda\) for the Box-Cox transformation. This syntax does not find the optimum \(\lambda\) that maximizes the LLF.

\section*{See Also \\ fminsearch}

\section*{busdate}

\section*{Purpose Next or previous business day}
```

Syntax Busday = busdate(Date, DirFlag, Holiday, Weekend)

```

\section*{Arguments}
\begin{tabular}{cl} 
Date & \begin{tabular}{l} 
Reference date. Enter scalar, vector, or matrix of \\
reference business dates as serial date numbers or \\
date strings.
\end{tabular} \\
DirFlag & \begin{tabular}{l} 
(Optional) String or cell array of strings of \\
business day convention with possible values: \\
follow (default), modifiedfollow, previous, \\
modifiedprevious. Also, DirFlag may be a scalar, \\
vector, or matrix of search directions, where Next is \\
DIREC = 1 (default) or Previous is DIREC \(=-1\).
\end{tabular} \\
Holiday & \begin{tabular}{l} 
(Optional) Vector of holidays and nontrading-day \\
dates. All dates in Holiday must be the same \\
format: either serial date numbers or date strings. \\
(Using serial date numbers improves performance.)
\end{tabular} \\
& \begin{tabular}{l} 
If Holiday is not specified, the non-trading day \\
default vector is determined by the routine holidays \\
function.
\end{tabular} \\
Weekend & \begin{tabular}{l} 
(Optional) Vector of length 7, containing 0 and 1, the \\
value 1 indicating weekend days. The first element \\
of this vector corresponds to Sunday. Thus, when
\end{tabular} \\
& \begin{tabular}{l} 
Saturday and Sunday form the weekend (default), \\
Weekend = [1 0 0 0 0 1].
\end{tabular}
\end{tabular}

Description Busday = busdate (Date, DirFlag, Holiday, Weekend) returns the scalar, vector, or matrix of the next or previous business day(s), depending on Holiday.

Use the function datestr to convert serial date numbers to formatted date strings.

\section*{Examples Example 1.}
```

Busday = busdate('3-Jul-2001', 1)
Busday =

```
    731037
datestr(Busday)
ans =
05-Jul-2001

Example 2. You can indicate that Saturday is a business day by appropriately setting the Weekend argument.
```

Weekend = [1 0 0 0 0 0 0];

```

July 4, 2003 falls on a Friday. Use busdate to verify that Saturday, July 5 , is actually a business day.
```

Date = datestr(busdate('3-Jul-2003', 1, [], Weekend))

```

See Also
holidays | isbusday

\section*{busdays}

Purpose Business days in serial date format
```

Syntax
bdates = busdays(sdate, edate, bdmode)
bdates = busdays(sdate, edate, bdmode, holvec)

```

\section*{Arguments}

\section*{Description}
\begin{tabular}{ll} 
sdate & Start date in string or serial date format. \\
edate & End date in string or serial date format. \\
bdmode & (Optional) Frequency of business days:
\end{tabular}
- DAILY, Daily, daily, D, d, 1 (default)
- WEEKLY, Weekly, weekly, W, w, 2
- MONTHLY, Monthly, monthly, M, m, 3
- QUARTERLY, Quarterly, quarterly, Q, q, 4
- SEMIANNUAL, Semiannual, semiannual, S, s, 5
- ANNUAL, Annual, annual, A, a, 6

Strings must be enclosed in single quotation marks.
holvec (Optional) Holiday dates vector in string or serial date format.
bdates = busdays(sdate, edate, bdmode) generates a vector of business days, bdates, in serial date format between the last business date of the period that contains the start date, and the last business date of period that contains the end date. If holvec is not supplied, the dates are generated based on United States holidays. If you do not supply bdmode, busdays generates a daily vector.

For example:
```

vec = datestr(busdays('1/2/01','1/9/01','weekly'))
vec =
05-Jan-2001
12-Jan-2001

```

The end of the week is considered to be a Friday. Between 1/2/01 (Monday) and 1/9/01 (Tuesday) there is only one end-of-week day, \(1 / 5 / 01\) (Friday).

Because \(1 / 9 / 01\) is part of following week, the following Friday (1/12/01) is also reported.
bdates = busdays(sdate, edate, bdmode, holvec) lets you supply a vector of holidays, holvec, used to generate business days. holvec can either be in serial date format or date string format. If you use this syntax, you need to supply the frequency bdmode.

The output, bdates, is a column vector of business dates in serial date format.

Setting holvec to ' ' (empty string) or [] (empty vector) results in BUSDAYS using a default holiday schedule. The default holiday schedule is the NYSE holiday schedule.

\section*{Purpose Candlestick chart}
```

Syntax candle(HighPrices, LowPrices, ClosePrices, OpenPrices)
candle(HighPrices, LowPrices, ClosePrices, OpenPrices,
Color, Dates, Dateform)

```

\section*{Arguments}
\begin{tabular}{ll} 
HighPrices & High prices for a security. A column vector. \\
LowPrices & Low prices for a security. A column vector. \\
ClosePrices & Closing prices for a security. A column vector. \\
OpenPrices & \begin{tabular}{l} 
Opening prices for a security. A column vector. \\
Color \\
\\
\\
\\
\\
\\
\\
(Optional) Candlestick color. A string. MATLAB \\
software supplies a default color if none is specified. \\
The default color differs depending on the background \\
color of the figure window. See ColorSpec in the
\end{tabular} \\
& \begin{tabular}{l} 
MATLAB documentation for color names.
\end{tabular} \\
Dates & \begin{tabular}{l} 
(Optional) Column vector of dates for user specified
\end{tabular} \\
& \begin{tabular}{l} 
X-axis tick labels.
\end{tabular} \\
& \begin{tabular}{l} 
(Optional) Date string format used as the \(x\)-axis \\
tick labels. (See datetick in the MATLAB \\
documentation.) You can specify a dateform only \\
when tsobj does not contain time-of-day data.
\end{tabular} \\
& \begin{tabular}{l} 
If tsobj contains time-of-day data, dateform is \\
restricted to 'dd-mmm-yyy HH:MM'.
\end{tabular} \\
&
\end{tabular}

\section*{Description}
candle(HighPrices, LowPrices, ClosePrices, OpenPrices) plots a candlestick chart given column vectors with the high, low, closing, and opening prices of a security.

If the closing price is greater than the opening price, the body (the region between the opening and closing price) is unfilled.

If the opening price is greater than the closing price, the body is filled. candle(HighPrices, LowPrices, ClosePrices, OpenPrices, Color, Dates, Dateform) plots a candlestick chart given column vectors with the high, low, closing, and opening prices of a security. In addition, the optional arguments Color, Dates, and Dateform specify the color of the candle box and the date string format used as the \(x\)-axis tick labels.

\section*{Examples}

Given HighPrices, LowPrices, ClosePrices, and OpenPrices as equal-size vectors of stock price data
```

candle(HighPrices, LowPrices, ClosePrices, OpenPrices, 'blue')

```
plots a candlestick chart with blue candles.
The following example shows a candlestick chart for the most recent 21 days in disney.mat:
```

load disney;
candle(dis_HIGH(end-20:end), dis_LOW(end-20:end), dis_CLOSE(end-20:end),...
dis_OPEN(end-20:end), 'b');

```


\section*{See Also}
bolling | candle | dateaxis | highlow | movavg | pointfig
```

Purpose Time series candle plot
Syntax candle(tsobj)
candle(tsobj, color)
candle(tsobj, color, dateform)
candle(tsobj, color, dateform, ParameterName, ParameterValue, ...)
hcdl = candle(tsobj, color, dateform, ParameterName,
ParameterValue, ...)

```

\section*{Arguments}
\begin{tabular}{ll} 
tsobj & \begin{tabular}{l} 
Financial time series object \\
color \\
(Optional) A three-element row vector representing \\
RGB or a color identifier. (See plot in the MATLAB \\
documentation.)
\end{tabular} \\
dateform & \begin{tabular}{l} 
(Optional) Date string format used as the \(x\)-axis \\
tick labels. (See datetick in the MATLAB \\
documentation.) You can specify a dateform only \\
when tsobj does not contain time-of-day data.
\end{tabular} \\
& \begin{tabular}{l} 
If tsobj contains time-of-day data, dateform is \\
restricted to 'dd-mmm-yyyy HH:MM'
\end{tabular}
\end{tabular}

\section*{Description}
candle(tsobj) generates a candle plot of the data in the financial time series object tsobj. tsobj must contain at least four data series representing the high, low, open, and closing prices. These series must have the names High, Low, Open, and Close (case-insensitive).
candle(tsobj, color) additionally specifies the color of the candle box.
candle(tsobj, color, dateform) additionally specifies the date string format used as the \(x\)-axis tick labels. See datestr for a list of date string formats.
candle(tsobj, color, dateform, ParameterName, ParameterValue, ...) indicates the actual name(s) of the required data series if the data series do not have the default names. ParameterName can be
- HighName: high prices series name
- LowName: low prices series name
- OpenName: open prices series name
- CloseName: closing prices series name
hcdl = candle(tsobj, color, dateform, ParameterName, ParameterValue, ...) returns the handle to the patch objects and the line object that make up the candle plot. hdcl is a three-element column vector representing the handles to the two patches and one line that forms the candle plot.

Examples
Create a candle plot for Disney stock for the dates March 31, 1998 through April 30, 1998:
```

load disney.mat
candle(dis('3/31/98::4/30/98'))
title('Disney 3/31/98 to 4/30/98')

```


See Also
candle | chartfts | highlow | plot

Purpose
Syntax

Description

\section*{Input Arguments}

Cash flow and time mapping for bond portfolio
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate, Settle, Maturity) [CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) [CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate, Settle, Maturity, 'ParameterName', 'ParameterValue ...)
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate, Settle, Maturity) returns matrices of cash flow amounts, cash flow dates, time factors, and cash flow flags for a portfolio of NUMBONDS fixed-income securities.
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face) returns matrices of cash flow amounts, cash flow dates, time factors, and cash flow flags for a portfolio of NUMBONDS fixed-income securities defined using required and optional inputs.
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(CouponRate, Settle, Maturity, 'ParameterName', 'ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. Parametervalue is the value corresponding to 'ParameterName'. Specify parameter/value pairs in any order. Names are case-insensitive.

CouponRate
Decimal number indicating the annual percentage rate used to determine the coupons payable on a bond.
```

Settle

```

Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

\section*{Maturity}

Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

\section*{Period}

Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

\section*{Default: 2}

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis on page Glossary-1.
Default: 0
EndMonthRule
End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days.
- \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- 1 = set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1
IssueDate
Issue date for a bond.
Default: If you do not specify an IssueDate, the cash flow payment dates are determined from other inputs.

FirstCouponDate
Irregular or normal first coupon date.
Default: If you do not specify a FirstCouponDate, the effective start is the Settle date.

LastCouponDate

Irregular or normal last coupon date.
Default: If you do not specify a LastCouponDate, the effective date is the Settle date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date.

Default: If you do not specify StartDate, the effective start date is the Settle date.

\section*{Face}

Face or par value.
Default: 100

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.
AdjustCashFlowsBasis
Adjust the cash flows based on the actual period day count. NINST-by- 1 of logicals.

Default: False

\section*{BusinessDayConvention}

Require payment dates to be business dates. NINST-by- 1 cell array with possible choices of business day convention:
- actual
- follow
- modifiedfollow
- previous
- modifiedprevious

Default: actual
CompoundingFrequency
Compounding frequency for yield calculation.
Default: SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. If you use ISMA day counts and BUS/252, the specified basis are used.

Default: SIA bases use the actual/actual day count to compute discount factors.

Holidays
Holidays used for business day convention. NHOLIDAYS-by-1 of MATLAB date numbers.

Default: If no dates are specified, holidays.m is used.

\section*{Output \\ Arguments}

\section*{CFlowAmounts}

Cash flow matrix of a portfolio of bonds. Each row represents the cash flow vector of a single bond. Each element in a column represents a specific cash flow for that bond.

CFlowDates

Cash flow date matrix of a portfolio of bonds. Each row represents a single bond in the portfolio. Each element in a column represents a cash flow date of that bond.

\section*{TFactors}

Matrix of time factors for a portfolio of bonds. Each row corresponds to the vector of time factors for each bond. Each element in a column corresponds to the specific time factor associated with each cash flow of a bond. Time factors help determine the present value of a stream of cash flows. The term time factor refers to the exponent \(T F\) in the discounting equation
\[
P V=\frac{C F}{\left(1+\frac{z}{f}\right)^{T F}},
\]
where:
\(P V=\quad\) Present value of a cash flow.
\(C F=\quad\) Cash flow amount.
\(z=\quad\) Risk-adjusted annualized rate or yield corresponding to a given cash flow. The yield is quoted on a semiannual basis.
\(f=\quad\) Frequency of quotes for the yield.
\(T F=\quad\) Time factor for a given cash flow. Time is measured in semiannual periods from the settlement date to the cash flow date. In computing time factors, use SIA actual/actual day count conventions for all time factor calculations.

CFlowFlags
Matrix of cash flow flags for a portfolio of bonds. Each row corresponds to the vector of cash flow flags for each bond. Each
element in a column corresponds to the specific flag associated with each cash flow of a bond. Flags identify the type of each cash flow (for example, nominal coupon cash flow, front, or end partial, or "stub" coupon, maturity cash flow).
\begin{tabular}{l|l}
\hline Flag & Cash Flow Type \\
\hline 0 & Accrued interest due on a bond at settlement. \\
\hline 1 & \begin{tabular}{l} 
Initial cash flow amount smaller than normal due \\
to a "stub" coupon period. A stub period is created \\
when the time from issue date to first coupon date \\
is shorter than normal.
\end{tabular} \\
\hline 2 & \begin{tabular}{l} 
Larger than normal initial cash flow amount \\
because the first coupon period is longer than \\
normal.
\end{tabular} \\
\hline \(\mathbf{3}\) & Nominal coupon cash flow amount. \\
\hline 4 & \begin{tabular}{l} 
Normal maturity cash flow amount (face value \\
plus the nominal coupon amount).
\end{tabular} \\
\hline 5 & \begin{tabular}{l} 
End "stub" coupon amount (last coupon period is \\
abnormally short and actual maturity cash flow \\
is smaller than normal).
\end{tabular} \\
\hline 6 & \begin{tabular}{l} 
Larger than normal maturity cash flow because \\
the last coupon period longer than normal.
\end{tabular} \\
\hline 7 & \begin{tabular}{l} 
Maturity cash flow on a coupon bond when the \\
bond has less than one coupon period to maturity.
\end{tabular} \\
\hline 8 & \begin{tabular}{l} 
Smaller than normal maturity cash flow when the \\
bond has less than one coupon period to maturity.
\end{tabular} \\
\hline 9 & \begin{tabular}{l} 
Larger than normal maturity cash flow when the \\
bond has less than one coupon period to maturity.
\end{tabular} \\
\hline 10 & Maturity cash flow on a zero coupon bond. \\
\hline
\end{tabular}

Definitions

Examples

The elements contained in the cfamounts cash flow matrix, time factor matrix, and cash flow flag matrix correspond to the cash flow dates for each security. The first element of each row in the cash flow matrix is the accrued interest payable on each bond. This accrued interest is zero in the case of all zero coupon bonds. cfamounts determines all cash flows and time mappings for a bond whether or not the coupon structure contains odd first or last periods. All output matrices are padded with NaNs as necessary to ensure that all rows have the same number of elements.

Compute the cash flow structure and time factors for a bond portfolio containing a corporate bond paying interest quarterly and a Treasury bond paying interest semiannually:
```

Settle = '01-Nov-1993';
Maturity = ['15-Dec-1994';'15-Jun-1995'];
CouponRate= [0.06; 0.05];
Period = [4; 2];
Basis = [1; 0];
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = ...
cfamounts(CouponRate,Settle, Maturity, Period, Basis)

```

This returns:

CFlowAmounts =
\begin{tabular}{rrrrrr}
-0.7667 & 1.5000 & 1.5000 & 1.5000 & 1.5000 & 101.5000 \\
-1.8989 & 2.5000 & 2.5000 & 2.5000 & 102.5000 & NaN \\
& & & & & \\
CFlowDates = & & & & \\
728234 & 728278 & 728368 & 728460 & 728552 & 728643 \\
728234 & 728278 & 728460 & 728643 & 728825 & NaN
\end{tabular}

\footnotetext{
TFactors =
}
\begin{tabular}{llllll}
0 & 0.2404 & 0.7403 & 1.2404 & 1.7403 & 2.2404
\end{tabular}
```

llllll
CFlowFlags =
0
0}

```

Compute the cash flow structure and time factors for a bond portfolio containing a corporate bond paying interest quarterly and a Treasury bond paying interest semiannually. Use parameter/value pairs for the following optional input arguments: Period, Basis, BusinessDayConvention, and AdjustCashFlowsBasis:
```

Settle = '01-Jun-2010';
Maturity = ['15-Dec-2011';'15-Jun-2012'];
CouponRate= [0.06; 0.05];
Period = [4; 2];
Basis = [1; 0];
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = ...
cfamounts(CouponRate,Settle, Maturity, 'Period',Period, ...
'Basis', Basis, 'AdjustCashFlowsBasis', true,...
'BusinessDayConvention','modifiedfollow')

```

This returns:

CFlowAmounts =
\begin{tabular}{rrrrrrrr}
-1.2667 & 1.5000 & 1.5000 & 1.5000 & 1.5000 & 1.5000 & 1.5000 & 101.5000 \\
-2.3077 & 2.4932 & 2.5068 & 2.4932 & 2.5068 & 102.5000 & NaN & NaN
\end{tabular}
\begin{tabular}{rrrrrrr}
734290 & 734304 & 734396 & 734487 & 734577 & 734669 & 734761 \\
734290 & 734304 & 734487 & 734669 & 734852 & 735035 & NaN
\end{tabular}

Column 8

734852
NaN

TFactors \(=\)
\begin{tabular}{rrrrrrrr}
0 & 0.0769 & 0.5761 & 1.0769 & 1.5761 & 2.0769 & 2.5761 & 3.0769 \\
0 & 0.0769 & 1.0769 & 2.0769 & 3.0769 & 4.0769 & NaN & NaN
\end{tabular}

CFlowFlags =
\begin{tabular}{llllllll}
0 & 3 & 3 & 3 & 3 & 3 & 3 & 4
\end{tabular}
\begin{tabular}{ll} 
References & \begin{tabular}{l} 
Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John \\
Wiley \& Sons, 2002.
\end{tabular} \\
& \begin{tabular}{l} 
Mayle, Jan, "Standard Securities Calculations Methods: Fixed Income \\
Securities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994.
\end{tabular} \\
See Also & \begin{tabular}{l} 
Stigum, Marcia, and Franklin Robinson, Money Market and Bond \\
Calculations, McGraw-Hill, 1996.
\end{tabular} \\
& \begin{tabular}{l} 
accrfrac | cfdates | cftimes | cfamounts | cpncount | cpndaten | \\
cpndatenq | cpndatep | cpndatepq | cpndaysn | cpndaysp
\end{tabular}
\end{tabular}

\title{
Purpose Cash flow convexity \\ Syntax CFlowConvexity \(=\) cfconv(CashFlow, Yield)
}

\section*{Arguments}
\[
\begin{array}{ll}
\text { CashFlow } & \text { A vector of real numbers. } \\
\text { Yield } & \text { Periodic yield. A scalar. Enter as a decimal fraction. }
\end{array}
\]

\section*{Description CFlowConvexity = cfconv(CashFlow, Yield) returns the convexity of a cash flow in periods.}

\section*{Examples \\ Given a cash flow of nine payments of \(\$ 2.50\) and a final payment \(\$ 102.50\), with a periodic yield of \(2.5 \%\)}
```

CashFlow = [2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 102.5];
Convex = cfconv(CashFlow, 0.025)
Convex =
90.4493 (periods)

```

See Also bndconvp | bndconvy | bnddurp | bnddury | cfdur

\section*{Purpose}

Cash flow dates for fixed-income security
Syntax CFlowDates \(=\) cfdates(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date numbers \\
or date strings. \\
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are \(0,1,2\) \\
(default), \(3,4,6\), and 12.
\end{tabular} \\
Period & (Optional) Day-count basis of the instrument. \\
Basis & A vector of integers. \\
& - \(0=\) actual/actual (default) \\
& - \(1=30 / 360\) (SIA) \\
& - \(2=\) actual/360 \\
& - \(3=\) actual/365 \\
& - \(4=30 / 360\) (PSA) \\
& - \(5=30 / 360\) (ISDA) \\
& - \(6=30 / 360\) (European) \\
& - \(7=\) actual/365 (Japanese) \\
& - \(8=\) actual/actual (ISMA) \\
- \(9=\) actual/360 (ISMA) \\
- \(10=\) actual/365 (ISMA)
\end{tabular}

\section*{cfdates}
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\) actual/365 (ISDA)
- 13 = BUS/252
\begin{tabular}{ll} 
& \begin{tabular}{l} 
For more information, seebasis on page \\
Glossary-1.
\end{tabular} \\
EndMonthRule & \begin{tabular}{l} 
(Optional) End-of-month rule. A vector. \\
This rule applies only when Maturity is an \\
end-of-month date for a month having 30 or \\
fewer days. 0 = ignore rule, meaning that a \\
bond's coupon payment date is always the \\
same numerical day of the month. 1 = set rule \\
on (default), meaning that a bond's coupon \\
payment date is always the last actual day of \\
the month.
\end{tabular} \\
IssueDate & \begin{tabular}{l} 
(Optional) Date when a bond was issued. \\
FirstCouponDate \\
(Optional) Date when a bond makes its first \\
coupon payment. When FirstCouponDate
\end{tabular} \\
& \begin{tabular}{l} 
and LastCouponDate are both specified,
\end{tabular} \\
& FirstCouponDate takes precedence in \\
determining the coupon payment structure.
\end{tabular}
date. If you do not specify StartDate, the effective start date is the Settle date.

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if Maturity contains N dates, then Settle must contain N dates or a single date.

\section*{Description}

\section*{Examples}

CFlowDates = cfdates(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns a matrix of cash flow dates for a bond or set of bonds. cfdates determines all cash flow dates for a bond whether or not the coupon payment structure is normal or the first and/or last coupon period is long or short.

CFlowDates is an N-row matrix of serial date numbers, padded with NaNs as necessary to ensure that all rows have the same number of elements. Use the function datestr to convert serial date numbers to formatted date strings.

Note The cash flow flags for a portfolio of bonds were formerly available as the cfdates second output argument, CFlowFlags. You can now use cfamounts to get these flags. If you specify a CFlowFlags argument, cfdates displays a message directing you to use cfamounts.
```

CFlowDates = cfdates('14 Mar 1997', '30 Nov 1998', 2, 0, 1)
CFlowDates =
729541 729724 729906 730089
datestr(CFlowDates)
ans =

```

\section*{cfdates}
```

31-May-1997
30-Nov-1997
31-May-1998
30-Nov-1998

```

Given three securities with different maturity dates and the same default arguments
```

Maturity = ['30-Sep-1997'; '31-Oct-1998'; '30-Nov-1998'];
CFlowDates = cfdates('14-Mar-1997', Maturity)
CFlowDates =

| 729480 | 729663 | NaN | NaN |
| ---: | ---: | ---: | ---: |
| 729510 | 729694 | 729875 | 730059 |
| 729541 | 729724 | 729906 | 730089 |

```

Look at the cash-flow dates for the last security.
```

datestr(CFlowDates(3,:))
ans =
31-May-1997
30-Nov-1997
31-May-1998
30-Nov-1998

```

\section*{See Also}
accrfrac | cfamounts | cftimes | cpncount | cpndaten | cpndatenq | cpndatep | cpndatepq | cpndaysn | cpndaysp | cpnpersz
Purpose
Cash-flow duration and modified duration
Syntax [Duration, ModDuration] = cfdur(CashFlow, Yield)

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline Description & [Duration, ModDuration] = cfdur(CashFlow, Yield) calculates the duration and modified duration of a cash flow in periods. \\
\hline Examples & Given a cash flow of nine payments of \(\$ 2.50\) and a final payment \(\$ 102.50\), with a periodic yield of \(2.5 \%\) \\
\hline & CashFlow=[2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 2.5 102.5]; \\
\hline & [Duration, ModDuration] = cfdur(CashFlow, 0.025) \\
\hline & Duration = \\
\hline & 8.9709 (periods) \\
\hline & ModDuration \(=\) \\
\hline & 8.7521 (periods) \\
\hline
\end{tabular}

\footnotetext{
See Also bndconvp | bndconvy | bnddurp | bnddury | cfconv
}

\section*{cfport}

\author{
Purpose Portfolio form of cash flow amounts \\ Syntax \\ [CFBondDate, AllDates, AllTF, IndByBond] = cfport(CFlowAmounts, CFlowDates, TFactors)
}

\section*{Arguments}
\begin{tabular}{ll} 
CFlowAmounts & \begin{tabular}{l} 
Number of bonds (NUMBONDS) by number of cash \\
flows (NUMCFS) matrix with entries listing cash \\
flow amounts corresponding to each date in \\
CFlowDates.
\end{tabular} \\
CFlowDates & \begin{tabular}{l} 
NUMBONDS-by-NUMCFS matrix with rows listing \\
cash flow dates for each bond and padded with
\end{tabular} \\
TFactors & \begin{tabular}{l} 
NaNs.
\end{tabular} \\
& \begin{tabular}{l} 
(Optional) NUMBONDS-by-NUMCFS matrix with \\
entries listing the time between settlement and \\
the cash flow date measured in semiannual \\
coupon periods.
\end{tabular}
\end{tabular}

\section*{Description}
[CFBondDate, AllDates, AllTF, IndByBond] = cfport(CFlowAmounts, CFlowDates, TFactors) computes a vector of all cash flow dates of a bond portfolio, and a matrix mapping the cash flows of each bond to those dates. Use the matrix for pricing the bonds against a curve of discount factors.

CFBondDate is a NUMBONDS by number of dates (NUMDATES) matrix of cash flows indexed by bond and by date in AllDates. Each row contains a bond's cash flow values at the indices corresponding to entries in AllDates. Other indices in the row contain zeros.

AllDates is a NUMDATES-by- 1 list of all dates that have any cash flow from the bond portfolio.

AllTF is a NUMDATES-by-1 list of time factors corresponding to the dates in AllDates. If TFactors is not entered, AllTF contains the number of days from the first date in AllDates.

IndByBond is a NUMBONDS-by-NUMCFS matrix of indices. The \(i\) th row contains a list of indices into AllDates where the \(i\) th bond has cash flows. Since some bonds have more cash flows than others, the matrix is padded with NaNs.

Use cfamounts to calculate the cash flow amounts, cash flow dates, and time factors for each of two bonds. Then use cfplot (available at /finance/findemos/cfplot.m) to plot the cash flow diagram.
```

Settle = '03-Aug-1999';
Maturity = ['15-Aug-2000';'15-Dec-2000'];
CouponRate= [0.06; 0.05];
Period = [3;2];
Basis = [1;0];
[CFlowAmounts, CFlowDates, TFactors] = cfamounts(CouponRate,...
Settle, Maturity, Period, Basis);
cfplot(CFlowDates,CFlowAmounts)
xlabel('Numeric Cash Flow Dates')
ylabel('Bonds')
title('Cash Flow Diagram')

```

\section*{cfport}


Finally, call cfport to map the cash flow amounts to the cash flow dates.
Each row in the resultant CFBondDate matrix represents a bond. Each column represents a date on which one or more of the bonds has a cash flow. A 0 means the bond did not have a cash flow on that date. The dates associated with the columns are listed in AllDates. For example, the first bond had a cash flow of 2.000 on 730347 . The second bond had no cash flow on this date.

For each bond, IndByBond indicates the columns of CFBondDate, or dates in AllDates, for which a bond has a cash flow.
```

[CFBondDate, AllDates, AllTF, IndByBond] = ...
cfport(CFlowAmounts, CFlowDates, TFactors)
CFBondDate =
-1.8000 2.0000 2.0000 2.0000 0 102.0000 0

```
```

    -0.6694
                                    0.5000
                                2.5000
                                    0102.5000
    AllDates =
7 3 0 3 3 5
730347
730469
7 3 0 5 9 1
730652
7 3 0 7 1 3
730835
AllTF =
0
0.0663
0.7322
1.3989
1.7322
2.0663
2.7322
IndByBond =

| 1 | 2 | 3 | 4 | 6 |
| :--- | :--- | :--- | :--- | :--- |


| 1 | 3 | 5 | 7 | NaN |
| :--- | :--- | :--- | :--- | :--- |

```

\section*{See Also}
cfamounts

Purpose Time factors corresponding to bond cash flow dates
```

Syntax [TFactors] = cftimes(Settle, Maturity)
[TFactors] = cftimes(Settle, Maturity
Period, Basis, EndMonthRule,
IssueDate, FirstCouponDate, LastCouponDate, StartDate)
[TFactors] = cftimes(Settle, Maturity,
'ParameterName', 'ParameterValue ...)

```

\section*{Description}

Input Arguments
[TFactors] = cftimes(Settle, Maturity) determines the time factors corresponding to the cash flows of a bond or set of bonds.
[TFactors] = cftimes(Settle, Maturity Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) determines the time factors corresponding to the cash flows of a bond or set of bonds, including optional inputs.
[TFactors] = cftimes(Settle, Maturity, 'ParameterName', 'ParameterValue ...) accepts optional inputs as one or more comma-separated parameter/value pairs. 'ParameterName' is the name of the parameter inside single quotes. Parametervalue is the value corresponding to 'ParameterName'. Specify parameter/value pairs in any order. Names are case-insensitive.

Settle
Settlement date. A vector of serial date numbers or date strings. Settle must be earlier than Maturity.

Maturity
Maturity date. A vector of serial date numbers or date strings.

\section*{Ordered Input or Parameter-Value Pairs}

Enter the following inputs using an ordered syntax or as parameter/value pairs. You cannot mix ordered syntax with parameter/value pairs.

Period

Coupons per year of the bond. A vector of integers. Values are \(0,1,2,3,4,6\), and 12 .

\section*{Default: 2}

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0

EndMonthRule

End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month. \(1=\) set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

Default: 1
IssueDate
Issue date for a bond.

\section*{FirstCouponDate}

Irregular or normal first coupon date.

\section*{LastCouponDate}

Irregular or normal last coupon date.

\section*{StartDate}

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify StartDate, the effective start date is the Settle date.

\section*{Parameter-Value Pairs}

Enter the following inputs only as parameter/value pairs.
CompoundingFrequency
Compounding frequency for yield calculation. By default, SIA bases (0-7) and BUS/252 use a semiannual compounding convention and ISMA bases (8-12) use an annual compounding convention.

\section*{DiscountBasis}

Basis used to compute the discount factors for computing the yield. The default behavior is for SIA bases to use the actual/actual day
count to compute discount factors. If you use ISMA day counts and BUS/252, the specified bases are used.

\section*{Output \\ Arguments}

\section*{Definitions}

Examples

\section*{References}

Krgin, Dragomir, Handbook of Global Fixed Income Calculations, John Wiley \& Sons, 2002.

Mayle, Jan, "Standard Securities Calculations Methods: Fixed Income Securities Formulas for Analytic Measures", SIA, Vol 2, Jan 1994.

Stigum, Marcia, and Franklin Robinson, Money Market and Bond Calculations, McGraw-Hill, 1996.

See Also
accrfrac | cfdates | cfamounts | cpncount | cpndaten | cpndatenq | cpndatep | cpndatepq | cpndaysn | cpndaysp | date2time

\section*{Purpose}

Syntax

\section*{Arguments}
\begin{tabular}{ll} 
highp & High price (vector) \\
lowp & Low price (vector) \\
closep & Closing price (vector) \\
tvolume & Volume traded (vector) \\
tsobj & Financial time series object
\end{tabular}

\section*{Description}

Chaikin oscillator
```

chosc = chaikosc(highp, lowp, closep, tvolume)

```
chosc = chaikosc(highp, lowp, closep, tvolume)
chosc = chaikosc([highp lowp closep tvolume])
chosc = chaikosc([highp lowp closep tvolume])
choscts = chaikosc(tsobj)
choscts = chaikosc(tsobj)
choscts = chaikosc(tsobj, ParameterName, ParameterValue, ... )
```

choscts = chaikosc(tsobj, ParameterName, ParameterValue, ... )

```

Financial time series object

The Chaikin oscillator is calculated by subtracting the 10 -period exponential moving average of the Accumulation/Distribution (A/D) line from the three-period exponential moving average of the A/D line.
chosc \(=\) chaikosc(highp, lowp, closep, tvolume) calculates the Chaikin oscillator (vector), chosc, for the set of stock price and volume traded data (tvolume). The prices that must be included are the high (highp), low (lowp), and closing (closep) prices.
chosc \(=\) chaikosc([highp lowp closep tvolume]) accepts a four-column matrix as input.
choscts = chaikosc(tsobj) calculates the Chaikin Oscillator, choscts, from the data contained in the financial time series object tsobj. tsobj must at least contain data series with names High, Low, Close, and Volume. These series must represent the high, low, and closing prices, plus the volume traded. choscts is a financial time series object with the same dates as tsobj but only one series named ChaikOsc.

\section*{chaikosc}
choscts \(=\) chaikosc(tsobj, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are
- HighName: high prices series name
- LowName: low prices series name
- CloseName: closing prices series name
- VolumeName: volume traded series name

Parameter values are the strings that represent the valid parameter names.

Examples Compute the Chaikin oscillator for Disney stock and plot the results.
```

load disney.mat
dis_CHAIKosc = chaikosc(dis)
plot(dis_CHAIKosc)
title('Chaikin Oscillator for Disney')

```


References

See Also

Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 91-94.

Purpose Chaikin volatility
Syntax chvol = chaikvolat(highp, lowp)
chvol = chaikvolat([highp lowp])
chvol = chaikvolat(high, lowp, nperdiff, manper)
chvol = chaikvolat([high lowp], nperdiff, manper)
chvts = chaikvolat(tsobj)
chvts = chaikvolat(tsobj, nperdiff, manper, ParameterName, ParameterValue, ...)

\section*{Arguments}
\begin{tabular}{ll} 
highp & High price (vector). \\
lowp & Low price (vector). \\
nperdiff & Period difference (vector). Default = 10. \\
manper & \begin{tabular}{l} 
Length of exponential moving average in periods \\
(vector). Default \(=10\).
\end{tabular} \\
tsobj & Financial time series object.
\end{tabular}

Description chvol = chaikvolat(highp, lowp) calculates the Chaikin volatility from the series of stock prices, highp and lowp. The vector chvol contains the Chaikin volatility values, calculated on a 10 -period exponential moving average and 10 -period difference.
chvol = chaikvolat([highp lowp]) accepts a two-column matrix as the input.
chvol = chaikvolat(high, lowp, nperdiff, manper) manually sets the period difference nperdiff and the length of the exponential moving average manper in periods.
chvol = chaikvolat([high lowp], nperdiff, manper) accepts a two-column matrix as the first input.
chvts \(=\) chaikvolat(tsobj) calculates the Chaikin volatility from the financial time series object tsobj. The object must contain at least two series named High and Low, representing the high and low prices per period. chvts is a financial time series object containing the Chaikin volatility values, based on a 10-period exponential moving average and 10 -period difference. chvts has the same dates as tsobj and a series called ChaikVol.
chvts = chaikvolat (tsobj, nperdiff, manper, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are
- HighName: high prices series name
- LowName: low prices series name

Parameter values are the strings that represent the valid parameter names.
nperdiff, the period difference, and manper, the length of the exponential moving average in periods, can also be set with this form of chaikvolat.

\section*{Examples}

Compute the Chaikin volatility for Disney stock and plot the results:
```

load disney.mat
dis_CHAIKvol = chaikvolat(dis)
plot(dis_CHAIKvol)
title('Chaikin Volatility for Disney')

```


References

See Also

Achelis, Steven B., Technical Analysis from A to Z, Second Edition, McGraw-Hill, 1995, pp. 304-305.

\section*{Purpose Interactive display}

Syntax chartfts(tsobj)

Description chartfts(tsobj) produces a figure window that contains one or more plots. You can use the mouse to observe the data at a particular time point of the plot.

Examples Create a financial time series object from the supplied data file ibm9599.dat:
```

    ibmfts = ascii2fts('ibm9599.dat', 1, 3, 2);
    ```

Chart the financial time series object ibmfts:
```

chartfts(ibmfts)

```

With the Zoom feature set off, a mouse click on the indicator line displays object data in a pop-up box.


With the Zoom feature set on, mouse clicks indicate the area of the chart to zoom.


You can find a tutorial on using chartfts in "Visualizing Financial Time Series Objects" on page 9-18. See "Zoom Tool" on page 9-21 for details on performing the zoom. Also see "Combine Axes Tool" on page 9-24 for information about combining axes for specified plots.

See Also candle | highlow | plot

\section*{Portfolio.checkFeasibility}

Superclasses AbstractPortfolio
Purpose \(\quad\) Check feasibility of input portfolios against a portfolio object
Syntax status = checkFeasibility(obj, pwgt)
Description status = checkFeasibility (obj, pwgt) to check the feasibility of input portfolios against a portfolio object.

Tips

Input
Arguments
obj
- The constraint tolerance to assess whether a constraint is satisfied is obtained from the hidden property obj.defaultTolCon.

A portfolio object [Portfolio].
pwgt
Portfolios to be checked [NumAssets-by-NumPortsmatrix].

\section*{Output \\ Arguments \\ status \\ Row vector of NumPorts indicators that are true if portfolio is feasible and false otherwise.}

> Note By definition, any portfolio set must be nonempty and bounded. If the set is empty, no portfolios can be feasible. Use estimateBounds to test for nonempty and bounded sets.

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Examples Given portfolio \(p\), determine if \(p\) is feasible:
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontier;
p.checkFeasibility(pwgt)
ans =

```
    \(\begin{array}{llllllllll}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{array}\)
See Also
estimateBounds

Tutorials . "Validating the Portfolio Problem" on page 4-73

\section*{chfield}

Purpose Change data series name
Syntax newfts = chfield(oldfts, oldname, newname)

\section*{Arguments}

\section*{Description}

See Also fieldnames \| isfield \| rmfield

\title{
Purpose \\ Syntax \\ DesignSUR = convert2sur(Design, Group)
}

\section*{Arguments}

Design A matrix or a cell array that depends on the number of data series NUMSERIES.
- If NUMSERIES = 1 , convert2sur returns the Design matrix.
- If NUMSERIES > 1, Design is a cell array with NUMSAMPLES cells, where each cell contains a NUMSERIES-by-NUMPARAMS matrix of known values.

Group Contains information about how data series are to be grouped, with separate parameters for each group. Specify groups either by series or by groups:
- To identify groups by series, construct an index vector that has NUMSERIES elements. Element i \(=1\), ..., NUMSERIES in the vector, and has the index \(j=1, \ldots\), NUMGROUPS of the group in which series \(i\) is a member.
- To identify groups by groups, construct a cell array with NUMGROUPS elements. Each cell contains a vector with the indexes of the series that populate a given group.

In either case, the number of series is NUMSERIES and the number of groups is NUMGROUPS, with \(1 \leq\) NUMGROUPS \(\leq\) NUMSERIES.

\section*{Description}

\section*{Examples}

DesignSUR = convert2sur(Design, Group) converts a multivariate normal regression model into a seemingly unrelated regression model with a specified grouping of the data series. DesignSUR is either a matrix or a cell array that depends on the value of NUMSERIES:
- If NUMSERIES = 1, DesignSUR = Design, which is a NUMSAMPLES-by-NUMPARAMS matrix.
- If NUMSERIES > 1 and NUMGROUPS groups are to be formed, Design is a cell array with NUMSAMPLES cells, where each cell contains a NUMSERIES-by-(NUMGROUPS * NUMPARAMS) matrix of known values.

The original collection of parameters that are common to all series are replicated to form collections of parameters for each group.

This example has ten series in three groups, and two model parameters. Suppose

Group 1 has series \(1,3,4,8\).
Group 2 has series 2, 6, 10 .
Group 3 has series 5, 7, 9 .
Either:
```

Group = [ 1, 2, 1, 1, 3, 2, 3, 1, 3, 2];

```
or
```

Group = cell(3,1);
Group{1} = [1, 3, 4, 8];
Group{2} = [2, 6, 10];
Group{3} = [5, 7, 9];

```

A regression with DesignSUR would have \(3 \times 2=6\) model parameters.
```

Purpose Convert to specified frequency
Syntax }\begin{array}{l}{\mathrm{ newfts = convertto(oldfts, newfreq)}}<br>{\mathrm{ newfts = convertto(oldfts, newfreq, 'param1','value1','param2',}}<br>{}<br>{}<br>{}

```

\section*{Arguments}
\begin{tabular}{ll} 
oldfts & Name of an existing financial time series object. \\
newfreq & 1, DAILY, Daily, daily, D, d \\
& 2, WEEKLY, Weekly, weekly, W, w \\
& 3, MONTHLY, Monthly, monthly, M, m \\
& 4, QUARTERLY, Quarterly, quarterly, Q, q \\
& 5, SEMIANNUAL, Semiannual, semiannual, s, s \\
& 6, ANNUAL, Annual, annual, A, a
\end{tabular}

\section*{Description}

See Also
convertto converts a financial time series of any frequency to one of a specified frequency.
newfts \(=\) convertto(oldfts, newfreq) converts the object oldfts to the new time series object newfts with the frequency newfreq.

Refer to the documentation for each frequency conversion function to determine the valid parameter/value pairs.
```

toannual | todaily | tomonthly | toquarterly | tosemi | toweekly

```

Purpose Correlation coefficients
```

Syntax r = corrcoef(X)
r = corrcoef(X,Y),

```

\section*{Arguments}
\begin{tabular}{ll}
X & \begin{tabular}{l} 
Matrix where each row is an observation and each \\
column is a variable.
\end{tabular} \\
Y & \begin{tabular}{l} 
Matrix where each row is an observation and each \\
column is a variable.
\end{tabular}
\end{tabular}

\section*{Description}
corrcoef for financial time series objects is based on the MATLAB corrcoef function. See corrcoef in the MATLAB documentation. \(r=\operatorname{corrcoef}(X)\) calculates a matrix \(r\) of correlation coefficients for an array \(X\), in which each row is an observation and each column is a variable.
\(r=\operatorname{corrcoef}(X, Y)\), where \(X\) and \(Y\) are column vectors, is the same as \(r=c o r r c o e f([X Y])\). corrcoef converts \(X\) and \(Y\) to column vectors if they are not; that is, \(r=\operatorname{corrcoef}(X, Y)\) is equivalent to \(r=\operatorname{corrcoef}([X(:) \quad Y(:)])\) in that case.

If \(c\) is the covariance matrix, \(c=\operatorname{cov}(X)\), then corrcoef \((X)\) is the matrix whose ( \(i, j\) ) 'th element is ci, \(j / \mathrm{sqrt}(\mathrm{ci}, i * \mathrm{c}(j, j)\) ).
\([r, p]=\operatorname{corrcoef}(\ldots)\) also returns \(p\), a matrix of \(p\)-values for testing the hypothesis of no correlation. Each p-value is the probability of getting a correlation as large as the observed value by random chance, when the true correlation is zero. If \(p(i, j)\) is less than 0.05 , then the correlation \(r(i, j)\) is significant.
[ \(\mathrm{r}, \mathrm{p}, \mathrm{rlo}, \mathrm{rup}]=\operatorname{corrcoef}(\ldots)\) also returns matrices \(r l o\) and rup, of the same size as \(r\), containing lower and upper bounds for a \(95 \%\) confidence interval for each coefficient.
[...]=corrcoef(...,'PARAM1',VAL1,'PARAM2',VAL2,...) specifies additional parameters and their values. Valid parameters are:
- 'alpha' - A number between 0 and 1 to specify a confidence level of \(100^{*}(1-\mathrm{ALPHA}) \%\). Default is 0.05 for \(95 \%\) confidence intervals.
- 'rows ' - Either 'all' (default) to use all rows, 'complete' to use rows with no NaN values, or 'pairwise' to compute r(i,j) using rows with no NaN values in column \(i\) or \(j\).

The \(p\)-value is computed by transforming the correlation to create a \(t\)-statistic having \(N-2\) degrees of freedom, where \(N\) is the number of rows of X. The confidence bounds are based on an asymptotic normal distribution of \(0.5 * \log ((1+r) /(1-\mathrm{r}))\), with an approximate variance equal to \(1 /(\mathrm{N}-3)\). These bounds are accurate for large samples when \(X\) has a multivariate normal distribution. The 'pairwise' option can produce an \(r\) matrix that is not positive definite.

\section*{Examples}

Generate random data having correlation between column 4 and the other columns.
```

x = randn(30,4); % uncorrelated data
x(:,4) = sum(x,2); % introduce correlation
f = fints((today:today+29)', x); % create a fints object using x
[r,p] = corrcoef(x) % compute sample correlation and p-values
[i,j] = find(p<0.05); % find significant correlations
[i,j] % display their (row,col) indices

```

Note Class support for inputs \(\mathrm{X}, \mathrm{Y}\) : float: double and single.

\footnotetext{
See Also cov \| std \| var
}

Purpose Convert standard deviation and correlation to covariance
Syntax ExpCovariance \(=\operatorname{corr2cov}(E x p S i g m a, ~ E x p C o r r C)\)

\section*{Arguments}

\section*{Description}

Examples
```

ExpSigma = [0.5 2.0];
ExpCorrC = [1.0 -0.5
-0.5 1.0];
ExpCovariance = corr2cov(ExpSigma, ExpCorrC)

```

Expected results:
ExpCovariance =
\[
\begin{array}{rr}
0.2500 & -0.5000 \\
-0.5000 & 4.0000
\end{array}
\]

See Also corrcoef | cov | cov2corr | ewstats | std

\section*{Purpose Covariance matrix}

Syntax
\(\operatorname{cov}(X)\)
\(\operatorname{cov}(X, Y)\)

\section*{Arguments}
\begin{tabular}{ll}
\(X\) & Financial times series object. \\
\(Y\) & Financial times series object.
\end{tabular}

\section*{Description}
cov for financial time series objects is based on the MATLAB cov function. See cov in the MATLAB documentation.

If \(X\) is a financial time series object with one series, \(\operatorname{cov}(X)\) returns the variance. For a financial time series object containing multiple series, where each row is an observation, and each series a variable, \(\operatorname{cov}(X)\) is the covariance matrix.
\(\operatorname{diag}(\operatorname{cov}(X))\) is a vector of variances for each series and sqrt \((\operatorname{diag}(\operatorname{cov}(X)))\) is a vector of standard deviations.
\(\operatorname{cov}(X, Y)\), where \(X\) and \(Y\) are financial time series objects with the same number of elements, is equivalent to \(\operatorname{cov}([X(:) \quad Y(:)])\).
\(\operatorname{cov}(X)\) or \(\operatorname{cov}(X, Y)\) normalizes by \((N-1)\) if \(N>1\), where \(N\) is the number of observations. This makes \(\operatorname{cov}(X)\) the best unbiased estimate of the covariance matrix if the observations are from a normal distribution. For \(N=1\), cov normalizes by \(N\).
\(\operatorname{cov}(X, 1)\) or \(\operatorname{cov}(X, Y, 1)\) normalizes by \(N\) and produces the second moment matrix of the observations about their mean. \(\operatorname{cov}(X, Y, 0)\) is the same as \(\operatorname{cov}(X, Y)\) and \(\operatorname{cov}(X, 0)\) is the same as \(\operatorname{cov}(X)\). The mean is removed from each column before calculating the result.
```

Examples To create a covariance matrix for the following dates:

```
```

dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
A = [-1 1 2 ; -2 3 1 ; 4 0 3]
A = [-1 1 2 ; -2 3 1 ; 4 0 3]
f = fints(dates, A);
f = fints(dates, A);
c = cov(f)
c = cov(f)
c =
c =
    10.3333 -4.1667 3.0000
    10.3333 -4.1667 3.0000
    -4.1667 2.3333 -1.5000
    -4.1667 2.3333 -1.5000
    3.0000 -1.5000 1.0000
```

    3.0000 -1.5000 1.0000
    ```

See Also corrcoef | mean | std | var

\section*{Purpose}

Convert covariance to standard deviation and correlation coefficient
Syntax
[ExpSigma, ExpCorrC] = cov2corr(ExpCovariance)

\section*{Arguments}

ExpCovariance \(n\)-by- \(n\) covariance matrix; for example, from cov or ewstats. \(n\) is the number of random processes.

\section*{Description}
[ExpSigma, ExpCorrC] = cov2corr(ExpCovariance) converts covariance to standard deviations and correlation coefficients.

ExpSigma is a 1-by- \(n\) vector with the standard deviation of each process.
ExpCorrC is an \(n\)-by- \(n\) matrix of correlation coefficients.
```

ExpSigma(i) = sqrt(ExpCovariance(i,i))

```
\(\operatorname{ExpCorrC}(i, j)=\operatorname{ExpCovariance}(i, j) /(E x p S i g m a(i) * E x p S i g m a(j))\)

\section*{Examples}
\[
\begin{aligned}
\text { ExpCovariance }= & {\left[\begin{array}{ll}
0.25 & -0.5 \\
& -0.5 \\
4.0
\end{array}\right] ; }
\end{aligned}
\]
[ExpSigma, ExpCorrC] = cov2corr(ExpCovariance)
Expected results:

> ExpSigma =
\[
0.5000 \quad 2.0000
\]

ExpCorrC =
\begin{tabular}{rr}
1.0000 & -0.5000 \\
-0.5000 & 1.0000
\end{tabular}

\footnotetext{
See Also corr2cov \| corrcoef \| cov \| ewstats \| std
}

\section*{Purpose}

Coupon payments remaining until maturity

\author{
Syntax NumCouponsRemaining = cpncount(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)
}

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date numbers \\
or date strings.
\end{tabular} \\
Period & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are 0, 1, 2 \\
(default), 3, 4, 6, and 12.
\end{tabular} \\
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the instrument. \\
A vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252
\(\left.\begin{array}{ll} & \begin{array}{l}\text { For more information, seebasis on page } \\ \text { Glossary-1. }\end{array} \\ \text { EndMonthRule } & \text { (Optional) End-of-month rule. A vector. } \\ \text { This rule applies only when Maturity is an } \\ \text { end-of-month date for a month having } 30 \text { or } \\ \text { fewer days. 0 = ignore rule, meaning that a } \\ \text { bond's coupon payment date is always the } \\ \text { same numerical day of the month. 1 = set rule } \\ \text { on (default), meaning that a bond's coupon } \\ \text { payment date is always the last actual day of } \\ \text { the month. }\end{array}\right\}\)

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

\section*{Description NumCouponsRemaining = cpncount(Settle, Maturity, Period, Basis, EndMonthRule) returns the whole number of coupon payments between the settlement and maturity dates for a coupon bond or set of bonds.}

\section*{Examples}
```

NumCouponsRemaining = cpncount('14 Mar 1997', '30 Nov 2000',...
2, 0, 0)
n =
8

```

Given three coupon bonds with different maturity dates and the same default arguments
```

Maturity = ['30 Sep 2000'; '31 Oct 2001'; '30 Nov 2002'];
NumCouponsRemaining = cpncount('14 Sep 1997', Maturity)
NumCouponsRemaining =

```
7
9
11

\section*{See Also}
accrfrac | cfamounts | cfdates | cftimes | cpndaten | cpndatenq | cpndatep | cpndatepq | cpndaysn | cpndaysp | cpnpersz

Purpose Next coupon date for fixed-income security
```

Syntax NextCouponDate = cpndaten(Settle, Maturity, Period, Basis,
EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)

```

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date numbers \\
or date strings.
\end{tabular} \\
Period & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are 0, 1, 2 \\
(default), 3, 4, 6, and 12.
\end{tabular} \\
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the instrument. \\
A vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\mathrm{actual} / 365\) (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
\begin{tabular}{ll} 
EndMonthRule & \begin{tabular}{l} 
(Optional) End-of-month rule. A vector. \\
This rule applies only when Maturity is an \\
end-of-month date for a month having 30 or \\
fewer days. 0 = ignore rule, meaning that a \\
bond's coupon payment date is always the \\
same numerical day of the month. 1 = set rule \\
on (default), meaning that a bond's coupon \\
payment date is always the last actual day of \\
the month. \\
(Optional) Date when a bond was issued.
\end{tabular} \\
IssueDate & \begin{tabular}{l} 
(Optional) Date when a bond makes its first
\end{tabular} \\
FirstCouponDate \\
& \begin{tabular}{l} 
coupon payment. When FirstCouponDate \\
and LastCouponDate are both specified,
\end{tabular} \\
& \begin{tabular}{l} 
FirstCouponDate takes precedence in \\
determining the coupon payment structure.
\end{tabular} \\
& \begin{tabular}{l} 
(Optional) Last coupon date of a bond before \\
the maturity date. In the absence of a specified
\end{tabular} \\
& \begin{tabular}{l} 
FirstCouponDate, a specified LastCouponDate \\
determines the coupon structure of the bond.
\end{tabular} \\
The coupon structure of a bond is truncated \\
at the LastCouponDate regardless of where it
\end{tabular}

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must
be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

\author{
Description
}

\section*{Examples}
```

NextCouponDate = cpndaten('14 Mar 1997', '30 Nov 2000', 2, 0, 0);
datestr(NextCouponDate)
ans =
30-May-1997
NextCouponDate = cpndaten('14 Mar 1997', '30 Nov 2000', 2, 0, 1);
datestr(NextCouponDate)
ans =
31-May-1997
Maturity = ['30 Sep 2000'; '31 Oct 2000'; '30 Nov 2000'];
NextCouponDate = cpndaten('14 Mar 1997', Maturity);
datestr(NextCouponDate)
ans =
31-Mar-1997

```

30-Apr-1997
31-May-1997
See Also \(\begin{aligned} & \text { accrfrac | cfamounts | cfdates | cftimes | cpncount | cpndatenq | } \\ & \text { cpndatep | cpndatepq | cpndaysn | cpndaysp | cpnpersz }\end{aligned}\)

Purpose Next quasi coupon date for fixed income security

\author{
Syntax NextQuasiCouponDate = cpndatenq(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)
}

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date numbers \\
or date strings.
\end{tabular} \\
Period & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are 0, 1, 2 \\
(default), 3, 4, 6, and 12.
\end{tabular} \\
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the instrument. \\
A vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual \(/ 360\)
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

IssueDate (Optional) Date when a bond was issued.
FirstCouponDate
(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.
LastCouponDate (Optional) Last coupon date of a bond before the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars,
or empty matrices. Fill unspecified entries in input vectors with the value NaN. Dates can be serial date numbers or date strings.

\section*{Description}

Examples

NextQuasiCouponDate = cpndatenq(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate) determines the next quasi coupon date for a portfolio of NUMBONDS fixed income securities whether or not the first or last coupon is normal, short, or long. For zero coupon bonds cpndatenq returns quasi coupon dates as if the bond had a semiannual coupon structure. Successive quasi coupon dates determine the length of the standard coupon period for the fixed income security of interest and do not necessarily coincide with actual coupon payment dates.

Outputs are NUMBONDS-by- 1 vectors.
If Settle is a coupon date, this function never returns the settlement date. It returns the quasi coupon date strictly after settlement.

NextQuasiCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date string.

Given a pair of bonds with the characteristics
```

Settle = char('30-May-1997','10-Dec-1997');
Maturity = char('30-Nov-2002','10-Jun-2004');

```

Compute NextCouponDate for this pair of bonds.
```

NextCouponDate = cpndaten(Settle, Maturity);
datestr(NextCouponDate)
ans =
31-May-1997
10-Jun-1998

```

Compute the next quasi coupon dates for these two bonds.
```

NextQuasiCouponDate = cpndatenq(Settle, Maturity);
datestr(NextQuasiCouponDate)
ans =
31-May-1997
10-Jun-1998

```

Because no FirstCouponDate has been specified, the results are identical.

Now supply an explicit FirstCouponDate for each bond.
```

FirstCouponDate = char('30-Nov-1997','10-Dec-1998');

```

Compute the next coupon dates.
```

NextCouponDate = cpndaten(Settle, Maturity, 2, 0, 1, [],...
FirstCouponDate);
datestr(NextCouponDate)
ans =
30-Nov-1997
10-Dec-1998

```

The next coupon dates are identical to the specified first coupon dates.
Now recompute the next quasi coupon dates.
```

NextQuasiCouponDate = cpndatenq(Settle, Maturity, 2, 0, 1, [],...
FirstCouponDate);
datestr(NextQuasiCouponDate)
ans =

```
```

31-May-1997
10-Jun-1998

```

These results illustrate the distinction between actual coupon payment dates and quasi coupon dates. FirstCouponDate (and LastCouponDate, as well), when specified, is associated with an actual coupon payment and also serves as the synchronization date for determining all quasi coupon dates. Since each bond in this example pays semiannual coupons, and the first coupon date occurs more than six months after settlement, each will have an intermediate quasi coupon date before the actual first coupon payment occurs.

See Also accrfrac | cfamounts | cfdates | cftimes | cpncount | cpndaten | cpndatep | cpndatepq | cpndaysn | cpndaysp | cpnpersz

\section*{Purpose}

Previous coupon date for fixed-income security

\author{
Syntax \\ PreviousCouponDate = cpndatep(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)
}

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity. \\
Maturity date. A vector of serial date numbers \\
or date strings. \\
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are \(0,1,2\) \\
(default), \(3,4,6\), and 12.
\end{tabular} \\
Period & (Optional) Day-count basis of the instrument. \\
A vector of integers.
\end{tabular}
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252
\begin{tabular}{ll} 
& \begin{tabular}{l} 
For more information, seebasis on page \\
Glossary-1.
\end{tabular} \\
EndMonthRule & (Optional) End-of-month rule. A vector. \\
This rule applies only when Maturity is an \\
end-of-month date for a month having 30 or \\
fewer days. 0 = ignore rule, meaning that a \\
bond's coupon payment date is always the \\
same numerical day of the month. 1 = set rule \\
on (default), meaning that a bond's coupon \\
payment date is always the last actual day of \\
the month.
\end{tabular}

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must
be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

\author{
Description
}

Examples
PreviousCouponDate = cpndatep(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate) returns the previous coupon date on or before settlement for a portfolio of bonds. This function finds the previous coupon date whether or not the coupon structure is synchronized with the maturity date.
For zero coupon bonds the previous coupon date is the issue date, if available. However, if the issue date is not supplied, the previous coupon date for zero coupon bonds is the previous quasi coupon date calculated as if the frequency is semiannual.

PreviousCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date string.
```

PreviousCouponDate = cpndatep('14 Mar 1997', '30 Jun 2000',...
2, 0, 0);
datestr(PreviousCouponDate)
ans =
30-Dec-1996
PreviousCouponDate = cpndatep('14 Mar 1997', '30 Jun 2000',...
2, 0, 1);
datestr(PreviousCouponDate)
ans =
31-Dec-1996
Maturity = ['30 Apr 2000'; '31 May 2000'; '30 Jun 2000'];
PreviousCouponDate = cpndatep('14 Mar 1997', Maturity);

```
datestr(PreviousCouponDate)
ans =

31-Oct-1996
30-Nov-1996
31-Dec-1996
See Also
accrfrac | cfamounts | cfdates | cftimes | cpncount | cpndaten | cpndatenq | cpndatepq | cpndaysn | cpndaysp|cpnpersz

\section*{Purpose}

Previous quasi coupon date for fixed income security
Syntax \(\quad \begin{aligned} & \text { PreviousQuasiCouponDate }=~ c p n d a t e p q(S e t t l e, ~ M a t u r i t y, ~ P e r i o d, ~\end{aligned}\)
Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate)

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date numbers \\
or date strings.
\end{tabular} \\
Period & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are 0, 1, 2 \\
(default), 3, 4, 6, and 12.
\end{tabular} \\
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the instrument. A \\
vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

IssueDate (Optional) Date when a bond was issued.
FirstCouponDate
(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.

LastCouponDate (Optional) Last coupon date of a bond before the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date.

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars,
or empty matrices. Fill unspecified entries in input vectors with the value NaN. Dates can be serial date numbers or date strings.

\section*{Description}

\section*{Examples}

PreviousQuasiCouponDate = cpndatepq(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate) determines the previous quasi coupon date on or before settlement for a set of NUMBONDS fixed income securities. This function finds the previous quasi coupon date for a bond with a coupon structure in which the first or last period is either normal, short, or long (whether or not the coupon structure is synchronized to maturity). For zero coupon bonds this function returns quasi coupon dates as if the bond had a semiannual coupon structure.

The term "previous quasi coupon date" refers to the previous coupon date for a bond calculated as if no issue date were specified. Although the issue date is not actually a coupon date, when issue date is specified, the previous actual coupon date for a bond is normally calculated as being either the previous coupon date or the issue date, whichever is greater. This function always returns the previous quasi coupon date regardless of issue date. If the settlement date is a coupon date, this function returns the settlement date.

PreviousQuasiCouponDate is returned as a serial date number. The function datestr converts a serial date number to a formatted date string.

Given a pair of bonds with the characteristics
```

Settle = char('30-May-1997','10-Dec-1997');
Maturity = char('30-Nov-2002','10-Jun-2004');

```

With no FirstCouponDate explicitly supplied, compute the PreviousCouponDate for this pair of bonds.
```

PreviousCouponDate = cpndatep(Settle, Maturity);
datestr(PreviousCouponDate)

```
```

ans =
30-Nov-1996
10-Dec-1997

```

Note that since the settlement date for the second bond is also a coupon date, cpndatep returns this date as the previous coupon date.

Now establish a FirstCouponDate and IssueDate for this pair of bonds.
```

FirstCouponDate = char('30-Nov-1997','10-Dec-1998');
IssueDate = char('30-May-1996', '10-Dec-1996');

```

Recompute the PreviousCouponDate for this pair of bonds.
```

PreviousCouponDate = cpndatep(Settle, Maturity, 2, 0, 1, ...
IssueDate, FirstCouponDate);
datestr(PreviousCouponDate)
ans =
30-May-1996
10-Dec-1996

```

Since both of these bonds settled before the first coupon had been paid, cpndatep returns the IssueDate as the PreviousCouponDate.

Using the same data, compute PreviousQuasiCouponDate.
```

PreviousQuasiCouponDate = cpndatepq(Settle, Maturity, 2, 0, 1,..
IssueDate, FirstCouponDate);
datestr(PreviousQuasiCouponDate)
ans =

```
\(30-\) Nov-1996

For the first bond the settlement date is not a normal coupon date. The PreviousQuasiCouponDate is the coupon date before or on the settlement date. Since the coupon structure is synchronized to FirstCouponDate, the previous quasi coupon date is \(30-\) Nov-1996. PreviousQuasiCouponDate disregards IssueDate and FirstCouponDate in this case. For the second bond the settlement date (10-Dec-1997) occurs on a date when a coupon would normally be paid in the absence of an explicit FirstCouponDate. cpndatepq returns this date as PreviousQuasiCouponDate.

See Also
accrfrac | cfamounts | cfdates | cftimes | cpncount | cpndaten | cpndatenq | cpndatep | cpndaysn | cpndaysp | cpnpersz

Purpose Number of days to next coupon date
```

Syntax NumDaysNext = cpndaysn(Settle, Maturity, Period, Basis,
EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate,
StartDate)

```

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date numbers \\
or date strings.
\end{tabular} \\
Period & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are 0, 1, 2 \\
(default), 3, 4, 6, and 12.
\end{tabular} \\
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the instrument. \\
A vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual \(/ 360\)
- 3 = actual \(/ 365\)
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
\begin{tabular}{ll} 
EndMonthRule & \begin{tabular}{l} 
(Optional) End-of-month rule. A vector. \\
This rule applies only when Maturity is an \\
end-of-month date for a month having 30 or \\
fewer days. 0 = ignore rule, meaning that a \\
bond's coupon payment date is always the \\
same numerical day of the month. 1 = set rule \\
on (default), meaning that a bond's coupon \\
payment date is always the last actual day of \\
the month.
\end{tabular} \\
(Optional) Date when a bond was issued. \\
IssueDate & \begin{tabular}{l} 
(Optional) Date when a bond makes its first \\
coupon payment. When FirstCouponDate
\end{tabular} \\
and LastCouponDate are both specified,
\end{tabular}

Required arguments must be number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

\section*{Description}

\section*{Examples}
```

NumDaysNext = cpndaysn('14 Sep 2000', '30 Jun 2001', 2, 0, 0)
NumDaysNext =
1 0 7
NumDaysNext = cpndaysn('14 Sep 2000', '30 Jun 2001', 2, 0, 1)
NumDaysNext =
1 0 8
Maturity = ['30 Apr 2001'; '31 May 2001'; '30 Jun 2001'];
NumDaysNext = cpndaysn('14 Sep 2000', Maturity)
NumDaysNext =

```
    47
    77
    108

See Also
accrfrac | cfamounts | cftimes | cfdates | cpncount | cpndaten | cpndatenq | cpndatep | cpndatepq | cpndaysp | cpnpersz

\section*{Purpose}

Number of days since previous coupon date

\author{
Syntax \\ NumDaysPrevious = cpndaysp(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate)
}

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date numbers \\
or date strings.
\end{tabular} \\
Period & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are 0, 1, 2 \\
(default), 3, 4, 6, and 12.
\end{tabular} \\
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the instrument. \\
A vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\) actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
\begin{tabular}{|c|c|}
\hline EndMonthRule & (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. \\
\hline IssueDate & (Optional) Date when a bond was issued. \\
\hline FirstCouponDate & (Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. \\
\hline LastCouponDate & (Optional) Last coupon date of a bond before the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date. \\
\hline StartDate & (Future implementation) \\
\hline
\end{tabular}
(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
(Optional) Date when a bond was issued.
(Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure.
(Optional) Last coupon date of a bond before the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's date.

StartDate (Future implementation)

Required arguments must be a number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

\section*{Description}

\section*{Examples}

NumDaysPrevious = cpndaysp(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the number of days between the previous coupon date and the settlement date for a bond or set of bonds. When the coupon frequency is 0 (a zero coupon bond), the previous coupon date is calculated as if the frequency were semiannual.
```

NumDaysPrevious = cpndaysp('14 Mar 2000', '30 Jun 2001', 2, 0, 0)
NumDaysPrevious =
7 5
NumDaysPrevious = cpndaysp('14 Mar 2000', '30 Jun 2001', 2, 0, 1)
NumDaysPrevious =
7 4
Maturity = ['30 Apr 2001'; '31 May 2001'; '30 Jun 2001'];
NumDaysPrevious = cpndaysp('14 Mar 2000', Maturity)
NumDaysPrevious =
135
105
7 4
accrfrac | cfamounts | cfdates | cftimes | cpncount | cpndaten | cpndatenq | cpndatep | cpndatepq | cpndaysn | cpnpersz

```

\section*{See Also}

Purpose Number of days in coupon period
Syntax \(\quad \begin{aligned} & \text { NumDaysPeriod = cpnpersz(Settle, Maturity, Period, Basis, } \\ & \text { EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, } \\ & \text { StartDate) }\end{aligned}\)

Arguments
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings. Settle must be \\
earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. A vector of serial date numbers \\
or date strings.
\end{tabular} \\
Period & \begin{tabular}{l} 
(Optional) Coupons per year of the bond. A \\
vector of integers. Allowed values are 0, 1, 2 \\
(default), 3, 4, 6, and 12.
\end{tabular} \\
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the instrument. \\
A vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- \(10=\) actual/365 (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- \(12=\mathrm{actual} / 365\) (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
\begin{tabular}{|c|c|}
\hline EndMonthRule & (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. \\
\hline IssueDate & (Optional) Date when a bond was issued. \\
\hline FirstCouponDate & (Optional) Date when a bond makes its first coupon payment. When FirstCouponDate and LastCouponDate are both specified, FirstCouponDate takes precedence in determining the coupon payment structure. \\
\hline LastCouponDate & (Optional) Last coupon date of a bond before the maturity date. In the absence of a specified FirstCouponDate, a specified LastCouponDate determines the coupon structure of the bond. The coupon structure of a bond is truncated at the LastCouponDate regardless of where it falls and will be followed only by the bond's maturity cash flow date. \\
\hline StartDate & (Future implementation) \\
\hline
\end{tabular}

Required arguments must be a number of bonds (NUMBONDS)-by-1 or 1-by-NUMBONDS conforming vectors or scalars. Optional arguments must be either NUMBONDS-by-1 or 1-by-NUMBONDS conforming vectors, scalars, or empty matrices.

\section*{Description}

\section*{Examples}

NumDaysPeriod = cpnpersz(Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate) returns the number of days in the coupon period containing the settlement date. For zero coupon bonds coupon dates are computed as if the bonds have a semiannual coupon structure.
```

NumDaysPeriod = cpnpersz('14 Sep 2000', '30 Jun 2001', 2, 0, 0)
NumDaysPeriod =
1 8 3
NumDaysPeriod = cpnpersz('14 Sep 2000', '30 Jun 2001', 2, 0, 1)
NumDaysPeriod =
184
Maturity = ['30 Apr 2001'; '31 May 2001'; '30 Jun 2001'];
NumDaysPeriod = cpnpersz('14 Sep 2000', Maturity)
NumDaysPeriod =

```
    184
    183
    184
See Also accrfrac | cfamounts | cfdates | cpncount | cpndaten | cpndatenq
| cpndatep | cpndatepq | cpndaysn | cpndaysp
\begin{tabular}{ll} 
Purpose & Create trading calendars \\
Syntax & \begin{tabular}{l} 
createholidays(Filename, Codefile, InfoFile, TargetDir, \\
IncludeWkds, Wprompt, NoGUI)
\end{tabular}
\end{tabular}

\section*{Arguments}
\begin{tabular}{ll} 
Filename & The data file name. \\
Codefile & The code file name. \\
InfoFile & The info file name. \\
TargetDir & \begin{tabular}{l} 
The target folder where to write the new \\
holidays.m files.
\end{tabular} \\
IncludeWkds & \begin{tabular}{l} 
Option to include weekends in the holiday list. \\
Values are:
\end{tabular}
\end{tabular} Values are:
- 0 - Do not include weekends in the holiday list.
- 1 - Include weekends in the holiday list.

Wprompt

NoGUI

Option to prompt for the file location for each holiday.m file that is created. Values are:
- 0 - Do not prompt for the file location.
- 1 - Prompt for the file location.

Run createholidays without displaying the Trading Calendars graphical user interface. Values are:
- 0 - Display the GUI.
- 1 - Do not display the GUI.

\title{
Description \\ \\ Examples \\ \\ Examples \\ ```
createholidays('FinancialCalendar\My_datafile.csv',... \\ 'FinancialCalendar\My_codesfile.csv',... \\ 'FinancialCalendar\My_infofile.csv','c:\work',1,1,1)
``` \\ will create holidays*.m files from My_datafile.csv in the folder \(\mathrm{c}: \backslash\) work. Weekends will be included in the holidays list based on the input flag INCLUDEWDKS \(=1\).
}

Note To use createholidays, you must obtain data, codes, and info files from http://www. FinancialCalendar.com trading calendars.

\author{
See Also holidays
}

\section*{Purpose}

Cumulative sum
Syntax
newfts \(=\) cumsum(oldfts)
newfts \(=\) cumsum(oldfts) calculates the cumulative sum of each individual time series data series in the financial time series object oldfts and returns the result in another financial time series object newfts. newfts contains the same data series names as oldfts.

Examples
Compute the cumulative sum for Disney stock and plot the results:
```

load disney.mat
cs_dis = cumsum(fillts(dis));
plot(cs_dis)
title('Cumulative Sum for Disney')

```


\section*{cumsum}

See Also cumsum
Purpose Decimal currency values to fractional values
Syntax Fraction = cur2frac(Decimal, Denominator)
Description Fraction = cur2frac(Decimal, Denominator) converts decimalcurrency values to fractional values. Fraction is returned as a string.
Examples Fraction = cur2frac(12.125, 8)
returns Fraction \(=12.1\), a string.
See Also cur2str | frac2cur
Purpose Bank-formatted text
```

Syntax String = cur2str(Value, Digits)

```

Description String = cur2str(Value, Digits) returns the given value in bank format. By default, Digits = 2. A negative Digits rounds the value to the left of the decimal point. String is returned as a string with a leading dollar sign (\$). Negative numbers are displayed in parentheses.

\section*{Examples \\ String = cur2str(-8264, 2)}
returns String \(=(\$ 8264.00)\)
See Also cur2frac | frac2cur

\section*{Purpose Time and frequency from dates}

\author{
Synfax [TFactors, F] = date2time(Settle, Maturity, Compounding, Basis, EndMonthRule)
}

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. A vector of serial date \\
numbers or date strings.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
A vector of serial maturity dates.
\end{tabular} \\
Compounding & \begin{tabular}{l} 
Scalar value representing the rate at which \\
the input zero rates were compounded when \\
annualized. This argument determines the \\
formula for the discount factors:
\end{tabular}
\end{tabular}
- Compounding \(=1,2,3,4,6,12\)
- Disc \(=(1+Z / F)^{\wedge}(-T)\), where \(F\) is the compounding frequency, \(Z\) is the zero rate, and \(T\) is the time in periodic units, for example, \(T=F\) is one year.
- Compounding \(=365\)
- Disc \(=(1+Z / F)^{\wedge}(-T)\), where \(F\) is the number of days in the basis year and \(T\) is a number of days elapsed computed by basis.
- Compounding \(=-1\)
- Disc \(=\exp (-T * Z)\), where \(T\) is time in years.

\section*{date2time}

Basis
(Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
(Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. \(1=\) set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

\title{
Description [TFactors, F] = date2time(Settle, Maturity, Compounding, Basis, EndMonthRule) computes time factors appropriate to compounded rate quotes between the settlement and maturity dates. TFactors is a vector of time factors. \\ \(F\) is a scalar of related compounding frequencies. \\ date2time is the inverse of time2date.
}

See Also cftimes | disc2rate | rate2disc | time2date

\title{
Purpose Convert serial-date axis labels to calendar-date axis labels
}

Syntax dateaxis(Tickaxis, DateForm, StartDate)

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline Tickaxis & (Optional) Determines which axis tick labels- \(x, y\), or \(z\)-to replace. Enter as a string. Default \(=\) ' x '. \\
\hline DateForm & (Optional) Specifies which date format to use. Enter as an integer from 0 to 17. If no DateForm argument is entered, this function determines the date format based on the span of the axis limits. For example, if the difference between the axis minimum and maximum is less than 15 , the tick labels are converted to three-letter day-of-the-week abbreviations (DateForm = 8). See DateForm format descriptions below. \\
\hline StartDate & (Optional) Assigns the date to the first axis tick value. Enter as a string. The tick values are treated as serial date numbers. The default StartDate is the lower axis limit converted to the appropriate date number. For example, a tick value of 1 is converted to the date 01-Jan-0000. Entering StartDate as '06-apr-1999' assigns the date April 6, 1999 to the first tick value and the axis tick labels are set accordingly. \\
\hline
\end{tabular}

\section*{Description}
dateaxis(Tickaxis, DateForm, StartDate) replaces axis tick labels with date labels on a graphic figure.

See the MATLAB set command for information on modifying the axis tick values and other axis parameters.
\begin{tabular}{lll}
\hline DateForm & Format & Description \\
\hline 0 & \(01-\) Mar-1999 & day-month-year \\
& \(15: 45: 17\) & hour:minute:second \\
1 & \(01-\) mar-1999 & day-month-year \\
2 & \(03 / 01 / 99\) & month/day/year \\
3 & Mar & month, three letters \\
4 & M & month, single letter \\
5 & 3 & month \\
6 & \(03 / 01\) & month/day \\
7 & 1 & day of month \\
8 & Wed & day of week, three letters \\
9 & W & day of week, single letter \\
10 & 9999 & year, four digits \\
11 & Mar99 & year, two digits \\
12 & \(15: 45: 17\) & month year \\
13 & \(03: 45: 17\) & PM \\
14 & \(15: 45\) & hour:minute:second \\
15 & \(03: 45\) PM & hour:minute:second AM or PM \\
16 & \(95 / 03 / 01\) & hour:minute \\
17 & & hear:minute AM or PM \\
\hline
\end{tabular}

\section*{Examples dateaxis(' \(x\) ') or dateaxis}
converts the \(x\)-axis labels to an automatically determined date format.
dateaxis('y', 6)

\section*{dateaxis}
converts the \(y\)-axis labels to the month/day format.
```

dateaxis('x', 2, '03/03/1999')

```
converts the \(x\)-axis labels to the month/day/year format. The minimum \(x\)-tick value is treated as March 3, 1999.

See Also
bolling | candle | datenum | datestr | highlow | movavg | pointfig

\section*{Purpose Display date entries}
```

Syntax
datedisp(NumMat, DateForm)
CharMat = datedisp(NumMat, DateForm)

```

\section*{Arguments}
\begin{tabular}{ll} 
NumMat & Numeric matrix to display. \\
DateForm & \begin{tabular}{l} 
(Optional) Date format. See datestr for available \\
and default format flags.
\end{tabular}
\end{tabular}

\section*{Description datedisp(NumMat, DateForm) displays a matrix with the serial} dates formatted as date strings, using a matrix with mixed numeric entries and serial date number entries. Integers between datenum('01-Jan-1900') and datenum('01-Jan-2200') are assumed to be serial date numbers, while all other values are treated as numeric entries.

CharMat is a character array representing NumMat. If no output variable is assigned, the function prints the array to the display.

\section*{Examples}
```

NumMat = [730730, 0.03, 1200 730100;
730731, 0.05, 1000 NaN]
NumMat =
1.0e+05 *

| 7.3073 | 0.0000 | 0.0120 | 7.3010 |
| ---: | ---: | ---: | ---: |
| 7.3073 | 0.0000 | 0.0100 | NaN |

datedisp(NumMat)
01-Sep-2000 0.03 1200 11-Dec-1998

```
\[
02 \text {-Sep-2000 } 0.05 \quad 1000 \quad \mathrm{NaN}
\]

\section*{See Also \\ datenum | datestr}
\(\begin{array}{ll}\text { Purpose } & \text { Indices of date numbers in matrix } \\ \text { Syntax } & \text { Indices = datefind(Subset, Superset, Tolerance) }\end{array}\)
Arguments

\section*{Description}

\section*{Examples}
```

Superset = datenum(1999, 7, 1:31);
Subset = [datenum(1999, 7, 10); datenum(1999, 7, 20)];
Indices = datefind(Subset, Superset, 1)
Indices =

```

\section*{Purpose}

Date of day in future or past month
Syntax TargetDate = datemnth(StartDate, NumberMonths, DayFlag, Basis, EndMonthRule)

\section*{Arguments}
\begin{tabular}{ll} 
StartDate & \begin{tabular}{l} 
Enter as serial date numbers or date strings. \\
NumberMonths \\
DayFlag \\
(positive) or past (negative). Values must be \\
in integer form. \\
(Optional) Vector containing values that specify \\
how the actual day number for the target \\
date in future or past month is determined. 0 \\
(default) = day number should be the day in \\
the future or past month corresponding to the \\
actual day number of the start date. \(1=\) day \\
number should be the first day of the future or \\
past month. 2 = day number should be the last \\
day of the future or past month.
\end{tabular} \\
This flag has no effect if EndMonthRule is set \\
to 1.
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- \(2=\) actual/ 360
- \(3=\) actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
EndMonthRule (Optional) End-of-month rule. A vector. \(1=\) rule in effect, meaning that if you are beginning on the last day of a month, and the month has 30 or fewer days, you will end on the last actual day of the future or past month regardless of whether that month has \(28,29,30\) or 31 days) \(0=\) rule off (default), meaning that the rule is not in effect.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then NumberMonths must be an \(n\)-by- 1 vector of integers or a single integer. TargetDate is then an \(n\)-by- 1 vector of date numbers.

\section*{Description}

TargetDate = datemnth(StartDate, NumberMonths, DayFlag, Basis, EndMonthRule) returns the serial date number of the target date in the future or past.

Use datestr to convert serial date numbers to formatted date strings.

\section*{Examples}
```

Day = datemnth('3 jun 2001', 6, 0, 0, 0)
Day =
7 3 1 1 8 8
datestr(Day)
ans =
03-Dec-2001
Day = datemnth('3 jun 2001', 6, 1, 0, 1); datestr(Day)
ans =
01-Dec-2001
Day = datemnth('31 jan 2001', 5, 0, 0, 0); datestr(Day)
ans =
30-Jun-2001
Day = datemnth('31 jan 2001', 5, 1, 0, 0); datestr(Day)
ans =
01-Jun-2001
Day = datemnth('31 jan 2001', 5, 1, 0, 1); datestr(Day)
ans =
30-Jun-2001
Day = datemnth('31 jan 2001', 5, 2, 0, 1); datestr(Day)
ans =
30-Jun-2001
Months = [1; 3; 5; 7; 9];
Day = datemnth('31 jan 2001', Months); datestr(Day)
ans =
28-Feb-2001
30-Apr-2001
30-Jun-2001
31-Aug-2001
31-0ct-2001

```

See Also
datestr | datevec | days360 | days365 | daysact | daysdif | wrkdydif

\section*{Purpose Create date number}

Syntax
```

N = datenum(V)
N = datenum(S, F)
N = datenum(S, F, P)
N = datenum([S, P, F])
N = datenum(Y, M, D)
N = datenum(Y, M, D, H, MN, S)
N = datenum(S)
N = datenum(S, P)

```

\section*{Description}
datenum is one of three conversion functions that enable you to express dates and times in any of three formats in MATLAB software: a string (or date string), a vector of date and time components (or date vector), or as a numeric offset from a known date in time (or serial date number). Here is an example of a date and time expressed in the three MATLAB formats:


A serial date number represents the whole and fractional number of days from a specific date and time, where datenum( 'Jan-1-0000 \(00: 00: 00\) ') returns the number 1 . (The year 0000 is merely a reference point and is not intended to be interpreted as a real year in time.)
\(\mathrm{N}=\) datenum( V ) converts one or more date vectors V to serial date numbers N . Input V can be an m -by- 6 or m -by- 3 matrix containing m full or partial date vectors respectively. A full date vector has six elements, specifying year, month, day, hour, minute, and second, in that order. A partial date vector has three elements, specifying year, month, and day, in that order. Each element of \(V\) must be a positive double-precision number. datenum returns a column vector of \(m\) date numbers, where \(m\) is the total number of date vectors in V .
\(\mathrm{N}=\) datenum(S, F) converts one or more date strings \(S\) to serial date numbers N using format string F to interpret each date string. Input S
can be a one-dimensional character array or cell array of date strings. All date strings in S must have the same format, and that format must match one of the date string formats shown in the help for the datestr function. datenum returns a column vector of \(m\) date numbers, where \(m\) is the total number of date strings in S. MATLAB software considers date string years that are specified with only two characters (e.g., '79') to fall within 100 years of the current year.

See the datestr reference page to find valid string values for \(F\). These values are listed in Table 1 in the column labeled "Dateform String." You can use any string from that column except for those that include the letter \(Q\) in the string (for example, 'QQ-YYYY'). Certain formats may not contain enough information to compute a date number. In these cases, hours, minutes, seconds, and milliseconds default to 0 , the month defaults to January, the day to 1 , and the year to the current year.
\(N=\) datenum(S, F, P) converts one or more date strings \(S\) to date numbers \(N\) using format \(F\) and pivot year \(P\). The pivot year is used in interpreting date strings that have the year specified as two characters. It is the starting year of the 100 -year range in which a two-character date string year resides. The default pivot year is the current year minus 50 years.
\(\mathrm{N}=\) datenum([S, P, F]) is the same as the syntax shown above, except the order of the last two arguments are switched.
\(\mathrm{N}=\operatorname{datenum}(\mathrm{Y}, \mathrm{M}, \mathrm{D})\) returns the serial date numbers for corresponding elements of the \(\mathrm{Y}, \mathrm{M}\), and D (year, month, day) arrays. \(Y, M\), and \(D\) must be arrays of the same size (or any can be a scalar) of type double. You can also specify the input arguments as a date vector, [Y M D].

For this and the following syntax, values outside the normal range of each array are automatically carried to the next unit. Values outside the normal range of each array are automatically carried to the next unit. For example, month values greater than 12 are carried to years. Month values less than 1 are set to be 1 . All other units can wrap and have valid negative values.
\(\mathrm{N}=\operatorname{datenum}(\mathrm{Y}, \mathrm{M}, \mathrm{D}, \mathrm{H}, \mathrm{MN}, \mathrm{S})\) returns the serial date numbers for corresponding elements of the Y, M, D, H, MN, and S (year, month, day, hour, minute, and second) array values. datenum does not accept milliseconds in a separate input, but as a fractional part of the seconds (S) input. Inputs Y, M, D, H, MN, and S must be arrays of the same size (or any can be a scalar) of type double. You can also specify the input arguments as a date vector, [Y M D H MN S].
\(\mathrm{N}=\) datenum(S) converts date string \(S\) into a serial date number. String \(S\) must be in one of the date formats \(0,1,2,6,13,14,15,16\), or 23 , as defined in the reference page for the datestr function. MATLAB software considers date string years that are specified with only two characters (e.g., ' 79 ') to fall within 100 years of the current year. If the format of date string \(S\) is known, use the syntax \(N=\) datenum ( \(S, F\) ).
\(N=\) datenum (S, P) converts date string S, using pivot year P. If the format of date string \(S\) is known, use the syntax \(N=\) datenum(S, \(F\), P).

Note The last two calling syntaxes are provided for backward compatibility and are significantly slower than the syntaxes that include a format argument F.

\section*{Examples Convert a date string to a serial date number:}
```

n = datenum('19-May-2001', 'dd-mmm-yyyy')
n =

```
    730990

Specifying year, month, and day, convert a date to a serial date number:
```

n = datenum(2001, 12, 19)

```
\(\mathrm{n}=\)

731204

Convert a date vector to a serial date number:
```

format bank
datenum('March 28, 2005 3:37:07.033 PM')
ans =
732399.65

```

Convert a date string to a serial date number using the default pivot year:
```

n = datenum('12-jun-17', 'dd-mmm-yy')
n =
736858

```

Convert the same date string to a serial date number using 1400 as the pivot year:
```

n = datenum('12-jun-17', 'dd-mmm-yy', 1400)
n =
5 1 7 7 1 2

```

Specify format 'dd.mm.yyyy' to be used in interpreting a nonstandard date string:
```

n = datenum('19.05.2000', 'dd.mm.yyyy')
n =

```
    730625
datedisp | datestr | datevec | daysact | now | today
\begin{tabular}{|c|c|}
\hline Purpose & Create date string \\
\hline \multirow[t]{5}{*}{Syntax} & S = datestr (V) \\
\hline & S = datestr (N) \\
\hline & \(S=\operatorname{datestr}(\mathrm{D}, \mathrm{F})\) \\
\hline & S = datestr (S1, F, P) \\
\hline & S = datestr(..., 'local') \\
\hline
\end{tabular}

Description
datestr is one of three conversion functions that enable you to express dates and times in any of three formats in MATLAB software: a string (or date string), a vector of date and time components (or date vector), or as a numeric offset from a known date in time (or serial date number). Here is an example of a date and time expressed in the three MATLAB formats:


A serial date number represents the whole and fractional number of days from 1-Jan-0000 to a specific date. The year 0000 is merely a reference point and is not intended to be interpreted as a real year in time.
\(\mathrm{S}=\) datestr \((\mathrm{V})\) converts one or more date vectors V to date strings S . Input V must be an m -by- 6 matrix containing m full (six-element) date vectors. Each element of \(V\) must be a positive double-precision number. datestr returns a column vector of \(m\) date strings, where \(m\) is the total number of date vectors in V .
\(S\) = datestr( \(N\) ) converts one or more serial date numbers \(N\) to date strings S . Input argument N can be a scalar, vector, or multidimensional array of positive double-precision numbers. datestr returns a column vector of \(m\) date strings, where \(m\) is the total number of date numbers in N .

S = datestr(D, F) converts one or more date vectors, serial date numbers, or date strings \(D\) into the same number of date strings \(S\). Input
argument \(F\) is a format number or string that determines the format of the date string output. Valid values for \(F\) are given in the table Standard MATLAB \({ }^{\circledR}\) Date Format Definitions on page 17-252, below. Input F may also contain a free-form date format string consisting of format tokens shown in the table Free-Form Date Format Specifiers on page 17-255, below.

Date strings with 2 -character years are interpreted to be within the 100 years centered around the current year.

S = datestr(S1, F, P) converts date string S1 to date string S, applying format \(F\) to the output string, and using pivot year \(P\) as the starting year of the 100-year range in which a two-character year resides. The default pivot year is the current year minus 50 years. All date strings in S 1 must have the same format.

S = datestr(..., 'local') returns the string in a localized format. The default is US English ('en_US'). This argument must come last in the argument sequence.

Note The vectorized calling syntax can offer significant performance improvement for large arrays.

\section*{Standard MATLAB Date Format Definitions}
\begin{tabular}{l|l|l}
\hline \begin{tabular}{l} 
dateform \\
(number)
\end{tabular} & dateform (string) & Example \\
\hline 0 & \begin{tabular}{l} 
'dd-mmm-yyyy \\
HH:MM:SS'
\end{tabular} & 01 -Mar-2000 15:45:17 \\
\hline 1 & 'dd-mmm-yyyy ' & 01 -Mar-2000 \\
\hline 2 & 'mm/dd/yy' & \(03 / 01 / 00\) \\
\hline 3 & \(' \mathrm{mmm}\) & Mar \\
\hline 4 & \(' \mathrm{~m}^{\prime}\) & M \\
\hline
\end{tabular}

Standard MATLAB Date Format Definitions (Continued)
\begin{tabular}{|c|c|c|}
\hline dateform (number) & dateform (string) & Example \\
\hline 5 & 'mm' & 03 \\
\hline 6 & 'mm/dd ' & 03/01 \\
\hline 7 & ' dd ' & 01 \\
\hline 8 & 'ddd ' & Wed \\
\hline 9 & 'd' & W \\
\hline 10 & 'yyyy ' & 2000 \\
\hline 11 & 'yy' & 00 \\
\hline 12 & 'mmmyy ' & Mar00 \\
\hline 13 & 'HH:MM:SS' & 15:45:17 \\
\hline 14 & 'HH:MM:SS PM' & 3:45:17 PM \\
\hline 15 & 'HH:Mm ' & 15:45 \\
\hline 16 & 'HH:MM PM' & 3:45 PM \\
\hline 17 & 'QQ-YY' & Q1-01 \\
\hline 18 & 'QQ' & Q1 \\
\hline 19 & 'dd/mm' & 01/03 \\
\hline 20 & 'dd/mm/yy ' & 01/03/00 \\
\hline 21 & \[
\begin{aligned}
& \text { 'mmm.dd, yyyy } \\
& \text { HH:MM:SS' }
\end{aligned}
\] & Mar.01,2000 15:45:17 \\
\hline 22 & 'mmm.dd, yyyy' & Mar.01,2000 \\
\hline 23 & 'mm/dd/yyyy' & 03/01/2000 \\
\hline 24 & 'dd/mm/yyyy' & 01/03/2000 \\
\hline 25 & 'yy/mm/dd' & 00/03/01 \\
\hline 26 & 'yyyy/mm/dd' & 2000/03/01 \\
\hline
\end{tabular}

Standard MATLAB Date Format Definitions (Continued)
\begin{tabular}{l|l|l}
\hline \begin{tabular}{l} 
dateform \\
(number)
\end{tabular} & dateform (string) & Example \\
\hline 27 & 'QQ-YYYY' & Q1-2001 \\
\hline 28 & 'mmmyyyy' & Mar2000 \\
\hline \begin{tabular}{l}
29 (ISO \\
8601 )
\end{tabular} & 'yyyy-mm-dd' & \(2000-03-01\) \\
\hline \begin{tabular}{l}
30 (ISO \\
8601 )
\end{tabular} & 'yyyymmddTHHMMSS' & \(20000301 \mathrm{T154517}\) \\
\hline 31 & 'yyyy-mm-dd HH:MM:SS' & \(2000-03-01 \quad 15: 45: 17\) \\
\hline
\end{tabular}

Note dateform numbers \(0,1,2,6,13,14,15,16\), and 23 produce a string suitable for input to datenum or datevec. Other date string formats do not work with these functions unless you specify a date form in the function call.

Note For date formats that specify only a time (for example., dateform numbers \(13,14,15\), and 16), MATLAB software sets the date to January 1 of the current year.

Time formats like 'h:m:s', 'h:m:s.s','h:m pm', ... can also be part of the input array S. If you do not specify a format string \(F\), or if you specify F as -1 , the date string format defaults to the following:

If \(S\) contains date information only, for example, 01-Mar-1995
16 If S contains time information only, for example, 03:45 PM

0
If \(S\) is a date vector, or a string that contains both date and time information, for example, 01-Mar-1995 03:45

The following table shows the string symbols to use in specifying a free-form format for the output date string. MATLAB interprets these symbols according to your computer's language setting and the current MATLAB language setting.

Note You cannot use more than one format specifier for any date or time field. For example, datestr( n , 'dddd dd mmm') specifies two formats for the day of the week, and thus returns an error.

\section*{Free-Form Date Format Specifiers}
\begin{tabular}{l|l|l}
\hline Symbol & Interpretation & Example \\
\hline yyyy & Show year in full. & 1990, 2002 \\
\hline yy & Show year in two digits. & 90,02 \\
\hline mmmm & \begin{tabular}{l} 
Show month using full \\
name.
\end{tabular} & March, December \\
\hline mmm & \begin{tabular}{l} 
Show month using first \\
three letters.
\end{tabular} & Mar, Dec \\
\hline mm & Show month in two digits. & 03, 12 \\
\hline m & \begin{tabular}{l} 
Show month using \\
capitalized first letter.
\end{tabular} & M, D \\
\hline dddd & Show day using full name. & Monday, Tuesday \\
\hline
\end{tabular}

Free-Form Date Format Specifiers (Continued)
\begin{tabular}{l|l|l}
\hline Symbol & Interpretation & Example \\
\hline ddd & \begin{tabular}{l} 
Show day using first \\
three letters.
\end{tabular} & Mon, Tue \\
\hline dd & Show day in two digits. & 05,20 \\
\hline d & \begin{tabular}{l} 
Show day using \\
capitalized first letter.
\end{tabular} & M, T \\
\hline HH & \begin{tabular}{l} 
Show hour in two digits \\
(no leading zeros when \\
free-form specifier AM or \\
PM is used (see last entry \\
in this table)).
\end{tabular} & 05,5 AM \\
\hline MM & \begin{tabular}{l} 
Show minute in two \\
digits.
\end{tabular} & 12,02 \\
\hline SS & Show second in two digits. & 07,59 \\
\hline FFF & \begin{tabular}{l} 
Show millisecond in three \\
digits.
\end{tabular} & .057 \\
\hline AM or PM & \begin{tabular}{l} 
Append AM or PM to date \\
string (see note below).
\end{tabular} & \(3: 45: 02\) PM \\
\hline
\end{tabular}

Note Free-form specifiers AM and PM from the table above are identical. They do not influence which characters are displayed following the time (AM versus PM), but only whether or not they are displayed. MATLAB software selects AM or PM based on the time entered.

Tips
A vector of three or six numbers could represent either a single date vector, or a vector of individual serial date numbers. For example, the vector [2000 1215114503 ] could represent either 11:45:03 on December 15, 2000 or a vector of date numbers \(2000,12,15\), and so
on. MATLAB uses the following general rule in interpreting vectors associated with dates:
- A 3- or 6 -element vector having a first element within an approximate range of 500 greater than or less than the current year is considered by MATLAB to be a date vector. Otherwise, it is considered to be a vector of serial date numbers.

To specify dates outside of this range as a date vector, first convert the vector to a serial date number using the datenum function as shown here:
```

datestr(datenum([[1400 12 15 11 45 03]), ...
'mmm.dd,yyyy HH:MM:SS')
ans =
Dec.15,1400 11:45:03

```

\section*{Examples}

Return the current date and time in a string using the default format, 0 :
```

datestr(now)
ans =
28-Mar-2005 15:36:23

```

Reformat the date and time, and also show milliseconds:
```

dt = datestr(now, 'mmmm dd, yyyy HH:MM:SS.FFF AM')
dt =
March 28, 2005 3:37:07.952 PM

```

Format the same showing only the date and in the \(\mathrm{mm} / \mathrm{dd} / \mathrm{yy}\) format. Note that you can specify this format either by number or by string.
```

datestr(now, 2) -or- datestr(now, 'mm/dd/yy')

```
```

ans =

```

03/28/05

Display the returned date string using your own format made up of symbols shown in the Free-Form Date Format Specifiers on page 17-255 table above.
```

datestr(now, 'dd.mm.yyyy')
ans =
28.03.2005

```

Convert a nonstandard date form into a standard MATLAB date form by first converting to a date number and then to a string:
```

datestr(datenum('28.03.2005', 'dd.mm.yyyy'), 2)
ans =
03/28/05

```

See Also
dateaxis | datedisp | datenum | datevec | daysact | now | today
```

Purpose Date components
Syntax }\quadV=\operatorname{datevec}(N
Date components

```
```

V = datevec(S, F)

```
V = datevec(S, F)
V = datevec(S, F, P)
V = datevec(S, F, P)
V = datevec(S, P, F)
V = datevec(S, P, F)
[Y, M, D, H, MN, S] = datevec(...)
[Y, M, D, H, MN, S] = datevec(...)
V = datevec(S)
V = datevec(S)
V = datevec(S, P)
```

V = datevec(S, P)

```

\section*{Description}
datevec is one of three conversion functions that enable you to express dates and times in any of three formats in MATLAB software: a string (or date string), a vector of date and time components (or date vector), or as a numeric offset from a known date in time (or serial date number). Here is an example of a date and time expressed in the three MATLAB formats:


A serial date number represents the whole and fractional number of days from 1-Jan-0000 to a specific date. The year 0000 is merely a reference point and is not intended to be interpreted as a real year in time.
\(\mathrm{V}=\) datevec \((\mathrm{N})\) converts one or more date numbers N to date vectors V . Input argument N can be a scalar, vector, or multidimensional array of positive date numbers. datevec returns an \(m\)-by- 6 matrix containing \(m\) date vectors, where \(m\) is the total number of date numbers in \(N\).
\(\mathrm{V}=\) datevec (S, F) converts one or more date strings \(S\) to date vectors V using format string F to interpret the date strings in S . Input argument \(S\) can be a cell array of strings or a character array where each row corresponds to one date string. All of the date strings in \(S\) must have the same format which must be composed of date format symbols according to the table "Free-Form Date Format Specifiers" in the datestr help.

Formats with 'Q' are not accepted by datevec. datevec returns an \(m\)-by- 6 matrix of date vectors, where \(m\) is the number of date strings in \(S\).
Certain formats may not contain enough information to compute a date vector. In those cases, hours, minutes, and seconds default to 0 , days default to 1 , months default to January, and years default to the current year. Date strings with two character years are interpreted to be within the 100 years centered around the current year.
\(\mathrm{V}=\) datevec \((\mathrm{S}, \mathrm{F}, \mathrm{P})\) converts the date string S to a date vector V using date format \(F\) and pivot year \(P\). The pivot year is the starting year of the 100 -year range in which a two-character year resides. The default pivot year is the current year minus 50 years.
\(V=\) datevec (S, P, F) is the same as the syntax shown above, except the order of the last two arguments are switched.
[Y, M, D, H, MN, S] = datevec (...) takes any of the two syntaxes shown above and returns the components of the date vector as individual variables. datevec does not return milliseconds in a separate output, but as a fractional part of the seconds (S) output.
\(\mathrm{V}=\) datevec \((\mathrm{S})\) converts date string \(S\) to date vector V . Input argument \(S\) must be in one of the date formats \(0,1,2,6,13,14,15,16\), or 23 as defined in the reference page for the datestr function. This calling syntax is provided for backward compatibility, and is significantly slower than the syntax which specifies the format string. If the format is known, the \(V=\) datevec ( \(\mathrm{S}, \mathrm{F}\) ) syntax is recommended.
\(\mathrm{V}=\) datevec \((\mathrm{S}, \mathrm{P})\) converts the date string S using pivot year \(P\). If the format is known, the \(V=\operatorname{datevec}(S, F, P)\) or \(V=\) datevec (S, \(P\), F) syntax should be used.

Note If more than one input argument is used, the first argument must be a date string or array of date strings.

When creating your own date vector, you need not make the components integers. Any components that lie outside their conventional ranges
affect the next higher component (so that, for instance, the anomalous June 31 becomes July 1). A zeroth month, with zero days, is allowed.

Note The vectorized calling syntax can offer significant performance improvement for large arrays.

\section*{Examples Obtain a date vector using a string as input:}
```

format short g
datevec('March 28, 2005 3:37:07.952 PM')
ans =
2005 3 3 28

```

Obtain a date vector using a serial date number as input:
```

t = datenum('March 28, 2005 3:37:07.952 PM')
t =
7.324e+005
datevec(t)
ans =
2005

```

Assign elements of the returned date vector:
```

[y, m, d, h, mn, s] = datevec('March 28, 2005 3:37:07.952 PM');
sprintf('Date: %d/%d/%d Time: %d:%d:%2.3f\n', m, d, y, h, mn, s)
ans =
Date: 3/28/2005 Time: 15:37:7.952

```

Use free-form date format 'dd.mm.yyyy' to indicate how you want a nonstandard date string interpreted:
```

datevec('28.03.2005', 'dd.mm.yyyy')
ans = 2005 3 28 0 0 0

```

See Also
datenum | datestr | now | today

\section*{Purpose}

Date of future or past workday
Syntax EndDate = datewrkdy(StartDate, NumberWorkDays, NumberHolidays)

\section*{Arguments}

\section*{Description}

\section*{Examples}
\begin{tabular}{ll} 
StartDate & \begin{tabular}{l} 
Start date vector. Enter as serial date numbers \\
or date strings.
\end{tabular} \\
NumberWorkDays & \begin{tabular}{l} 
Vector containing number of work or business \\
days in future (positive) or past (negative), \\
including the starting date.
\end{tabular} \\
NumberHolidays & \begin{tabular}{l} 
Vector containing values for the number \\
of holidays within NumberWorkDays. \\
NumberHolidays and NumberWorkDays must \\
have the same sign.
\end{tabular}
\end{tabular}

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then NumberWorkDays must be an \(n\)-by- 1 vector of integers or a single integer. EndDate is then an \(n\)-by- 1 vector of date numbers.

EndDate = datewrkdy(StartDate, NumberWorkDays, NumberHolidays) returns the serial number of the date a given number of workdays before or after the start date.
Use datestr to convert serial date numbers to formatted date strings.
```

Workday = datewrkdy('12-dec-2000', 16, 2);
datestr(Workday)
ans =
04-Jan-2001
NumDays = [16; 20; 44];

```
```

Workdays = datewrkdy('12-dec-2000', NumDays, 2);
datestr(Workdays)
ans =
4-Jan-2001
10-Jan-2001
13-Feb-2001

```

See Also busdate | holidays | isbusday | wrkdydif
Purpose Day of month
Syntax DayMonth = day(Date)
DayMonth = day(Date, F)
Description DayMonth \(=\) day (Date) returns the day of the month given a serialdate number or date string.DayMonth = day (Date, F) returns the day of the of the month, given aserial date number or date string, in a specified date format.
Examples DayMonth = day(730544)
or
DayMonth = day('2/28/00')
returns DayMonth = 28You can also use the F argument to designate a country-specific dateformat:
```

    DayMonth = day('28/02/00','dd/mm/yyyy')
    returns DayMonth = 28

```
See Also datevec | eomday | month | year
Purpose Days between dates based on 360-day year
```

Syntax NumDays = days360(StartDate, EndDate)

```

\section*{Arguments}
\[
\begin{array}{ll}
\text { StartDate } & \text { Enter as serial date numbers or date strings. } \\
\text { EndDate } & \text { Enter as serial date numbers or date strings. }
\end{array}
\]

Either input can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then EndDate must be an \(n\)-by- 1 vector of integers or a single integer. NumDays is then an \(n\)-by- 1 vector of date numbers.

\section*{Description NumDays = days360(StartDate, EndDate) returns the number of days between StartDate and EndDate based on a 360 -day year (that is, all months contain 30 days). If EndDate is earlier than StartDate, NumDays is negative.}

\section*{Examples}
```

NumDays = days360('15-jan-2000', '15-mar-2000')
NumDays =
6 0
MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days360('15-jan-2000', MoreDays)
NumDays =

```
    60
    90

\author{
References Addendum to Securities Industry Association, Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures, Vol. 2, Spring 1995.
}

\author{
See Also \\ days365 | daysact | daysdif | wrkdydif | yearfrac
}

\title{
Purpose Days between dates based on 360-day year (European)
}

Syntax NumDays = days360e(StartDate, EndDate)
Arguments

\section*{Description}

\section*{Examples}
\begin{tabular}{ll} 
StartDate & \begin{tabular}{l} 
Row vector, column vector, or scalar value in serial \\
date number or date string format.
\end{tabular} \\
EndDate & \begin{tabular}{l} 
Row vector, column vector, or scalar value in serial \\
date number or date string format.
\end{tabular}
\end{tabular}

Either input can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all.

NumDays = days360e(StartDate, EndDate) returns a vector or scalar value representing the number of days between StartDate and EndDate based on a 360 -day year (that is, all months contain 30 days). If EndDate is earlier than StartDate, NumDays is negative.

This day count convention is used primarily in Europe. Under this convention all months contain 30 days.

Example 1. Use this convention to find the number of days in the month of January.
```

StartDate = '1-Jan-2002';
EndDate = '1-Feb-2002';
NumDays = days360e(StartDate, EndDate)
NumDays =

```

Example 2. Use this convention to find the number of days in February during a leap year.
```

StartDate = '1-Feb-2000';
EndDate = '1-Mar-2000';
NumDays = days360e(StartDate, EndDate)
NumDays =

```
30

Example 3. Use this convention to find the number of days in February of a non-leap year.
```

StartDate = '1-Feb-2002';
EndDate = '1-Mar-2002';
NumDays = days360e(StartDate, EndDate)
NumDays =

```
30

See Also
days360 | days360isda | days360psa
\begin{tabular}{ll} 
Purpose & \begin{tabular}{l} 
Days between dates based on 360-day year (International Swap Dealer \\
Association (ISDA) compliant)
\end{tabular} \\
Syntax & NumDays = days360isda(StartDate, EndDate)
\end{tabular}

\section*{Arguments}

\section*{Description NumDays = days360isda(StartDate, EndDate) returns a vector or scalar value representing the number of days between StartDate and EndDate based on a 360 -day year (that is, all months contain 30 days). If EndDate is earlier than StartDate, NumDays is negative.}

Under this convention all months contain 30 days.

\section*{Examples}

Example 1. Use this convention to find the number of days in the month of January.
```

StartDate = '1-Jan-2002';
EndDate = '1-Feb-2002';
NumDays = days360isda(StartDate, EndDate)
NumDays =

```
30

Example 2. Use this convention to find the number of days in February during a leap year.
```

StartDate = '1-Feb-2000';
EndDate = '1-Mar-2000';
NumDays = days360isda(StartDate, EndDate)
NumDays =

```
30

Example 3. Use this convention to find the number of days in February of a non leap year.
```

StartDate = '1-Feb-2002';
EndDate = '1-Mar-2002';
NumDays = days360isda(StartDate, EndDate)
NumDays =

```
30

See Also days360 | days360e | days360psa
\begin{tabular}{ll} 
Purpose & \begin{tabular}{l} 
Days between dates based on 360-day year (Public Securities \\
Association (PSA) compliant)
\end{tabular} \\
Syntax & NumDays = days360psa(StartDate, EndDate)
\end{tabular}

\section*{Arguments}

Either input can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all.

\section*{Description}

\section*{Examples}

NumDays = days360psa(StartDate, EndDate) returns a vector or scalar value representing the number of days between StartDate and EndDate based on a 360 -day year (that is, all months contain 30 days). If EndDate is earlier than StartDate, NumDays is negative.

Under this payment convention all months contain 30 days. In both leap and non-leap years, if the StartDate is the last day of February, this day is considered to be day 30 of the month.

Example 1. Use this convention to find the number of days in between the last day of February and the first day of March during a leap year.
```

StartDate = '29-Feb-2000';
EndDate = '1-Mar-2000';
NumDays = days360psa(StartDate, EndDate)
NumDays =

```
    1

Example 2. Use this convention to find the number of days in between the last day of February and the first day of March during a non-leap year.
```

StartDate = '28-Feb-2002';
EndDate = '1-Mar-2002';
NumDays = days360psa(StartDate, EndDate)
NumDays =

```
    1

As expected, the number of days in both cases is the same. The convention always assumes that the last day of February is the 30th day.

See Also days360 | days360e | days360isda

\title{
Purpose Days between dates based on 365-day year
}
```

Syntax NumDays = days365(StartDate, EndDate)

```

\section*{Arguments}

StartDate Enter as serial date numbers or date strings.
EndDate Enter as serial date numbers or date strings.

Either input can contain multiple values, but if so, the other must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then EndDate must be an \(n\)-by- 1 vector of integers or a single integer. NumDays is then an \(n\)-by- 1 vector of date numbers.

\section*{Description NumDays \(=\) days365(StartDate, EndDate) returns the number of days between dates StartDate and EndDate based on a 365 -day year. (All months contain their actual number of days. February always contains 28 days.) If EndDate is earlier than StartDate, NumDays is negative. Enter dates as serial date numbers or date strings.}

\section*{Examples}
```

NumDays = days365('15-jan-2000', '15-mar-2000')
NumDays =
5 9
MoreDays = ['15-mar-2000'; '15-apr-2000'; '15-jun-2000'];
NumDays = days365('15-jan-2000', MoreDays)
NumDays =

```
    59

See Also days360 | daysact | daysdif | wrkdydif | yearfrac

Purpose Actual number of days between dates
Syntax NumDays = daysact (StartDate, EndDate)

\section*{Arguments}

\section*{Description}

\section*{Examples}

StartDate Enter as serial date numbers or date strings.
EndDate (Optional) Enter as serial date numbers or date strings.

Either input can contain multiple values, but if so, the other must then EndDate must be an \(n\)-row character array of date strings or a single date. NumDays is then an \(n\)-by- 1 vector of numbers. strings. NumDays is negative if EndDate is earlier than StartDate.

NumDays = daysact(StartDate) returns the actual number of days the base date 1 is 1-Jan-0000 A.D. See datenum for a similar function. contain the same number of values or a single value that applies to all. For example, if StartDate is an \(n\)-row character array of date strings,

NumDays = daysact(StartDate, EndDate) returns the actual number of days between two dates. Enter dates as serial date numbers or date between the MATLAB base date and StartDate. In MATLAB software,
```

NumDays = daysact('7-sep-2002', '25-dec-2002')
NumDays =
1 0 9
NumDays = daysact('9/7/2002')
NumDays =
731466
MoreDays = ['09/07/2002'; '10/22/2002'; '11/05/2002'];
NumDays = daysact(MoreDays, '12/25/2002')

```

NumDays = 109
64 50

\section*{See Also}
datenum | datevec | days360 | days365 | daysdif

Purpose Date away from starting date for any day-count basis
Syntax NumDays = daysadd(StartDate, NumDays, Basis)

\section*{Arguments}

StartDate Start date. Enter as serial date numbers or date strings.

NumDays Integer number of days from start date. Enter a negative integer for dates before start date.

Basis (Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

Note When using the 30/360 day-count basis, it is not always possible to find the exact date NumDays number of days away because of a known discontinuity in the method of counting days. A warning is displayed if this occurs.

\section*{Description NumDays = daysadd(StartDate, NumDays, Basis) returns a date NumDays number of days away from StartDate, using the given day-count basis.}

\section*{Examples}
```

NewDate = daysadd('01-Feb-2004', 31)
NewDate =

```
    732009
datestr(NewDate)
ans =
03-Mar-2004
NewDate = daysadd('01-Feb-2004', 31, 1)
NewDate =
    732008
datestr(NewDate)
ans =

\section*{References Stigum, Marcia L. and Franklin Robinson, Money Market and Bond Calculations, Richard D. Irwin, 1996, ISBN 1-55623-476-7}

\section*{See Also \\ daysdif}

\section*{Purpose}

Days between dates for any day-count basis
Syntax NumDays = daysdif(StartDate, EndDate, Basis)

Arguments
StartDate Enter as serial date numbers or date strings.
EndDate Enter as serial date numbers or date strings.
Basis (Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (PSA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ISMA)
- \(9=\) actual/360 (ISMA)
- \(10=\mathrm{actual} / 365\) (ISMA)
- \(11=30 / 360 \mathrm{E}\) (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

Any input argument can contain multiple values, but if so, the other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an n-row character array of date strings, then EndDate must be an n-row character array of date strings or a single date. NumDays is then an \(n\)-by- 1 vector of numbers.

\section*{Description}

\section*{Examples}

\section*{References}

See Also

NumDays = daysdif(StartDate, EndDate, Basis) returns the number of days between dates StartDate and EndDate using the given day-count basis. Enter dates as serial date numbers or date strings. Enter dates as serial date numbers or date strings. The first date (StartDate) is not included when determining the number of days between first and last date.

This function is a helper function for the bond pricing and yield functions. It is designed to make the code more readable and to eliminate redundant calls within if statements.
```

NumDays = daysdif('3/1/99', '3/1/00', 1)
NumDays =
360
MoreDays = ['3/1/2001'; '3/1/2002'; '3/1/2003'];
NumDays = daysdif('3/1/98', MoreDays)
NumDays =
1096
1461
1826

```

Stigum, Marcia L. and Franklin Robinson, Money Market and Bond Calculations, Richard D. Irwin, 1996, ISBN 1-55623-476-7.
datenum | days360 | days365 | daysact | daysadd | wrkdydif | yearfrac

\section*{Purpose}

Decimal to thirty-second quotation

\author{
Syntax \\ [OutNumber, Fractions] = dec2thirtytwo(InNumber, Accuracy)
}

\section*{Arguments}

\section*{Description}

Examples
InNumber Input number as a decimal fraction.
Accuracy (Optional) Rounding. Default \(=1\), round down to nearest thirty second. Other values are 2 (nearest half), 4 (nearest quarter) and 10 (nearest decile).
[OutNumber, Fractions] = dec2thirtytwo(InNumber, Accuracy) changes a decimal price quotation for a bond or bond future to a fraction with a denominator of 32 .

OutNumber is InNumber rounded downward to the closest integer.
Fractions is the fractional part in units of thirty-second with accuracy as prescribed by the input Accuracy.

Two bonds are quoted with decimal prices of 101.78 and 102.96. Convert these prices to fractions with a denominator of 32 .
```

InNumber = [101.78; 102.96];
[OutNumber, Fractions] = dec2thirtytwo(InNumber)
OutNumber =

```
    101
    102
Fractions =
    25

\section*{dec2thirtytwo}

\section*{See Also \\ thirtytwo2dec}

\section*{Purpose Fixed declining-balance depreciation schedule \\ ```
Syntax Depreciation = depfixdb(Cost, Salvage, Life, Period, Month)
```}

\section*{Arguments}

\section*{Description}

\section*{Examples A car is purchased for \(\$ 11,000\) with a salvage value of \(\$ 1500\) and a lifetime of eight years. To calculate the depreciation for the first five years}
```

    Depreciation = depfixdb(11000, 1500, 8, 5)
    ```
returns
```

Depreciation =

```
\begin{tabular}{lllll}
2425.08 & 1890.44 & 1473.67 & 1148.78 & 895.52
\end{tabular}

See Also depgendb | deprdv | depsoyd | depstln

\section*{depgendb}

Purpose General declining-balance depreciation schedule

\author{
Syntax Depreciation = depgendb(Cost, Salvage, Life, Factor)
}

\section*{Arguments}

\section*{Description}

\section*{Examples}

Depreciation = depgendb(Cost, Salvage, Life, Factor) calculates the declining-balance depreciation for each period.

A car is purchased for \(\$ 11,000\) and is to be depreciated over five years. The estimated salvage value is \(\$ 1000\). Using the double-declining-balance method, the function calculates the depreciation for each year and returns the remaining depreciable value at the end of the life of the car.
```

Depreciation = depgendb(11000, 1000, 5, 2)

```
returns
```

Depreciation =
4400.00 2640.00 1584.00 950.40 425.60

```
```

See Also
depfixdb | deprdv | depsoyd | depstln

```

\section*{Purpose \\ Syntax \\ Arguments}

Remaining depreciable value

\author{
Value = deprdv(Cost, Salvage, Accum)
}

\section*{Description}

\section*{Examples}

See Also

Purpose Sum of years' digits depreciation
Syntax Sum = depsoyd(Cost, Salvage, Life)

Arguments

Description Sum = depsoyd(Cost, Salvage, Life) calculates the depreciation for an asset using the sum of years' digits method. Sum is a 1-by-Life vector of depreciation values with each element corresponding to a year of the asset's life.

\section*{Examples}

The cost of an asset is \(\$ 13,000\) with a life of 10 years. The salvage value of the asset is \(\$ 1000\).
```

    Sum = depsoyd(13000, 1000, 10)'
    returns
Sum =
2181.82
1963.64
1745.45
1527.27
1309.09
1090.91
872.73
654.55
436.36
218.18

```

\section*{See Also depfixdb | depgendb | deprdv | depstln}

\section*{depstln}

Purpose Straight-line depreciation schedule
```

Syntax
Depreciation = depstln(Cost, Salvage, Life)

```

\section*{Arguments}
\begin{tabular}{ll} 
Cost & Cost of the asset. \\
Salvage & Salvage value of the asset. \\
Life & Depreciable life of the asset in years.
\end{tabular}
Description \begin{tabular}{l} 
Depreciation \(=\) depstln(Cost, Salvage, Life) calculates \\
straight-line depreciation for an asset.
\end{tabular}

\section*{Examples \\ The cost of an asset is \(\$ 13,000\) with a life of 10 years. The salvage value of the asset is \(\$ 1000\).}
```

    Depreciation = depstln(13000, 1000, 10)
    ```
returns
Depreciation =
1200
See Also depfixdb | depgendb | deprdv | depsoyd

\section*{Purpose Differencing}

\section*{Syntax newfts \(=\operatorname{diff}(0 l d f t s)\)}

Description diff computes the differences of the data series in a financial time series object. It returns another time series object containing the difference.
newfts \(=\) diff(oldfts) computes the difference of all the data in the data series of the object oldfts and returns the result in the object newfts. newfts is a financial time series object containing the same data series (names) as the input oldfts.

\section*{See Also \\ diff}

\section*{disc2zero}

Purpose Zero curve given discount curve
```

Syntax [ZeroRates, CurveDates] = disc2zero(DiscRates, CurveDates, Settle,
Compounding, Basis)

```

\section*{Arguments}
\begin{tabular}{ll} 
DiscRates & \begin{tabular}{l} 
Column vector of discount factors, as decimal \\
fractions. In aggregate, the factors in DiscRates \\
constitute a discount curve for the investment \\
horizon represented by CurveDates.
\end{tabular} \\
CurveDates & \begin{tabular}{l} 
Column vector of maturity dates (as serial date \\
numbers) that correspond to the discount factors in \\
DiscRates.
\end{tabular} \\
Settle & \begin{tabular}{l} 
Serial date number that is the common settlement \\
date for the discount rates in DiscRates.
\end{tabular} \\
Compounding & \begin{tabular}{l} 
(Optional) Output compounding. A scalar that sets \\
the compounding frequency per year for annualizing \\
the output zero rates. Allowed values are:
\end{tabular}
\end{tabular}

1 Annual compounding
2 Semiannual compounding (default)
3 Compounding three times per year
4 Quarterly compounding
6 Bimonthly compounding
12 Monthly compounding
365 Daily compounding
\[
\begin{aligned}
& \text {-1 Continuous compounding } \\
& \text { Basis (Optional) Day-count basis for annualizing the } \\
& \text { output zero rates. } \\
& \text { - } 0=\text { actual/actual (default) } \\
& \text { - } 1=30 / 360 \text { (SIA) } \\
& \text { - } 2 \text { = actual/360 } \\
& \text { - } 3 \text { = actual/365 } \\
& \text { - } 4=30 / 360 \text { (PSA) } \\
& \text { - } 5=30 / 360 \text { (ISDA) } \\
& \text { - } 6=30 / 360 \text { (European) } \\
& \text { - } 7 \text { = actual/365 (Japanese) } \\
& \text { - } 8=\text { actual/actual (ISMA) } \\
& \text { - } 9=\text { actual/360 (ISMA) } \\
& \text { - } 10 \text { = actual/365 (ISMA) } \\
& \text { - } 11=30 / 360 \mathrm{E} \text { (ISMA) } \\
& \text { - } 12 \text { = actual/365 (ISDA) } \\
& \text { - } 13 \text { = BUS/252 }
\end{aligned}
\]

For more information, seebasis on page Glossary-1.

\section*{Description}
[ZeroRates, CurveDates] = disc2zero(DiscRates, CurveDates, Settle, Compounding, Basis) returns a zero curve given a discount curve and its maturity dates.

ZeroRates Column vector of decimal fractions. In aggregate, the rates in ZeroRates constitute a zero curve for the investment horizon represented by CurveDates. The zero rates are the yields to maturity on theoretical zero-coupon bonds.

CurveDates Column vector of maturity dates (as serial date numbers) that correspond to the zero rates. This vector is the same as the input vector CurveDates.

\section*{Examples Given discount factors DiscRates over a set of maturity dates CurveDates, and a settlement date Settle}
```

DiscRates = [0.9996
0.9947
0.9896
0.9866
0.9826
0.9786
0.9745
0.9665
0.9552
0.9466];
CurveDates = [datenum('06-Nov-2000')
datenum('11-Dec-2000')
datenum('15-Jan-2001')
datenum('05-Feb-2001')
datenum('04-Mar-2001')
datenum('02-Apr-2001')
datenum('30-Apr-2001')
datenum('25-Jun-2001')
datenum('04-Sep-2001')
datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');

```

Set daily compounding for the output zero curve, on an actual/365 basis.
Compounding = 365;
Basis = 3;

Execute the function
```

[ZeroRates, CurveDates] = disc2zero(DiscRates, CurveDates,...
Settle, Compounding, Basis)

```
which returns the zero curve ZeroRates at the maturity dates CurveDates.
```

ZeroRates =
0.0487
0.0510
0.0523
0.0524
0.0530
0.0526
0.0530
0.0532
0.0549
0.0536

```
CurveDates =
    730796
    730831
    730866
    730887
    730914
    730943
    730971
    731027
    731098
    731167

\section*{disc2zero}

For readability, DiscRates and ZeroRates are shown here only to the basis point. However, MATLAB software computed them at full precision. If you enter DiscRates as shown, ZeroRates may differ due to rounding.

\section*{See Also \\ zero2disc}

How To . "Term Structure of Interest Rates" on page 2-36
```

Purpose Bank discount rate of money market security
Syntax DiscRate = discrate(Settle, Maturity, Face, Price, Basis)

```

Arguments
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Enter as serial date numbers or date strings. \\
Settle must be earlier than Maturity.
\end{tabular} \\
Maturity & Enter as serial date numbers or date strings. \\
Face & Redemption (par, face) value. \\
Price & Price of the security. \\
Basis & (Optional) Day-count basis of the instrument. A \\
& vector of integers. \\
& - \(0=\) actual/actual (default) \\
& - \(1=30 / 360\) (SIA) \\
& - \(2=\) actual/360 \\
& - \(3=\) actual/365 \\
& - \(4=30 / 360\) (PSA) \\
& - \(5=30 / 360\) (ISDA) \\
& - \(6=30 / 360\) (European) \\
& - \(7=\) actual/365 (Japanese) \\
& - \(8=\) actual/actual (ISMA) \\
& - \(9=\) actual/360 (ISMA) \\
& - \(10=\) actual/365 (ISMA) \\
& - \(11=30 / 360 \mathrm{E}\) (ISMA) \\
& - \(12=\) actual/365 (ISDA)
\end{tabular}
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Description DiscRate = discrate(Settle, Maturity, Face, Price, Basis) finds the bank discount rate of a security. The bank discount rate normalizes by the face value of the security (for example, U. S. Treasury Bills) and understates the true yield earned by investors.
ExamplesDiscRate \(=\) discrate('12-jan-2000', '25-jun-2000', 100, 97.74, 0)returns
DiscRate =0.0501a discount rate of \(5.01 \%\).
References Mayle, Standard Securities Calculation Methods, Volumes I-II, 3rd edition. Formula 1.
See Also acrudisc | fvdisc | prdisc | ylddisc

Purpose
Least-squares regression with missing data

\section*{Syntax}
[Parameters, Covariance, Resid, Info] = ecmlsrmle(Data, Design, MaxIterations, TolParam, TolObj, Param0, CovarO, CovarFormat)

\section*{Arguments}

Design A matrix or a cell array that handles two model structures:
- If NUMSERIES = 1, Design is a NUMSAMPLES-by-NUMPARAMS matrix with known values. This structure is the standard form for regression on a single series.
- If NUMSERIES \(\geq 1\), Design is a cell array. The cell array contains either one or NUMSAMPLES cells. Each cell contains a NUMSERIES-by-NUMPARAMS matrix of known values.

If Design has a single cell, it is assumed to have the same Design matrix for each sample. If Design has more than one cell, each cell contains a Design matrix for each sample.

MaxIterations (Optional) Maximum number of iterations for the estimation algorithm. Default value is 100 .

\author{
TolParam
}

TolObj
(Optional) Convergence tolerance for estimation algorithm based on changes in model parameter estimates. Default value is sqrt(eps) which is about \(1.0 \mathrm{e}-8\) for double precision. The convergence test for changes in model parameters is
\[
\| \text { Param }_{k}-\text { Param }_{k-1} \|<\text { TolParam } \times\left(1+\left\|\operatorname{Param}_{k}\right\|\right)
\]
where Param represents the output Parameters, and iteration \(k=2,3, \ldots\). Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam \(\leq\) 0 and TolObj \(\leq 0\), do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests.
(Optional) Convergence tolerance for estimation algorithm based on changes in the objective function. Default value is eps \(\wedge 3 / 4\) which is about \(1.0 \mathrm{e}-12\) for double precision. The convergence test for changes in the objective function is
\[
\left|O b j_{k}-O b j_{k-1}\right|<T o l O b j \times\left(1+\left|O b j_{k}\right|\right)
\]
for iteration \(k=2,3, \ldots\). Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam \(\leq\) 0 and TolObj \(\leq 0\), do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests.
\(\left.\begin{array}{ll}\text { Param0 } & \begin{array}{l}\text { (Optional) NUMPARAMS-by-1 column vector that } \\
\text { contains a user-supplied initial estimate for the } \\
\text { parameters of the regression model. Default is } \\
\text { a zero vector. } \\
\text { (Optional) NUMSERIES-by-NUMSERIES matrix }\end{array} \\
\text { that contains a user-supplied initial or known } \\
\text { estimate for the covariance matrix of the } \\
\text { regression residuals. Default is an identity } \\
\text { matrix. }\end{array}\right\}\)\begin{tabular}{l} 
For covariance-weighted least-squares \\
calculations, this matrix corresponds with \\
weights for each series in the regression. \\
The matrix also serves as an initial guess for \\
the residual covariance in the expectation \\
conditional maximization (ECM) algorithm. \\
(Optional) String that specifies the format for \\
the covariance matrix. The choices are:
\end{tabular}

\section*{ecmlsrmle}
ecmlsrmle estimates a NUMPARAMS-by- 1 column vector of model parameters called Parameters, and a NUMSERIES-by-NUMSERIES matrix of covariance parameters called Covariance.
ecmlsrmle(Data, Design) with no output arguments plots the log-likelihood function for each iteration of the algorithm.

To summarize the outputs of ecmlsrmle:
- Parameters is a NUMPARAMS-by-1 column vector of estimates for the parameters of the regression model.
- Covariance is a NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the regression model's residuals. For least-squares models, this estimate may not be a maximum likelihood estimate except under special circumstances.
- Resid is a NUMSAMPLES-by-NUMSERIES matrix of residuals from the regression.

Another output, Info, is a structure that contains additional information from the regression. The structure has these fields:
- Info. Obj - A variable-extent column vector, with no more than MaxIterations elements, that contains each value of the objective function at each iteration of the estimation algorithm. The last value in this vector, Obj (end), is the terminal estimate of the objective function. If you do least-squares, the objective function is the least-squares objective function.
- Info.PrevParameters - NUMPARAMS-by-1 column vector of estimates for the model parameters from the iteration just prior to the terminal iteration.
- Info. PrevCovariance - NUMSERIES-by-NUMSERIES matrix of estimates for the covariance parameters from the iteration just prior to the terminal iteration.

\footnotetext{
Notes
}

If doing covariance-weighted least-squares, Covar0 should usually be a diagonal matrix. Series with greater influence should have smaller
diagonal elements in Covar0 and series with lesser influence should have larger diagonal elements. Note that if doing CWLS, Covar0 need not be a diagonal matrix even if CovarFormat = 'diagonal'.

You can configure Design as a matrix if NUMSERIES = 1 or as a cell array if NUMSERIES \(\geq 1\).
- If Design is a cell array and NUMSERIES \(=1\), each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-by-NUMPARAMS matrix.

These points concern how Design handles missing data:
- Although Design should not have NaN values, ignored samples due to NaN values in Data are also ignored in the corresponding Design array.
- If Design is a 1-by-1 cell array, which has a single Design matrix for each sample, no NaN values are permitted in the array. A model with this structure must have NUMSERIES \(\geq\) NUMPARAMS with rank(Design\{1\}) = NUMPARAMS.
- ecmlsrmle is more strict than mvnrmle about the presence of NaN values in the Design array.

Use the estimates in the optional output structure Info for diagnostic purposes.

\section*{Examples}

\section*{References}

Roderick J. A. Little and Donald B. Rubin, Statistical Analysis with Missing Data, 2nd ed., John Wiley \& Sons, Inc., 2002.

\section*{ecmlsrmle}

Xiao-Li Meng and Donald B. Rubin, "Maximum Likelihood Estimation via the ECM Algorithm," Biometrika, Vol. 80, No. 2, 1993, pp. 267-278.

Joe Sexton and Anders Rygh Swensen, "ECM Algorithms that Converge at the Rate of EM," Biometrika, Vol. 87, No. 3, 2000, pp. 651-662.
A. P. Dempster, N.M. Laird, and D. B. Rubin, "Maximum Likelihood from Incomplete Data via the EM Algorithm," Journal of the Royal Statistical Society, Series B, Vol. 39, No. 1, 1977, pp. 1-37.

\footnotetext{
See Also
ecmlsrobj | ecmmvnrmle | mvnrmle
}

\section*{Purpose}

Log-likelihood function for least-squares regression with missing data

\author{
Syntax \\ Objective = ecmlsrobj(Data, Design, Parameters, Covariance)
}

\section*{Arguments}

\section*{Description}

Notes

\section*{Examples}

See Also

Objective = ecmlsrobj(Data, Design, Parameters, Covariance) computes a least-squares objective function based on current parameter estimates with missing data. Objective is a scalar that contains the least-squares objective function.
ecmlsrobj requires that Covariance be positive-definite.
Note that
ecmlsrobj(Data, Design, Parameters) = ecmmvnrobj(Data, ... Design, Parameters, IdentityMatrix)
where IdentityMatrix is a NUMSERIES-by-NUMSERIES identity matrix.
You can configure Design as a matrix if NUMSERIES \(=1\) or as a cell array if NUMSERIES \(\geq 1\).
- If Design is a cell array and NUMSERIES \(=1\), each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-by-NUMPARAMS matrix.

See "Multivariate Normal Regression" on page 7-17, "Least-Squares Regression" on page 7-18, "Covariance-Weighted Least Squares" on page 7-19, "Feasible Generalized Least Squares" on page 7-20, and "Seemingly Unrelated Regression" on page 7-21.
```

ecmlsrmle | mvnrmle | mvnrobj

```

\section*{Purpose}

Fisher information matrix for multivariate normal regression model

\section*{Syntax}

Fisher = ecmmvnrfish(Data, Design, Covariance, Method, MatrixFormat, CovarFormat)

\section*{Arguments}
- If NUMSERIES \(\geq 1\), Design is a cell array. The cell array contains either one or NUMSAMPLES cells. Each cell contains a NUMSERIES-by-NUMPARAMS matrix of known values.

If Design has a single cell, it is assumed to have the same Design matrix for each sample. If Design has more than one cell, each cell contains a Design matrix for each sample.
\(\left.\begin{array}{ll}\text { Covariance } & \begin{array}{l}\text { NUMSERIES-by-NUMSERIES matrix of estimates } \\ \text { for the covariance of the residuals of the } \\ \text { regression. } \\ \text { (Optional) String that identifies method of } \\ \text { calculation for the information matrix: }\end{array} \\ \text { Method } & \\ \text { - hessian - Default method. Use the expected } \\ \text { Hessian matrix of the observed log-likelihood } \\ \text { function. This method is recommended since } \\ \text { the resultant standard errors incorporate the } \\ \text { increased uncertainties due to missing data. }\end{array}\right\}\)

\section*{Description}

Fisher = ecmmvnrfish(Data, Design, Covariance, Method, MatrixFormat, CovarFormat) computes a Fisher information matrix based on current maximum likelihood or least-squares parameter estimates that account for missing data.

Fisher is a NUMPARAMS-by-NUMPARAMS Fisher information matrix or Hessian matrix. The size of NUMPARAMS depends on MatrixFormat and on current parameter estimates. If MatrixFormat = 'full',

NUMPARAMS = NUMSERIES * (NUMSERIES + 3)/2

If MatrixFormat = 'paramonly',
NUMPARAMS = NUMSERIES

Note ecmmvnrfish operates slowly if you calculate the full Fisher information matrix.

\author{
Examples See "Multivariate Normal Regression" on page 7-17, "Least-Squares Regression" on page 7-18, "Covariance-Weighted Least Squares" on page 7-19, "Feasible Generalized Least Squares" on page 7-20, and "Seemingly Unrelated Regression" on page 7-21.
}

\author{
See Also \\ ecmnmle | ecmnstd
}

Purpose

\section*{Syntax}

\section*{Arguments}
- If NUMSERIES \(\geq 1\), Design is a cell array. The cell array contains either one or NUMSAMPLES cells. Each cell contains a NUMSERIES-by-NUMPARAMS matrix of known values.

If Design has a single cell, it is assumed to have the same Design matrix for each sample. If Design has more than one cell, each cell contains a Design matrix for each sample.
(Optional) Maximum number of iterations for the estimation algorithm. Default value is 100 .

TolParam

Tol0bj

Paramo
(Optional) Convergence tolerance for estimation algorithm based on changes in model parameter estimates. Default value is sqrt(eps) which is about \(1.0 \mathrm{e}-8\) for double precision. The convergence test for changes in model parameters is
\[
\left\|\operatorname{Param}_{k}-\operatorname{Param}_{k-1}\right\|<\text { TolParam } \times\left(1+\left\|\operatorname{Param}_{k}\right\|\right)
\]
where Param represents the output Parameters, and iteration \(k=2,3, \ldots\). Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam \(\leq\) 0 and TolObj \(\leq 0\), do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests.
(Optional) Convergence tolerance for estimation algorithm based on changes in the objective function. Default value is eps \(\wedge 3 / 4\) which is about \(1.0 \mathrm{e}-12\) for double precision. The convergence test for changes in the objective function is
\[
\left|\mathrm{Obj}_{k}-\mathrm{Obj}_{k-1}\right|<\operatorname{TolObj} \times\left(1+\left|\mathrm{Obj}_{k}\right|\right)
\]
for iteration \(k=2,3, \ldots\). Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam \(\leq\) 0 and TolObj \(\leq 0\), do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests.
(Optional) NUMPARAMS-by- 1 column vector that contains a user-supplied initial estimate for the parameters of the regression model.
\begin{tabular}{ll} 
Covar0 & \begin{tabular}{l} 
(Optional) NUMSERIES-by-NUMSERIES matrix \\
that contains a user-supplied initial or known \\
estimate for the covariance matrix of the \\
regression residuals.
\end{tabular} \\
CovarFormat \(\quad\)\begin{tabular}{l} 
(Optional) String that specifies the format for \\
the covariance matrix. The choices are:
\end{tabular}
\end{tabular}
- 'full' - Default method. Compute the full covariance matrix.
- 'diagonal' - Force the covariance matrix to be a diagonal matrix.

\section*{Description}
[Parameters, Covariance, Resid, Info] = ecmmvnrmle(Data, Design, MaxIterations, TolParam, TolObj, Param0, Covar0, CovarFormat) estimates a multivariate normal regression model with missing data. The model has the form
\[
\text { Data }_{k} \square N\left(\text { Design }_{k} \times \text { Parameters, Covariance }\right)
\]
for samples \(k=1, \ldots\), NUMSAMPLES.
ecmmvnrmle estimates a NUMPARAMS-by- 1 column vector of model parameters called Parameters, and a NUMSERIES-by-NUMSERIES matrix of covariance parameters called Covariance.
ecmmvnrmle(Data, Design) with no output arguments plots the log-likelihood function for each iteration of the algorithm.

To summarize the outputs of ecmmvnrmle:
- Parameters is a NUMPARAMS-by- 1 column vector of estimates for the parameters of the regression model.
- Covariance is a NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the regression model's residuals.
- Resid is a NUMSAMPLES-by-NUMSERIES matrix of residuals from the regression. For any missing values in Data, the corresponding residual is the difference between the conditionally imputed value for Data and the model, that is, the imputed residual.

Note The covariance estimate Covariance cannot be derived from the residuals.

Another output, Info, is a structure that contains additional information from the regression. The structure has these fields:
- Info. Obj - A variable-extent column vector, with no more than MaxIterations elements, that contains each value of the objective function at each iteration of the estimation algorithm. The last value in this vector, Obj (end), is the terminal estimate of the objective function. If you do maximum likelihood estimation, the objective function is the log-likelihood function.
- Info.PrevParameters - NUMPARAMS-by-1 column vector of estimates for the model parameters from the iteration just prior to the terminal iteration.nfo. PrevCovariance - NUMSERIES-by-NUMSERIES matrix of estimates for the covariance parameters from the iteration just prior to the terminal iteration.

\section*{Notes}
ecmmvnrmle does not accept an initial parameter vector, since the parameters are estimated directly from the first iteration onward.

You can configure Design as a matrix if NUMSERIES = 1 or as a cell array if NUMSERIES \(\geq 1\).
- If Design is a cell array and NUMSERIES \(=1\), each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-by-NUMPARAMS matrix.

These points concern how Design handles missing data:
- Although Design should not have NaN values, ignored samples due to NaN values in Data are also ignored in the corresponding Design array.
- If Design is a 1-by-1 cell array, which has a single Design matrix for each sample, no NaN values are permitted in the array. A model with this structure must have NUMSERIES \(\geq\) NUMPARAMS with rank(Design\{1\}) = NUMPARAMS.
- ecmmvnrmle is more strict than mvnrmle about the presence of NaN values in the Design array.

Use the estimates in the optional output structure Info for diagnostic purposes.
\begin{tabular}{ll} 
References & \begin{tabular}{l} 
Roderick J. A. Little and Donald B. Rubin, Statistical Analysis with \\
\\
Missing Data, 2nd ed., John Wiley \& Sons, Inc., 2002. \\
\\
\\
\\
\\
Xiao-Li Meng and Donald B. Rubin, "Maximum Likelihood Estimation \\
via the ECM Algorithm," Biometrika, Vol. 80, No. 2, 1993, pp. 267-278.
\end{tabular} \\
& \begin{tabular}{l} 
Joe Sexton and Anders Rygh Swensen, "ECM Algorithms that Converge \\
at the Rate of EM," Biometrika, Vol. 87, No. 3, 2000, pp. 651-662.
\end{tabular} \\
& \begin{tabular}{l} 
A. P. Dempster, N.M. Laird, and D. B. Rubin, "Maximum Likelihood \\
from Incomplete Data via the EM Algorithm," Journal of the Royal
\end{tabular} \\
& Statistical Society, Series B, Vol. 39, No. 1, 1977, pp. 1-37.
\end{tabular}
\begin{tabular}{ll} 
Purpose & \begin{tabular}{l} 
Log-likelihood function for multivariate normal regression with missing \\
data
\end{tabular} \\
Syntax & \begin{tabular}{l} 
Objective \(=\) ecmmvnrobj (Data, Design, Parameters, Covariance, \\
CovarFormat)
\end{tabular}
\end{tabular}

\section*{Arguments}
\begin{tabular}{ll} 
Data & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES \\
samples of a NUMSERIES-dimensional random vector. \\
Missing values are represented as NaNs. Only \\
samples that are entirely NaNs are ignored. (To \\
ignore samples with at least one NaN, use mvnrmle.)
\end{tabular} \\
Design & \begin{tabular}{l} 
A matrix or a cell array that handles two model \\
structures:
\end{tabular} \\
& - If NUMSERIES = 1, Design is a \\
& NUMSAMPLES-by-NUMPARAMS matrix with \\
& known values. This structure is the standard form \\
& for regression on a single series. \\
& - If NUMSERIES \(\geq 1\), Design is a cell array. The cell \\
& array contains either one or NUMSAMPLES cells. \\
& Each cell contains a NUMSERIES-by-NUMPARAMS \\
& matrix of known values.
\end{tabular}

Covariance NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the residuals of the regression.

CovarFormat (Optional) String that specifies the format for the covariance matrix. The choices are:
- 'full' - Default method. The covariance matrix is a full matrix.
- 'diagonal' - The covariance matrix is a diagonal matrix.
\begin{tabular}{ll} 
Description & \begin{tabular}{l} 
Objective = ecmmvnrobj (Data, Design, Parameters, \\
Covariance, CovarFormat) computes a log-likelihood function based \\
on current maximum likelihood parameter estimates with missing data. \\
Objective is a scalar that contains the least-squares objective function.
\end{tabular} \\
Notes & \begin{tabular}{l} 
You can configure Design as a matrix if NUMSERIES = 1 or as a cell \\
array if NUMSERIES \(\geq 1\).
\end{tabular} \\
& \begin{tabular}{l} 
- If Design is a cell array and NUMSERIES \(=1\), each cell contains a \\
\\
NUMPARAMS row vector.
\end{tabular} \\
- If Design is a cell array and NUMSERIES > 1, each cell contains a \\
NUMSERIES-by-NUMPARAMS matrix.
\end{tabular}

\section*{Purpose}

Evaluate standard errors for multivariate normal regression model

\section*{Syntax}
[StdParameters, StdCovariance] = ecmmvnrstd(Data, Design, Covariance, Method, CovarFormat)

\section*{Arguments}
\begin{tabular}{ll} 
Data & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES \\
samples of a NUMSERIES-dimensional random vector. \\
Missing values are represented as NaNs. Only \\
samples that are entirely NaNs are ignored. (To \\
ignore samples with at least one NaN, use mvnrstd.)
\end{tabular} \\
Design & \begin{tabular}{l} 
A matrix or a cell array that handles two model \\
structures:
\end{tabular} \\
& - If NUMSERIES = 1, Design is a \\
& NUMSAMPLES-by-NUMPARAMS matrix with \\
& known values. This structure is the standard form \\
& for regression on a single series.
\end{tabular}
Method (Optional) String that identifies method of calculation for the information matrix:
- hessian - Default method. Use the expected Hessian matrix of the observed log-likelihood function. This method is recommended since the resultant standard errors incorporate the increased uncertainties due to missing data.
- fisher - Use the Fisher information matrix.

CovarFormat (Optional) String that specifies the format for the covariance matrix. The choices are:
- 'full' - Default method. The covariance matrix is a full matrix.
- 'diagonal' - The covariance matrix is a diagonal matrix.

\section*{Description}
[StdParameters, StdCovariance] = ecmmvnrstd(Data, Design, Covariance, Method, CovarFormat) evaluates standard errors for a multivariate normal regression model with missing data. The model has the form

Data \(_{k} \square N\left(\right.\) Design \(_{k} \times\) Parameters, Covariance \()\)
for samples \(k=1, \ldots\), NUMSAMPLES.
ecmmvnrstd computes two outputs:
- StdParameters is a NUMPARAMS-by-1 column vector of standard errors for each element of Parameters, the vector of estimated model parameters.
- StdCovariance is a NUMSERIES-by-NUMSERIES matrix of standard errors for each element of Covariance, the matrix of estimated covariance parameters.

Note ecmmvnrstd operates slowly when you calculate the standard errors associated with the covariance matrix Covariance.
\begin{tabular}{|c|c|}
\hline Notes & \begin{tabular}{l}
You can configure Design as a matrix if NUMSERIES = 1 or as a cell array if NUMSERIES \(\geq 1\). \\
- If Design is a cell array and NUMSERIES \(=1\), each cell contains a NUMPARAMS row vector. \\
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-by-NUMPARAMS matrix.
\end{tabular} \\
\hline References & Roderick J. A. Little and Donald B. Rubin, Statistical Analysis with Missing Data, 2nd ed., John Wiley \& Sons, Inc., 2002. \\
\hline Examples & See "Multivariate Normal Regression" on page 7-17, "Least-Squares Regression" on page 7-18, "Covariance-Weighted Least Squares" on page 7-19, "Feasible Generalized Least Squares" on page 7-20, and "Seemingly Unrelated Regression" on page 7-21. \\
\hline See Also & ecmmvnrmle | ecmmvnrstd | mvnrmle \\
\hline
\end{tabular}

Purpose Fisher information matrix
Syntax Fisher = ecmnfish(Data, Covariance, InvCovariance, MatrixFormat)

\section*{Arguments}
\begin{tabular}{ll} 
Data & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix of observed \\
multivariate normal data
\end{tabular} \\
Covariance & \begin{tabular}{l} 
NUMSERIES-by-NUMSERIES matrix with \\
covariance estimate of Data
\end{tabular} \\
InvCovariance & \begin{tabular}{l} 
(Optional) Inverse of covariance matrix: \\
inv(Covariance)
\end{tabular} \\
MatrixFormat & \begin{tabular}{l} 
(Optional) String that identifies parameters \\
included in the Fisher information matrix. If \\
MatrixFormat = [] or ' ', the default method
\end{tabular} \\
& \begin{tabular}{l} 
full is used. The parameter choices are
\end{tabular}
\end{tabular}
- full - (Default) Compute full Fisher information matrix.
- meanonly - Compute only components of the Fisher information matrix associated with the mean.
```

Description
Fisher = ecmnfish(Data, Covariance, InvCovariance,
MatrixFormat) computes a NUMPARAMS-by-NUMPARAMS Fisher
information matrix based on current parameter estimates, where

```
```

    NUMPARAMS = NUMSERIES*(NUMSERIES + 3)/2
    ```
    NUMPARAMS = NUMSERIES*(NUMSERIES + 3)/2
if MatrixFormat = 'full' and
if MatrixFormat = 'full' and
    NUMPARAMS = NUMSERIES
```

    NUMPARAMS = NUMSERIES
    ```
if MatrixFormat = 'meanonly'.
The data matrix has NaNs for missing observations. The multivariate normal model has
```

NUMPARAMS = NUMSERIES + NUMSERIES*(NUMSERIES + 1)/2

```
distinct parameters. Therefore, the full Fisher information matrix is of size NUMPARAMS-by-NUMPARAMS. The first NUMSERIES parameters are estimates for the mean of the data in Mean, and the remaining NUMSERIES* (NUMSERIES + 1)/2 parameters are estimates for the lower-triangular portion of the covariance of the data in Covariance, in row-major order.
If MatrixFormat = 'meanonly', the number of parameters is reduced to NUMPARAMS = NUMSERIES, where the Fisher information matrix is computed for the mean parameters only. In this format, the routine executes fastest.

This routine expects the inverse of the covariance matrix as an input. If you do not pass in the inverse, the routine computes it. You can obtain an approximation for the lower-bound standard errors of estimation of the parameters from
```

Stderr = (1.0/sqrt(NumSamples)) .* sqrt(diag(inv(Fisher)));

```

Because of missing information, these standard errors may be smaller than the estimated standard errors derived from the expected Hessian matrix. To see the difference, compare to standard errors calculated with ecmnhess.

See Also ecmnhess \| ecmnmle

Purpose Hessian of negative log-likelihood function
Syntax Hessian = ecmnhess(Data, Covariance, InvCovariance, MatrixFormat)

\section*{Arguments}
\begin{tabular}{ll} 
Data & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix of observed \\
multivariate normal data
\end{tabular} \\
Covariance & \begin{tabular}{l} 
NUMSERIES-by-NUMSERIES matrix with \\
covariance estimate of Data
\end{tabular} \\
InvCovariance & \begin{tabular}{l} 
(Optional) Inverse of covariance matrix: \\
inv(Covariance)
\end{tabular} \\
MatrixFormat & \begin{tabular}{l} 
(Optional) String that identifies parameters \\
included in the Hessian matrix. If
\end{tabular} \\
& \begin{tabular}{l} 
MatrixFormat = [] or ', the default method \\
full is used. The parameter choices are
\end{tabular} \\
& \begin{tabular}{l} 
- full - (Default) Compute full Hessian \\
\\
matrix.
\end{tabular} \\
& - meanonly - Compute only components of \\
the Hessian matrix associated with the \\
mean.
\end{tabular}

\section*{Description \\ Hessian = ecmnhess(Data, Covariance, InvCovariance, MatrixFormat) computes a NUMPARAMS -by-NUMPARAMS Hessian matrix of the observed negative log-likelihood function based upon current parameter estimates, where}
```

    NUMPARAMS = NUMSERIES*(NUMSERIES + 3)/2
    if MatrixFormat = 'full' and

```
```

    NUMPARAMS = NUMSERIES
    if MatrixFormat = 'meanonly'.

```
This routine is very slow for NUMSERIES > 10 or NUMSAMPLES > 1000.

The data matrix has NaNs for missing observations. The multivariate normal model has
```

NUMPARAMS = NUMSERIES + NUMSERIES*(NUMSERIES + 1)/2

```
distinct parameters. Therefore, the full Hessian is a NUMPARAMS-by-NUMPARAMS matrix.

The first NUMSERIES parameters are estimates for the mean of the data in Mean and the remaining NUMSERIES* (NUMSERIES + 1)/2 parameters are estimates for the lower-triangular portion of the covariance of the data in Covariance, in row-major order.

If MatrixFormat = 'meanonly', the number of parameters is reduced to NUMPARAMS = NUMSERIES, where the Hessian is computed for the mean parameters only. In this format, the routine executes fastest.

This routine expects the inverse of the covariance matrix as an input. If you do not pass in the inverse, the routine computes it.

The equation
```

Stderr = (1.0/sqrt(NumSamples)) .* sqrt(diag(inv(Hessian)));

```
provides an approximation for the observed standard errors of estimation of the parameters.

Because of the additional uncertainties introduced by missing information, these standard errors may be larger than the estimated standard errors derived from the Fisher information matrix. To see the difference, compare to standard errors calculated from ecmnfish.
ecmnfish | ecmnmle

\section*{ecmninit}
\[
\begin{array}{ll}
\text { Purpose } & \text { Initial mean and covariance } \\
\text { Syntax } & {[\text { Mean, Covariance }]=\text { ecmninit(Data, InitMethod) }}
\end{array}
\]

\section*{Arguments}

\section*{Description}

\section*{Algorithms}

> Data NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES samples of a NUMSERIES-dimensional random vector. Missing values are indicated by NaNs.

> InitMethod (Optional) String that identifies one of three defined initialization methods to compute initial estimates for the mean and covariance of the data. If InitMethod = [] or ' ', the default method nanskip is used. The initialization methods are
- nanskip - (Default) Skip all records with NaNs.
- twostage - Estimate mean. Fill NaNs with the mean. Then estimate the covariance.
- diagonal - Form a diagonal covariance.
[Mean, Covariance] = ecmninit(Data, InitMethod) creates initial mean and covariance estimates for the function ecmnmle. Mean is a NUMSERIES-by- 1 column vector estimate for the mean of Data. Covariance is a NUMSERIES-by-NUMSERIES matrix estimate for the covariance of Data.

\section*{Model}

The general model is
\(Z \square N(\) Mean, Covariance \()\),
where each row of Data is an observation of \(Z\).
Each observation of \(Z\) is assumed to be iid (independent, identically distributed) multivariate normal, and missing values are assumed to be missing at random (MAR).

\section*{Initialization Methods}

This routine has three initialization methods that cover most cases, each with its advantages and disadvantages.

\section*{nanskip}

The nanskip method works well with small problems (fewer than 10 series or with monotone missing data patterns). It skips over any records with NaNs and estimates initial values from complete-data records only. This initialization method tends to yield fastest convergence of the ECM algorithm. This routine switches to the twostage method if it determines that significant numbers of records contain NaN .

\section*{twostage}

The twostage method is the best choice for large problems (more than 10 series). It estimates the mean for each series using all available data for each series. It then estimates the covariance matrix with missing values treated as equal to the mean rather than as NaNs. This initialization method is quite robust but tends to result in slower convergence of the ECM algorithm.

\section*{diagonal}

The diagonal method is a worst-case approach that deals with problematic data, such as disjoint series and excessive missing data (more than \(33 \%\) missing data). Of the three initialization methods, this method causes the slowest convergence of the ECM algorithm.

Purpose Mean and covariance of incomplete multivariate normal data

\author{
Syntax \\ [Mean, Covariance] = ecmnmle(Data, InitMethod, MaxIterations, Tolerance, MeanO, Covar0)
}

\section*{Arguments}
\begin{tabular}{ll} 
Data & NUMSAMPLES-by-NUMSERIES matrix \\
with NUMSAMPLES samples of a \\
& NUMSERIES-dimensional random vector. \\
& Missing values are indicated by NaNs. A sample \\
is also called an observation or a record. \\
InitMethod & \begin{tabular}{l} 
(Optional) String that identifies one of three \\
defined initialization methods to compute \\
initial estimates for the mean and covariance of \\
\\
the data. If InitMethod = [] or '' the default \\
method nanskip is used. The initialization
\end{tabular} \\
methods are:
\end{tabular}
- nanskip - (Default) Skip all records with NaNs.
- twostage - Estimate mean. Fill NaNs with mean. Then estimate covariance.
- diagonal - Form a diagonal covariance.

Note If you supply Mean0 and Covaro, InitMethod is not executed.
\[
\begin{array}{ll}
\text { MaxIterations } & \text { (Optional) Maximum number of iterations } \\
\text { for the expectation conditional maximization } \\
& (\mathrm{ECM}) \text { algorithm. Default }=50 .
\end{array}
\]
\begin{tabular}{|c|c|}
\hline Tolerance & (Optional) Convergence tolerance for the ECM algorithm (Default = 1.0e-8.) If Tolerance \(\leq 0\), perform maximum iterations specified by MaxIterations and do not evaluate the objective function at each step unless in display mode, as described below. \\
\hline Meano & (Optional) Initial NUMSERIES-by-1 column vector estimate for the mean. If you leave Mean0 unspecified ([ ]), the method specified by InitMethod is used. If you specify Mean0, you must also specify Covaro. \\
\hline Covaro & (Optional) Initial NUMSERIES-by-NUMSERIES matrix estimate for the covariance, where the input matrix must be positive-definite. If you leave Covaro unspecified ([]), the method specified by InitMethod is used. If you specify Covar0, you must also specify Mean0. \\
\hline
\end{tabular}

\section*{Description}
[Mean, Covariance] = ecmnmle(Data, InitMethod, MaxIterations, Tolerance, Mean0, Covar0) estimates the mean and covariance of a data set. If the data set has missing values, this routine implements the ECM algorithm of Meng and Rubin [2] with enhancements by Sexton and Swensen [3]. ECM stands for expectation conditional maximization, a conditional maximization form of the EM algorithm of Dempster, Laird, and Rubin [4].
This routine has two operational modes.

\section*{Display Mode}

With no output arguments, this mode displays the convergence of the ECM algorithm. It estimates and plots objective function values for each iteration of the ECM algorithm until termination, as shown in the following plot.


Display mode can determine MaxIter and Tolerance values or serve as a diagnostic tool. The objective function is the negative log-likelihood function of the observed data and convergence to a maximum likelihood estimate corresponds with minimization of the objective.

\section*{Estimation Mode}

With output arguments, this mode estimates the mean and covariance via the ECM algorithm.

\section*{Examples}

\section*{Algorithms}

To see an example of how to use ecmnmle, run the demo program ecmguidemo.

\section*{Model}

The general model is
\[
Z \square N(\text { Mean, Covariance }),
\]
where each row of Data is an observation of \(Z\).
Each observation of \(Z\) is assumed to be iid (independent, identically distributed) multivariate normal, and missing values are assumed to be missing at random (MAR). See Little and Rubin [1] for a precise definition of MAR.

This routine estimates the mean and covariance from given data. If data values are missing, the routine implements the ECM algorithm of Meng and Rubin [2] with enhancements by Sexton and Swensen [3].
If a record is empty (every value in a sample is NaN ), this routine ignores the record because it contributes no information. If such records exist in the data, the number of nonempty samples used in the estimation is \(\leq\) NumSamples.

The estimate for the covariance is a biased maximum likelihood estimate (MLE). To convert to an unbiased estimate, multiply the covariance by Count/(Count - 1), where Count is the number of nonempty samples used in the estimation.

\section*{Requirements}

This routine requires consistent values for NUMSAMPLES and NUMSERIES with NUMSAMPLES > NUMSERIES. It must have enough nonmissing values to converge. Finally, it must have a positive-definite covariance matrix. Although the references provide some necessary and sufficient conditions, general conditions for existence and uniqueness of solutions in the missing-data case do not exist. The main failure mode is an ill-conditioned covariance matrix estimate. Nonetheless, this routine works for most cases that have less than \(15 \%\) missing data (a typical upper bound for financial data).

\section*{Initialization Methods}

This routine has three initialization methods that cover most cases, each with its advantages and disadvantages. The ECM algorithm always converges to a minimum of the observed negative log-likelihood
function. If you override the initialization methods, you must ensure that the initial estimate for the covariance matrix is positive-definite.

The following is a guide to the supported initialization methods.

\section*{nanskip}

The nanskip method works well with small problems (fewer than 10 series or with monotone missing data patterns). It skips over any records with NaNs and estimates initial values from complete-data records only. This initialization method tends to yield fastest convergence of the ECM algorithm. This routine switches to the twostage method if it determines that significant numbers of records contain NaN .

\section*{twostage}

The twostage method is the best choice for large problems (more than 10 series). It estimates the mean for each series using all available data for each series. It then estimates the covariance matrix with missing values treated as equal to the mean rather than as NaNs. This initialization method is quite robust but tends to result in slower convergence of the ECM algorithm.

\section*{diagonal}

The diagonal method is a worst-case approach that deals with problematic data, such as disjoint series and excessive missing data (more than \(33 \%\) of data missing). Of the three initialization methods, this method causes the slowest convergence of the ECM algorithm. If problems occur with this method, use display mode to examine convergence and modify either MaxIterations or Tolerance, or try alternative initial estimates with Mean0 and Covar0. If all else fails, try
```

MeanO = zeros(NumSeries);
CovarO = eye(NumSeries,NumSeries);

```

Given estimates for mean and covariance from this routine, you can estimate standard errors with the companion routine ecmnstd.

\section*{Convergence}

The ECM algorithm does not work for all patterns of missing values. Although it works in most cases, it can fail to converge if the covariance becomes singular. If this occurs, plots of the log-likelihood function tend to have a constant upward slope over many iterations as the log of the negative determinant of the covariance goes to zero. In some cases, the objective fails to converge due to machine precision errors. No general theory of missing data patterns exists to determine these cases. An example of a known failure occurs when two time series are proportional wherever both series contain nonmissing values.

\author{
References
}

See Also
ecmnfish | ecmnhess | ecmninit | ecmnobj | ecmnstd

Purpose Multivariate normal negative log-likelihood function
Syntax Objective = ecmnobj(Data, Mean, Covariance, CholCovariance)

\section*{Arguments}
\begin{tabular}{ll} 
Data & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix of observed \\
multivariate normal data
\end{tabular} \\
Mean & \begin{tabular}{l} 
NUMSERIES-by-1 column vector with mean \\
estimate of Data
\end{tabular} \\
Covariance & \begin{tabular}{l} 
NUMSERIES-by-NUMSERIES matrix with \\
covariance estimate of Data
\end{tabular} \\
CholCovariance & \begin{tabular}{l} 
(Optional) Cholesky decomposition of \\
covariance matrix: chol(Covariance)
\end{tabular}
\end{tabular}

Description Objective = ecmnobj(Data, Mean, Covariance, CholCovariance) computes the value of the observed negative log-likelihood function over the data given current estimates for the mean and covariance of the data.

The data matrix has NaNs for missing observations. The inputs Mean and Covariance are current estimates for model parameters.

This routine expects the Cholesky decomposition of the covariance matrix as an input. The routine computes the Cholesky decomposition if you do not explicitly specify it.

See Also chol | ecmnmle

\section*{Purpose}

Standard errors for mean and covariance of incomplete data
Syntax
[StdMean, StdCovariance] = ecmnstd(Data, Mean, Covariance, Method)

\section*{Arguments}
\begin{tabular}{ll} 
Data & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES \\
samples of a NUMSERIES-dimensional random vector. \\
Missing values are indicated by NaNs.
\end{tabular} \\
Mean & \begin{tabular}{l} 
NUMSERIES-by-1 column vector of \\
maximum-likelihood parameter estimates for \\
the mean of Data using the expectation conditional \\
maximization (ECM) algorithm
\end{tabular} \\
Covariance \(\quad\)\begin{tabular}{l} 
NUMSERIES-by-NUMSERIES matrix of \\
maximum-likelihood covariance estimates for \\
the covariance of Data using the ECM algorithm
\end{tabular} \\
Method & \begin{tabular}{l} 
(Optional) String indicating method of estimation for \\
standard error calculations. The methods are:
\end{tabular} \\
& \begin{tabular}{l} 
- hessian - (Default) Hessian of the observed \\
\\
negative log-likelihood function.
\end{tabular} \\
- fisher - Fisher information matrix.
\end{tabular}

\section*{Description}
[StdMean, StdCovariance] = ecmnstd(Data, Mean, Covariance, Method) computes standard errors for mean and covariance of incomplete data.

StdMean is a NUMSERIES-by-1 column vector of standard errors of estimates for each element of the mean vector Mean.

StdCovariance is a NUMSERIES-by-NUMSERIES matrix of standard errors of estimates for each element of the covariance matrix Covariance.

Use this routine after estimating the mean and covariance of Data with ecmnmle. If the mean and distinct covariance elements are treated as the parameter \(\theta\) in a complete-data maximum-likelihood estimation, then as the number of samples increases, \(\theta\) attains asymptotic normality such that
\[
\theta-E[\theta] \square N\left(0, I^{-1}(\theta)\right),
\]
where \(E[\theta]\) is the mean and \(I(\theta)\) is the Fisher information matrix.
With missing data, the Hessian \(H(\theta)\) is a good approximation for the Fisher information (which can only be approximated when data is missing).

It is usually advisable to use the default Method since the resultant standard errors incorporate the increased uncertainty due to missing data. In particular, standard errors calculated with the Hessian are generally larger than standard errors calculated with the Fisher information matrix.

Note This routine is very slow for NUMSERIES > 10 or NUMSAMPLES > 1000.

\section*{See Also}
Purpose Effective rate of return
Syntax Return = effrr(Rate, NumPeriods)
Arguments
Description Return \(=\) effrr(Rate, NumPeriods) calculates the annual effectiverate of return. Compounding continuously returns Return equivalentto (e^Rate-1).
Examples Find the effective annual rate of return based on an annual percentage rate of \(9 \%\) compounded monthly.
Return \(=\operatorname{effrr}(0.09,12)\)
returns
Return =
0.0938 or \(9.38 \%\)
See Also nomrr

Purpose
Compute expected lower partial moments for normal asset returns

\section*{Syntax}
```

elpm(Mean, Sigma)
elpm(Mean, Sigma, MAR)
elpm(Mean, Sigma, MAR, Order)
Moment = elpm(Mean, Sigma, MAR, Order)

```

\section*{Arguments}
\begin{tabular}{ll} 
Mean & \begin{tabular}{l} 
NUMSERIES vector with mean returns for a collection \\
of NUMSERIES assets.
\end{tabular} \\
Sigma & \begin{tabular}{l} 
NUMSERIES vector with standard deviation of returns \\
for a collection of NUMSERIES assets.
\end{tabular} \\
MAR & \begin{tabular}{l} 
(Optional) Scalar minimum acceptable return \\
(default MAR = 0). This is a cutoff level of return such \\
that all returns above MAR contribute nothing to the \\
lower partial moment.
\end{tabular} \\
Order & \begin{tabular}{l} 
(Optional) Either a scalar or a NUMORDERS vector \\
of nonnegative integer moment orders. If no order \\
specified, default Order = 0, which is the shortfall \\
probability. This function will not work for negative \\
or noninteger orders.
\end{tabular}
\end{tabular}

Description Given NUMSERIES asset returns with a vector of mean returns in a NUMSERIES vector Mean, a vector of standard deviations of returns in a NUMSERIES vector Sigma, a scalar minimum acceptable return MAR, and one or more nonnegative integer moment orders in a NUMORDERS vector Order, compute expected lower partial moments (elpm) relative to MAR for each asset in a NUMORDERS-by-NUMSERIESmatrix Moment.

The output, Moment, is a NUMORDERS-by-NUMSERIES matrix of expected lower partial moments with NUMORDERS Orders and NUMSERIES series,
that is, each row contains expected lower partial moments for a given order.

Note To compute upper partial moments, just reverse the signs of both the input Mean and MAR (do not reverse the signs of either Sigma or the output). This function computes expected lower partial moments with the mean and standard deviation of normally distributed asset returns. To compute sample lower partial moments from asset returns which have no distributional assumptions, use lpm.

\section*{Examples}

See "Expected Lower Partial Moments Example" on page 5-15.
See Also 1pm

Purpose Compute expected maximum drawdown for Brownian motion
Syntax EDD = emaxdrawdown(Mu, Sigma, T)

\section*{Arguments}

\section*{Description}

EDD = emaxdrawdown(Mu, Sigma, T) computes the expected maximum drawdown for a Brownian motion for each time period in \(T\) using the following equation:
\[
d X(t)=\mu d t+\sigma d W(t)
\]

If the Brownian motion is geometric with the stochastic differential equation
\[
d S(t)=\mu_{0} S(t) d t+\sigma_{0} S(t) d W(t)
\]
then use Ito's lemma with \(X(t)=\log (S(t))\) such that
\[
\begin{aligned}
\mu & =\mu_{0}-0.5 \sigma_{0}^{2} \\
\sigma & =\sigma_{0}
\end{aligned}
\]
converts it to the form used here.
The output argument ExpDrawdown is computed using an interpolation method. Values are accurate to a fraction of a basis point. Maximum drawdown is nonnegative since it is the change from a peak to a trough.

\begin{abstract}
Note To compare the actual results from maxdrawdown with the expected results of emaxdrawdown, set the Format input argument of maxdrawdown to either of the nondefault values ('arithmetic' or 'geometric '). These are the only two formats emaxdrawdown supports.
\end{abstract}

\section*{Examples See "Expected Maximum Drawdown Example" on page 5-21.}

References Malik Magdon-Ismail, Amir F. Atiya, Amrit Pratap, and Yaser S. Abu-Mostafa, "On the Maximum Drawdown of a Brownian Motion," Journal of Applied Probability, Volume 41, Number 1, March 2004, pp. 147-161.

\author{
See Also \\ maxdrawdown
}

Purpose Last date entry

\section*{Syntax \\ end}

Description
end returns the index to the last date entry in a financial time series object.

Examples Consider a financial time series object called fts:
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|l|}{\begin{tabular}{l}
desc: DJI30MAR94.dat \\
freq: Daily (1)
\end{tabular}} \\
\hline 'dates: (20)' & 'Open: & (20) ' \\
\hline '04-Mar-1994' & [ & \(3830.9]\) \\
\hline '07-Mar-1994' & [ & \(3851.7]\) \\
\hline '08-Mar-1994' & [ & 3858.5] \\
\hline '09-Mar-1994' & [ & 3854 ] \\
\hline '10-Mar-1994' & [ & \(3852.6]\) \\
\hline 11-Mar-1994' & [ & \(3832.6]\) \\
\hline '14-Mar-1994' & [ & 3870.3] \\
\hline 16-Mar-1994' & [ & 3851 ] \\
\hline '17-Mar-1994' & [ & \(3853.6]\) \\
\hline '18-Mar-1994' & [ & \(3865.4]\) \\
\hline '21-Mar-1994' & [ & 3878.4 ] \\
\hline '22-Mar-1994' & [ & 3865.7 ] \\
\hline '23-Mar-1994' & [ & \(3868.9]\) \\
\hline '24-Mar-1994' & [ & \(3849.9]\) \\
\hline '25-Mar-1994' & [ & 3827.1 ] \\
\hline '28-Mar-1994' & [ & 3776.5] \\
\hline '29-Mar-1994' & [ & \(3757.2]\) \\
\hline '30-Mar-1994' & [ & \(3688.4]\) \\
\hline '31-Mar-1994' & [ & 3639.7 ] \\
\hline
\end{tabular}

The command fts(15:end)returns
ans \(=\)
desc: DJI30MAR94.dat
freq: Daily (1)
'dates: (6)'
'24-Mar-1994'
'25-Mar-1994'
'28-Mar-1994'
[ \(29-M a r-1994 '\)
'30-Mar-1994'
'31-Mar-1994'

See Also subsasgn | subsref | end
Purpose Last date of month
```

Syntax DayMonth = eomdate(Date)
DayMonth = eomdate(Year, Month)

```

Description DayMonth = eomdate(Date) returns the serial date number of the last date of the month for the given Date. Enter Date as a four-digit integer or a date string.

DayMonth = eomdate(Year, Month) returns the serial date number of the last date of the month for the given year and month. Enter Year as a four-digit integer; enter Month as an integer from 1 through 12.

Either input argument can contain multiple values, but if so, the other input must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by- \(n\) vector of integers, then Month must be a 1-by- \(n\) vector of integers or a single integer. DayMonth is then a 1 -by- \(n\) vector of date numbers.

Use the function datestr to convert serial date numbers to formatted date strings or datenum to convert date and time to serial date number.

\section*{Examples}
```

DayMonth = eomdate(2001, 2)
DayMonth =
7 3 0 9 1 0
datestr(DayMonth)
ans =
28-Feb-2001
Year = [2002 2003 2004 2005];
DayMonth = eomdate(Year, 2)
DayMonth =
731275 731640 732006 732371
datestr(DayMonth)

```
ans =
28-Feb-2002
28-Feb-2003
29-Feb-2004
28-Feb-2005
See Also day | eomday | lbusdate | month | year

Purpose Last day of month

\section*{Syntax \(\quad E=\operatorname{eomday}(Y, M)\)}

Description \(\quad E=\operatorname{eomday}(Y, M)\) returns the last day of the year and month given by corresponding elements of arrays Y and M .

Examples \(\quad\) Because 1996 is a leap year, the statement eomday \((1996,2)\) returns 29. To show all the leap years in this century, try:
```

y = 1900:1999;
E = eomday(y, 2);
y(find(E == 29))
ans =

```

```

        Columns 7 through 12
            1928 1932 1936 1940 1944 1948
        Columns 13 through 18
            1952 1956 1960 1964 1968 1972
        Columns }19\mathrm{ through 24
            1976 1980 1984 1988 1992 1996
    ```
See Also day | eomdate | month
```

Purpose Multiple financial times series object equality
Syntax tsobj_1 == tsobj_2
iseq = eq(tsobj_1, tsobj_2)

```

Arguments
\begin{tabular}{ll} 
tsobj_1 & Financial time series object. \\
tsobj_2 & Financial time series object.
\end{tabular}

\section*{Description tsobj_1 == tsobj_2 returns True (1) if both financial time series} objects have the same dates, frequencies, data series names, and data values. Otherwise, eq returns False (0).

Note The data series names are case-sensitive, but do not have to be in the same order within each object.

\author{
Examples Compare : \\ load disney \\ dis == dis \\ ans \(=\) \\ 1
}

See Also isequal

\section*{Portfolio.estimateAssetMoments}

Purpose Estimate mean and covariance of asset returns from data

Syntax

Description

Tips

Input
Arguments
obj = estimateAssetMoments(obj, AssetReturns)
obj = estimateAssetMoments(obj, AssetReturns, varargin)
obj = estimateAssetMoments(obj, AssetReturns) to estimate the mean and covariance of asset returns from data.
obj = estimateAssetMoments(obj, AssetReturns, varargin) to estimate mean and covariance of asset returns from data with additional options specified by one or more Name, Value pair arguments.

Use dot notation to estimate the mean and covariance of asset returns from data:
```

obj = obj.estimateAssetMoments(AssetReturns, varargin);

```
obj
A portfolio object [Portfolio].
AssetReturns
Either a matrix or fints object that contains asset data that can be converted to asset returns [NumSamples-by-NumAssets matrix].

Note This method estimates the mean and covariance of asset returns from either price or return data. Data can reside in a NumSamples-by-NumAssets matrix of NumSamples prices or returns. This matrix represents a given periodicity for a collection of NumAssets assets or a fints object with NumSamples observations and NumAssets time series.

\section*{Name-Value Pair Arguments for varargin}

Optional comma-separated pairs of Name, Value arguments, where Name is the argument name and Value is the corresponding value. Name must

\section*{Portfolio.estimateAssetMoments}
appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, , NameN, ValueN.

\section*{DataFormat}

If the input data are prices, these values can be converted into returns with the DataFormat flag, where the default format is assumed to be returns. Be careful using price data because portfolio optimization requires total returns and not simply price returns.

Acceptable values for DataFormat are:
- 'Returns' - Data in AssetReturns contains asset total returns.
- 'Prices' - Data in AssetReturns contains asset total return prices.

Default: 'Returns'
MissingData
To handle time series with missing data (indicated with NaN values), the MissingData flag either uses the ECM algorithm to obtain maximum likelihood estimates in the presences of NaN values or excludes samples with NaN values. Since the default is false, it is necessary to specify MissingData as true to use the ECM algorithm.

Acceptable values for MissingData are:
- false - Do not use ECM algorithm to handle NaN values (just exclude NaN values).
- true - Use ECM algorithm to handle NaN values.

\section*{Portfolio.estimateAssetMoments}

For more information on the ECM algorithm, see ecmnmle and Chapter 7, "Regression with Missing Data".

Default: false

\section*{GetAssetList}

If a fints object is passed into this method and the GetAssetList flag is true, the series names from the fints object are used as asset names in obj.AssetList.

If a matrix is passed and the GetAssetList flag is true, default asset names are created based on the AbstractPortfolio property defaultforAssetList, which is currently 'Asset'.
If the GetAssetList flag is false, no action occurs, which is the default behavior.

Acceptable values for GetAssetList are:
- false - Do not extract or create asset names.
- true - Extract or create asset names from fints object.

Default: false

\section*{Output obj \\ Arguments \\ Updated portfolio object [Portfolio].}

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Portfolio.estimateAssetMoments}
```

Examples
To illustrate using estimateAssetMoments, generate random samples of 120 observations of asset returns for four assets from the mean and covariance of asset returns in the variables $m$ and $C$ with portsim. The default behavior of portsim creates simulated data with estimated mean and covariance identical to the input moments $m$ and $C$. In addition to a return series created by portsim in the variable $X$, a price series is created in the variable Y :

```
```

m = [ 0.05; 0.1; 0.12; 0.18 ];

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
    0 0.0119 0.0336 0.1225 ];
m = m/12;
m = m/12;
C = C/12;
C = C/12;
X = portsim(m', C, 120);
X = portsim(m', C, 120);
Y = ret2tick(X);
```

Y = ret2tick(X);

```

Given asset returns and prices in the variables \(X\) and \(Y\) from above, the following examples demonstrate equivalent ways to estimate asset moments for the portfolio object. A portfolio object is created in \(p\) with the moments of asset returns set directly in the constructor and a second portfolio object is created in \(q\) to obtain the mean and covariance of asset returns from asset return data in X usingestimateAssetMoments:
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
p = Portfolio('mean',m,'covar',C);
q = Portfolio;
q = q.estimateAssetMoments(X);

```

\section*{Portfolio.estimateAssetMoments}
```

[passetmean, passetcovar] = p.getAssetMoments
[qassetmean, qassetcovar] = q.getAssetMoments
passetmean =
0.0042
0.0083
0 . 0 1 0 0
0.0150
passetcovar =

| 0.0005 | 0.0003 | 0.0002 | 0 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0048 | 0.0028 |
| 0 | 0.0010 | 0.0028 | 0.0102 |

qassetmean =
0.0042
0.0083
0.0100
0.0150
qassetcovar =

| 0.0005 | 0.0003 | 0.0002 | 0.0000 |
| :--- | :--- | :--- | :--- |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0048 | 0.0028 |
| 0.0000 | 0.0010 | 0.0028 | 0.0102 |

```

Notice how either approach yields the same moments. The default behavior of estimateAssetMoments is to work with asset returns. If, instead, you have asset prices, such as in the variable \(Y\), estimateAssetMoments accepts a parameter name 'DataFormat' with a corresponding value set to 'prices' to indicate that the input to
the method is in the form of asset prices and not returns (the default parameter value for 'DataFormat' is 'returns'). The following example compares direct assignment of moments in the portfolio object \(p\) with estimated moments from asset price data in \(Y\) in the portfolio object q:
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
X = portsim(m', C, 120);
Y = ret2tick(X);
p = Portfolio('mean',m,'covar',C);
q = Portfolio;
q = q.estimateAssetMoments(Y, 'dataformat', 'prices');
[passetmean, passetcovar] = p.getAssetMoments
[qassetmean, qassetcovar] = q.getAssetMoments
passetmean =
0.0042
0.0083
0.0100
0.0150
passetcovar =

| 0.0005 | 0.0003 | 0.0002 | 0 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0048 | 0.0028 |

```

\section*{Portfolio.estimateAssetMoments}
```

    0}00.0010 0.0028 0.0102 
    qassetmean =
0.0042
0.0083
0 . 0 1 0 0
0.0150
qassetcovar =
0.0005 0.0003 0.0002 0.0000
0.0003 0.0024 0.0017 0.0010
0.0002 0.0017 0.0048
0.0000 0.0010 0.0028 0.0102

```

\section*{See Also}
setAssetMoments

\section*{Tutorials}
- "Working with Asset Returns and Moments of Asset Returns" on page 4-36
- "Estimating Asset Moments from Financial Time Series Data" on page 4-46
SuperclassesPurposeEstimate global lower and upper bounds for set of portfolios
Syntax [glb, gub, isbounded] = estimateBounds(obj) [glb, gub, isbounded] = estimateBounds(obj, obtainExactBounds)
Description
[glb, gub, isbounded] = estimateBounds(obj) to estimate the global lower and upper bounds for a given set of portfolios.
[glb, gub, isbounded] = estimateBounds(obj, obtainExactBounds) to estimate the global lower and upper bounds for a given set of portfolios with an additional option specified for obtainExactBounds.

\section*{Tips}

Input Arguments
- Use dot notation to estimate the global lower and upper bounds for a given set of portfolios:
[glb, gub, isbounded] = obj.estimateBounds;
- Estimated bounds are accurate in most cases to within \(1.0 e-8\). If you intend to use these bounds directly in a portfolio object, ensure that if you impose such bound constraints, a lower bound of 0 is probably preferable to a lower bound of, for example, 1.0e-10 for portfolio weights.
obj
A portfolio object [Portfolio].

\section*{obtainExactBounds}
(Optional) Boolean flag to specify whether to solve for all bounds or to accept specified bounds whenever available [logical]. If bounds are known, set obtainExactBounds to false to accept known bounds.

\section*{Portfolio.estimateBounds}

Default: True

\section*{Output \\ Arguments \\ glb}

Global lower bounds for portfolio set [vector]. gub

Global upper bounds for portfolio set [vector].
isbounded
Indicates if set is empty ([ ]), bounded (true), or unbounded (false).

Note By definition, any portfolio set must be nonempty and bounded:
- If the set is empty, isbounded \(=[\) ].
- If the set is nonempty and unbounded, isbounded = false.
- If the set is nonempty and bounded,isbounded = true.
- If the set is empty, glb and gub are set to NaN vectors.

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.
Examples Create an unbounded portfolio set as follows:
```

p = Portfolio('AInequality', [1 -1; 1 1 ], 'bInequality', 0);
[lb, ub, isbounded] = p.estimateBounds
lb =
-Inf
-Inf
ub =
1.0e-008 *
-0.3712
Inf
isbounded =

```
        0
estimateBounds returns (possibly infinite) bounds and sets the
isbounded flag to false. The result shows which assets are unbounded
so that you can apply bound constraints as necessary.
See Also

checkFeasibility
Tułorials - "Validating a Portfolio Set" on page 4-73

\section*{Portfolio.estimateFrontier}

Superclasses AbstractPortfolio
Purpose Estimate specified number of optimal portfolios over entire efficient frontier

Syntax

Description

Tips

Input obj
Arguments
A portfolio object [Portfolio].
NumPorts
(Optional) Number of points to obtain on the efficient frontier [scalar integer].

Note If no value is specified for NumPorts, the default value is obtained from the hidden property defaultNumPorts (current default value is 10 ). If NumPorts \(=1\), this method returns the portfolio specified by the hidden property defaultFrontierLimit (current default value is 'min').

Default: 10

\section*{Portfolio.estimateFrontier}

\section*{Output Arguments}

Optimal portfolios on the efficient frontier with specified number of portfolios spaced equally from minimum to maximum portfolio return [NumAssets-by-NumPorts matrix].
pbuy
Purchases relative to an initial portfolio for optimal portfolios on the efficient frontier [NumAssets-by-NumPorts matrix].
psell
Sales relative to an initial portfolio for optimal portfolios on the efficient frontier [NumAssets-by-NumPorts matrix].

Note If no initial portfolio is specified in obj. InitPort, that value is assumed to be 0 such that pbuy \(=\max (0\), pwgt) and psell \(=\max (0\), -pwgt).

\section*{Attributes}

\section*{Examples}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Obtain the default number of efficient portfolios over the entire range of the efficient frontier:
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;

```

\section*{Portfolio.estimateFrontier}
```

    0 0.0119 0.0336 0.1225 ];
    p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontier;
disp(pwgt);

| 0.8891 | 0.7215 | 0.5540 | 0.3865 | 0.2190 | 0.0515 | 0 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.0369 | 0.1289 | 0.2209 | 0.3129 | 0.4049 | 0.4969 | 0.4049 | 0.2314 | 0.0579 | 0 |
| 0.0404 | 0.0567 | 0.0730 | 0.0893 | 0.1056 | 0.1219 | 0.1320 | 0.1394 | 0.1468 | 0 |
| 0.0336 | 0.0929 | 0.1521 | 0.2113 | 0.2705 | 0.3297 | 0.4630 | 0.6292 | 0.7953 | 1.0000 |

```

Starting from the initial portfolio, estimateFrontier returns purchases and sales to get from your initial portfolio to each efficient portfolio on the efficient frontier. Given an initial portfolio in pwgt0, you can obtain purchases and sales:
```

pwgt0 = [ 0.3; 0.3; 0.2; 0.1 ];
p = p.setInitPort(pwgt0);
[pwgt, pbuy, psell] = p.estimateFrontier;
display(pwgt);
display(pbuy);
display(psell);
pwgt =

| 0.8891 | 0.7215 | 0.5540 | 0.3865 | 0.2190 | 0.0515 | 0 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0.0369 | 0.1289 | 0.2209 | 0.3129 | 0.4049 | 0.4969 | 0.4049 | 0.2314 | 0.0579 | 0 |
| 0.0404 | 0.0567 | 0.0730 | 0.0893 | 0.1056 | 0.1219 | 0.1320 | 0.1394 | 0.1468 | 0 |
| 0.0336 | 0.0929 | 0.1521 | 0.2113 | 0.2705 | 0.3297 | 0.4630 | 0.6292 | 0.7953 | 1.0000 |

```
pbuy \(=\)
\begin{tabular}{rrrrrrrrrr}
0.5891 & 0.4215 & 0.2540 & 0.0865 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0129 & 0.1049 & 0.1969 & 0.1049 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0521 & 0.1113 & 0.1705 & 0.2297 & 0.3630 & 0.5292 & 0.6953 & 0.9000
\end{tabular}
psell =
\begin{tabular}{rrrrrrrrrr}
0 & 0 & 0 & 0 & 0.0810 & 0.2485 & 0.3000 & 0.3000 & 0.3000 & 0.3000 \\
0.2631 & 0.1711 & 0.0791 & 0 & 0 & 0 & 0 & 0.0686 & 0.2421 & 0.3000 \\
0.1596 & 0.1433 & 0.1270 & 0.1107 & 0.0944 & 0.0781 & 0.0680 & 0.0606 & 0.0532 & 0.2000 \\
0.0664 & 0.0071 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{tabular}

If you do not have an initial portfolio, the purchase and sale weights assume that your initial portfolio is 0 .

\author{
See Also \\ estimateFrontierByReturn | estimateFrontierByRisk | estimateFrontierLimits \\ \footnotetext{
Tutorials \\ - "Estimate Efficient Frontiers" on page 4-88
}
}

\section*{Portfolio.estimateFrontierByReturn}
Superclasses AbstractPortfolio
Purpose Estimate optimal portfolios with targeted portfolio returns
Syntax [pwgt, pbuy, psell] = estimateFrontierByReturn(obj, TargetReturn)
Description [pwgt, pbuy, psell] = estimateFrontierByReturn(obj,
```TargetReturn) to estimate optimal portfolios with targeted portfolioreturns.
```

Tips Use dot notation to estimate optimal portfolios with targeted portfolio
Input

```Argumentsobjreturns:
```

```
[pwgt, pbuy, psell] = obj.estimateFrontierByReturn(TargetReturn);
```

```
[pwgt, pbuy, psell] = obj.estimateFrontierByReturn(TargetReturn);
```

A portfolio object [Portfolio].
TargetReturn
Target values for portfolio return [NumPorts vector].
Note TargetReturn specifies target returns for portfolios onthe efficient frontier. If any TargetReturn values are outsidethe range of returns for efficient portfolios, the TargetReturnis replaced with the minimum or maximum efficient portfolioreturn, depending upon whether the target return is below orabove the range of efficient portfolio returns.
Output
Arguments
pwgt

Optimal portfolios on the efficient frontier with specified target returns from TargetReturn that are [NumAssets-by-NumPorts matrix].

## Portfolio.estimateFrontierByReturn

## pbuy

Purchases relative to an initial portfolio for optimal portfolios on the efficient frontier that are [NumAssets-by-NumPorts matrix].

## psell

Sales relative to an initial portfolio for optimal portfolios on the efficient frontier that are [NumAssets-by-NumPorts matrix].

Note If no initial portfolio is specified in obj. InitPort, it is assumed to be 0 , such that pbuy $=\max (0$, pwgt $)$ and psell $=\max (0,-$ pwgt $)$.

## Attributes

## Examples To obtain efficient portfolios that have targeted portfolio returns, estimateFrontierByReturn accepts one or more target portfolio returns and obtains efficient portfolios with the specified returns. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio returns of $6 \%, 9 \%$, and $12 \%$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
```


## Portfolio.estimateFrontierByReturn

p = p.setDefaultConstraints;
pwgt = p.estimateFrontierByReturn([0.06, 0.09, 0.12]);
display(pwgt);
pwgt =
$0.8772 \quad 0.5032 \quad 0.1293$
$0.0434 \quad 0.2488 \quad 0.4541$
$0.0416 \quad 0.0780 \quad 0.1143$
$0.0378 \quad 0.1700 \quad 0.3022$
See Also estimateFrontier | estimateFrontierByRisk |
Tutorials

- "Estimate Efficient Frontiers" on page 4-88


## Portfolio.estimateFrontierByRisk

## Superclasses AbstractPortfolio

Purpose
Syntax
Description

Tips

Input
Arguments

## Output Arguments

Estimate optimal portfolios with targeted portfolio risks

```
[pwgt, pbuy, psell] = estimateFrontierByRisk(obj, TargetRisk)
```

[pwgt, pbuy, psell] = estimateFrontierByRisk(obj,
TargetRisk) to estimate optimal portfolios with targeted portfolio risks.

Use dot notation to estimate optimal portfolios with targeted portfolio risks:

```
[pwgt, pbuy, psell] = obj.estimateFrontierByRisk(TargetRisk);
```

obj

A portfolio object [Portfolio].
TargetRisk
Target values for portfolio risk [NumPorts vector].

Note If any TargetRisk values are outside the range of risks for efficient portfolios, the target risk is replaced with the minimum or maximum efficient portfolio risk, depending upon whether the target risk is below or above the range of efficient portfolio risks.
pwgt
Optimal portfolios on the efficient frontier with specified target risks from TargetRisk that are [NumAssets-by-NumPorts matrix].
pbuy
Purchases relative to an initial portfolio for optimal portfolios on the efficient frontier that are [NumAssets-by-NumPorts matrix].

## Portfolio.estimateFrontierByRisk

```
psell
```

Sales relative to an initial portfolio for optimal portfolios on the efficient frontier that are [NumAssets-by-NumPorts matrix].

Note If no initial portfolio is specified in obj. InitPort, it is assumed to be 0 such that so pbuy $=\max (0, \mathrm{pwgt})$ and $\mathrm{psell}=\max (0$, -pwgt).

## Attributes

## Examples

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

To obtain efficient portfolios that have targeted portfolio risks, estimateFrontierByRisk accepts one or more target portfolio risks and obtains efficient portfolios with the specified risks. Assume you have a universe of four assets where you want to obtain efficient portfolios with target portfolio risks of $12 \%, 14 \%$, and $16 \%$ :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
        0.00408 0.0289 0.0204 0.0119;
        0.00192 0.0204 0.0576 0.0336;
        0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontierByRisk([0.12, 0.14, 0.16]);
```


## Portfolio.estimateFrontierByRisk

| pwgt = |  |  |
| :---: | :---: | :---: |
| 0.3984 | 0.2659 | 0.1416 |
| 0.3064 | 0.3791 | 0.4474 |
| 0.0882 | 0.1010 | 0.1131 |
| 0.2071 | 0.2540 | 0.2979 |

See Also estimateFrontier | estimateFrontierByReturn |
Tutorials . "Estimate Efficient Frontiers" on page 4-88

## Portfolio.estimateFrontierLimits

Superclasses AbstractPortfolio
Purpose Estimate optimal portfolios at endpoints of efficient frontier

Description

Tips

Input
Arguments
Syntax $\quad[$ pwgt, pbuy, psell] $=$ estimateFrontierLimits(obj)
$[$ pwgt, pbuy, psell] $=$ estimateFrontierLimits(obj, Choice)
[pwgt, pbuy, psell] = estimateFrontierLimits(obj)
[pwgt, pbuy, psell] = estimateFrontierLimits(obj, Choice)
[pwgt, pbuy, psell] = estimateFrontierLimits(obj) to estimate the optimal portfolios at the endpoints of the efficient frontier.
[pwgt, pbuy, psell] = estimateFrontierLimits(obj, Choice) to estimate the optimal portfolios at the endpoints of the efficient frontier with an additional option specified for the Choice argument.

Use dot notation to estimate the optimal portfolios at the endpoints of the efficient frontier:
[pwgt, pbuy, psell] = obj.estimateFrontierLimits(Choice);
obj
A portfolio object [Portfolio].
Choice
Indicates which portfolios to obtain at the extreme ends of the efficient frontier [string].
Choice specifies various actions with default value [ ]. The options for Choice action are:

- [] - Compute both minimum-risk and maximum-return portfolios.
- 'Both' - Compute both minimum-risk and maximum-return portfolios.
- 'Min' - Compute minimum-risk portfolio only.
- 'Max' - Compute maximum-return portfolio only.


## Portfolio.estimateFrontierLimits

## Default: []

## Output pwgt <br> Arguments <br> Optimal portfolios at the endpoints of the efficient frontier TargetReturn that are [NumAssets-by-NumPorts matrix]. <br> pbuy <br> Purchases relative to an initial portfolio for optimal portfolios at the endpoints of the efficient frontier that are [NumAssets-by-NumPorts matrix]. <br> psell <br> Sales relative to an initial portfolio for optimal portfolios at the endpoints of the efficient frontier that are [ NumAssets-by-NumPorts matrix].

Note If no initial portfolio is specified in obj. InitPort, it is assumed to be 0 such that pbuy $=\max (0, \mathrm{pwgt})$ and psell $=\max (0,-\mathrm{pwg})$.

## Attributes

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

## Examples

Given portfolio p, estimateFrontierLimits obtains the endpoint portfolios:

## Portfolio.estimateFrontierLimits

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
        0.00408 0.0289 0.0204 0.0119;
        0.00192 0.0204 0.0576 0.0336;
        0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontierLimits;
disp(pwgt);
disp(pwgt);
    0.8891 0
    0.0369 0
    0.0404 0
    0.0336 1.0000
```

See Also estimateFrontier | estimateFrontierByReturn |
Tutorials . "Estimate Efficient Frontiers" on page 4-88

## Portfolio.estimatePortMoments

## Purpose Estimate moments of portfolio returns

```
Syntax
[prsk, pret] = estimatePortMoments(obj, pwgt)
```

Description

Tips

## Input <br> Arguments

Output
Arguments
prsk
Estimates for standard deviations of portfolio returns for each portfolio in pwgt [NumPorts vector].
pret
Estimates for means of portfolio returns for each portfolio in pwgt [NumPorts vector].

## Attributes

## Portfolio.estimatePortMoments

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

```
Examples Given portfolio p, use estimatePortMoments to show the range of risks and returns for efficient portfolios:
```

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontierLimits;
[prsk, pret] = p.estimatePortMoments(pwgt);
disp([prsk, pret]);
$0.0769 \quad 0.0590$
$0.3500 \quad 0.1800$

```

\section*{See Also}
```

estimatePortReturn | estimatePortRisk

```

Tutorials

- "Estimate Efficient Portfolios" on page 4-77

\section*{Portfolio.estimatePortReturn}

\section*{Superclasses AbstractPortfolio}

Purpose Estimate mean of portfolio returns (portfolio return)
Syntax pret = estimatePortReturn(obj, pwgt)
Description pret = estimatePortReturn(obj, pwgt) to estimate the mean of portfolio returns (portfolio return).
estimatePortReturn computes the mean of portfolio returns as the proxy for portfolio returns.

Note Depending upon whether costs have been set, the portfolio return is either gross or net portfolio returns.

Tips
Use dot notation to estimate the mean of portfolio returns (portfolio return):
```

pret = obj.estimatePortReturn(pwgt);

```

\section*{Input \\ Arguments}

\section*{Output \\ Arguments}
pret
Estimates for means of portfolio returns for each portfolio in pwgt [NumPorts vector].

\section*{Portfolio.estimatePortReturn}

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Examples Given portfolio p, use estimatePortReturn to estimate the mean of portfolio returns:}
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
0.00408 0.0289 0.0204 0.0119;
0.00192 0.0204 0.0576 0.0336;
0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
pwgt = p.estimateFrontierLimits;
pret = p.estimatePortReturn(pwgt);
disp(pret)
0.0590
0.1800

```
See Also \begin{tabular}{l} 
estimatePortRisk | estimateFrontierByReturn | \\
estimateFrontierByRisk
\end{tabular}
How To . "Obtaining Efficient Portfolios for Target Returns" on page 4-81

\section*{Portfolio.estimatePortRisk}

\section*{Superclasses AbstractPortfolio}

Purpose
Estimate standard deviation of portfolio returns (portfolio risk)
Syntax
prsk = estimatePortRisk(obj, pwgt)
Description

Tips

Input
Arguments

Output prsk
Arguments

\section*{Attributes}

Estimates for standard deviations of portfolio returns for each portfolio in pwgt [NumPorts vector].
Use dot notation to estimate the standard deviation of portfolio returns (portfolio risk):
```

prsk = obj.estimatePortRisk(pwgt);

```
obj
A portfolio object [Portfolio].
pwgt
A collection of portfolios[NumAssets-by-NumPorts matrix] where NumAssets is the number of asset in the universe and NumPorts is the number of portfolios in the collection of portfolios.
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}
false

\section*{Portfolio.estimatePortRisk}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.
```

Examples Given portfolio p, use estimatePortRisk to show the standard deviation of portfolio returns:

```
```

m = [ 0.05; 0.1; 0.12; 0.18 ];

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
C = [ 0.0064 0.00408 0.00192 0;
        0.00408 0.0289 0.0204 0.0119;
        0.00408 0.0289 0.0204 0.0119;
        0.00192 0.0204 0.0576 0.0336;
        0.00192 0.0204 0.0576 0.0336;
        0 0.0119 0.0336 0.1225 ];
        0 0.0119 0.0336 0.1225 ];
p = Portfolio;
p = Portfolio;
p = p.setAssetMoments(m, C);
p = p.setAssetMoments(m, C);
p = p.setDefaultConstraints;
p = p.setDefaultConstraints;
pwgt = p.estimateFrontierLimits;
pwgt = p.estimateFrontierLimits;
prsk = p.estimatePortRisk(pwgt);
prsk = p.estimatePortRisk(pwgt);
disp(prsk)
disp(prsk)
    0.0769
    0.0769
    0.3500
    0.3500
See Also estimatePortReturn | estimateFrontierByReturn |
How To . "Obtaining Efficient Portfolios for Target Risks" on page 4-83
```


## Purpose

Expected return and covariance from return time series

Syntax<br>[ExpReturn, ExpCovariance, NumEffObs] = ewstats(RetSeries, DecayFactor, WindowLength)

## Arguments

## Description

[ExpReturn, ExpCovariance, NumEffObs] = ewstats(RetSeries, DecayFactor, WindowLength) computes estimated expected returns, estimated covariance matrix, and the number of effective observations. These are maximum likelihood estimates which are generally biased.

ExpReturn is a 1 -by-NASSETS vector of estimated expected returns.
ExpCovariance is an NASSETS-by-NASSETS estimated covariance matrix. The standard deviations of the asset return processes are given by
STDVec = sqrt(diag(ExpCovariance))

The correlation matrix is

```
CorrMat = ExpCovariance./( STDVec*STDVec' )
```

NumEffobs is the number of effective observations $=$ (1-DecayFactor^WindowLength)/(1-DecayFactor).

A smaller DecayFactor or WindowLength emphasizes recent data more strongly but uses less of the available data set.

## Examples

```
RetSeries = [ 0.24 0.08
        0.15 0.13
        0.27 0.06
        0.14 0.13 ];
DecayFactor = 0.98;
[ExpReturn, ExpCovariance] = ewstats(RetSeries, DecayFactor)
ExpReturn =
    0.1995 0.1002
ExpCovariance =
    0.0032 -0.0017
    -0.0017 0.0010
```


## See Also

cov | mean
Purpose Exponential values
Syntax newfts $=\exp (t s o b j)$
Description newfts $=\exp (t s o b j)$ calculates the natural exponential (base e) of all the data in the data series of the financial time series object tsobj and returns the result in the object newfts.
See Also ..... $\log |\log 2| \log 10$

Purpose Data series extraction
Syntax ftse = extfield(tsobj, fieldnames)

## Arguments

$$
\begin{array}{ll}
\text { tsobj } & \text { Financial time series object } \\
\text { fieldnames } & \begin{array}{l}
\text { Data series to be extracted. A cell array if a list of } \\
\text { data series names (fieldnames) is supplied. A string } \\
\text { if only one is wanted. }
\end{array}
\end{array}
$$

## Description

ftse $=$ extfield(tsobj, fieldnames) extracts from tsobj the dates and data series specified by fieldnames into a new financial time series object ftse. ftse has all the dates in tsobj but contains a smaller number of data series.

## Examples

extfield is identical to referencing a field in the object. For example,

```
ftse = extfield(fts, 'Close')
```

is the same as

```
ftse = fts.Close
```

This function is the complement of the function rmfield.
See Also rmfield
$\begin{array}{ll}\text { Purpose } & \text { First business date of month } \\ \text { Syntax } & \text { Date }=\text { fbusdate(Year, Month, Holiday, Weekend) }\end{array}$

Arguments

| Year | Enter as four-digit integer. |
| :---: | :---: |
| Month | Enter as integer from 1 to 12. |
| Holiday | (Optional) Vector of holidays and nontrading-day dates. All dates in Holiday must be the same format either serial date numbers or date strings. (Using date numbers improves performance.) The holidays function supplies the default vector. |
| Weekend | (Optional) Vector of length 7, containing 0 and 1, the value 1 indicating weekend days. The first element of this vector corresponds to Sunday. Thus, when Saturday and Sunday form the weekend (default), then Weekend $=\left[\begin{array}{lllllll}1 & 0 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |

## Description

Date $=$ fbusdate(Year, Month, Holiday, Weekend) returns the serial date number for the first business date of the given year and month. Holiday specifies nontrading days.

Year and Month can contain multiple values. If one contains multiple values, the other must contain the same number of values or a single value that applies to all. For example, if Year is a 1 -by-n vector of integers, then Month must be a 1 -by-n vector of integers or a single integer. Date is then a 1 -by-n vector of date numbers.

Use the function datestr to convert serial date numbers to formatted date strings.

## fbusdate

```
Examples Example 1:
Date = fbusdate(2001, 11); datestr(Date)
ans =
01-Nov-2001
Year = [2002 2003 2004];
Date = fbusdate(Year, 11); datestr(Date)
ans =
01-Nov-2002
03-Nov-2003
01-Nov-2004
```

Example 2: You can indicate that Saturday is a business day by appropriately setting the Weekend argument.

```
Weekend = [1 0 0 0 0 0 0];
```

March 1, 2003, is a Saturday. Use fbusdate to check that this Saturday is actually the first business day of the month.

```
Date = datestr(fbusdate(2003, 3, [], Weekend))
Date =
01-Mar-2003
```

See Also busdate | eomdate | holidays | isbusday | lbusdate

## Purpose

Data from financial time series object
Syntax newfts = fetch(oldfts, StartDate, StartTime, EndDate, EndTime, delta, dmy_specifier, time_ref)

## Arguments

| oldfts | Existing financial time series object. |
| :--- | :--- |
| StartDate | First date in the range from which data is to <br> be extracted. |
| StartTime | Beginning time on each day. If you do not <br> require specific times or oldfts does not <br> contain time information, use []. If you specify <br> StartTime, you must also specify EndTime. |
| EndDate | Last date in the range from which data is to <br> be extracted. |
| EndTime | Ending time on each day. If you do not require <br> specific times or oldfts does not contain time <br> information, use [ ]. If you specify EndTime, |
| delta | you must also specify StartTime. <br> Skip interval. Can be any positive integer. |
| dmy_specifier | Units for the skip interval specified by <br> dmy_specifier. |
| Specifies the units for delta. Can be |  |

## Description

newfts = fetch(oldfts, StartDate, StartTime, EndDate, EndTime, delta, dmy_specifier, time_ref) requests data from a financial time series object beginning from the start date and/or start time to the end date and/or end time, skipping a specified number of days, months, or years.

Note If time information is present in oldfts, using [ ] for start or end times results in fetch returning all instances of a specific date.

Example 1. Create a financial time series object containing both dates and times:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
times]);
myFts = fints(dates_times,(1:6)',{'Data1'},1,'My first FINTS')
myFts =
    desc: My first FINTS
    freq: Daily (1)
        'dates: (6)' 'times: (6)' 'Data1: (6)
        '01-Jan-2001' '11:00' [ 1]
        ' " ' '12:00' [ 2]
        '02-Jan-2001' '11:00' [ 3]
        ' " ' '12:00' [ 4]
        '03-Jan-2001' '11:00' [ 5]
        ' " ' '12:00' [ 6]
```

To fetch all dates and times from this financial time series, enter

```
fetch(myFts,'01-Jan-2001',[],'03-Jan-2001',[],1,'d')
```

or

```
fetch(myFts,'01-Jan-2001','11:00','03-Jan-2001','12:00',1,'d')
```

These commands reproduce the entire time series shown above.
To fetch every other day's data, enter

```
fetch(myFts,'01-Jan-2001',[],'03-Jan-2001',[],2,'d')
```

This produces

```
ans =
    desc: My first FINTS
    freq: Daily (1)
    'dates: (4)' 'times: (4)' 'Data1: (4)'
    '01-Jan-2001' '11:00' [ 1]
    ' " ' '12:00' [ 2]
    '03-Jan-2001', '11:00' [12:00' [ [ [ [ [ 
```

Example 2. Create a financial time series object with time intervals of less than 1 hour:

```
dates2 = ['01-Jan-2001';'01-Jan-2001'; '01-Jan-2001';...
'02-Jan-2001'; '02-Jan-2001';'02-Jan-2001'];
times2 = ['11:00';'11:05';'11:06';'12:00';'12:05';'12:06'];
dates_times2 = cellstr([dates2, repmat(' ',size(dates2,1),1),...
times2]);
myFts2 = fints(dates_times2,(1:6)',{'Data1'},1,'My second
FINTS')
myFts2 =
```

| 'dates: (6)' | 'times: (6)' | 'Data1: | (6) ${ }^{\prime}$ |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' | [ | 1] |
| ' " ' | '11:05' | [ | 2] |
| " ' | '11:06' | [ | 3] |
| '02-Jan-2001' | '12:00' | [ | $4]$ |
| " ' | '12:05' | [ | 5] |
| ' " ' | '12:06 ' | [ | 6] |

Use fetch to extract data from this time series object at 5 -minute intervals for each day starting at 11:00 o'clock on January 1, 2001.

```
fetch(myFts2,'01-Jan-2001',[],'02-Jan-2001',[],1, 'd',5)
    desc: My second FINTS
    freq: Daily (1)
    'dates: (4)' 'times: (4)' 'Data1: (4)'
    '01-Jan-2001' '11:00' [ 1]
    ' " ' '11:05' [ 2]
    '02-Jan-2001' '12:00' [ 4]
    ' " ' '12:05' [ 5]
```

You can use this version of fetch to extract data at specific times. For example, to fetch data only at 11:06 and 12:06 from myFts2, enter

```
fetch(myFts2,'01-Jan-2001',[],'02-Jan-2001',[],1,'d',...
{'11:06';'12:06'})
ans =
    desc: My second FINTS
    freq: Daily (1)
    'dates: (2)' 'times: (2)' 'Data1: (2)'
```

| '01-Jan-2001' | '11:06' | $[$ | $3]$ |
| :---: | :---: | :---: | :---: |
| '02-Jan-2001' | $' 12: 06 '$ | $[$ | $6]$ |

See Also extfield | ftsbound | getfield | subsref

## fieldnames

## Purpose Get names of fields

```
Syntax fnames = fieldnames(tsobj)
fnames = fieldnames(tsobj, srsnameonly)
```


## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| srsnameonly | Field names returned: |
|  | $0=$ All field names (default). |
|  | $1=$ Data series names only. |

[^4][^5]
## Purpose

Fill missing values in time series

```
Syntax
newfts = fillts(oldfts, fill_method)
newfts = fillts(oldfts, fill_method, newdates)
newfts = fillts(oldfts, fill_method, newdates, {'T1','T2',...})
newfts = fillts(oldfts, fill_method, newdates, 'SPAN', {'TS','TE'},
delta)
newfts = fillts(... sortmode)
```


## Arguments

- pchip with extrapolation - 'pchipExtrap' - 'pe'
(See interp1 for a discussion of extrapolation.)
To fill with a constant, enter that constant.
A zero-order hold ('zero') fills a missing value with the value immediately preceding it. If the first value in the time series is missing, it remains a NaN.
$\left.\left.\begin{array}{ll}\text { newdates } & \begin{array}{l}\text { (Optional) Column vector of serial dates, a date } \\ \text { string, or a column cell array of date strings. } \\ \text { If oldfts contains time of day information, } \\ \text { newdates must be accompanied by a time } \\ \text { vector (newtimes). Otherwise, newdates is }\end{array} \\ \text { assumed to have times of '00:00' } .\end{array}\right\} \begin{array}{l}\text { First time, second time, start time, end time } \\ \text { T1, T2, TS, TE } \\ \text { delta } \\ \text { sortmode }\end{array} \begin{array}{l}\text { Time interval in minutes to span between the } \\ \text { start time and end time }\end{array}\right\}$


## Description

newfts = fillts(oldfts, fill_method) replaces missing values (represented by NaN ) in the financial time series object oldfts with real values, using either a constant or the interpolation process indicated by fill_method.
newfts = fillts(oldfts, fill_method, newdates) replaces all the missing values on the specified dates newdates added to the financial time series oldfts with new values. The values can be a single constant or values obtained through the interpolation process designated by fill_method. If any of the dates in newdates exists in oldfts, the existing one has precedence.

```
newfts = fillts(oldfts, fill_method, newdates,
```

\{'T1', 'T2', ...\}) additionally allows the designation of specific times of day for addition or replacement of data.
newfts = fillts(oldfts, fill_method, newdates, 'SPAN', \{'TS', 'TE'\}, delta) is similar to the previous format except that you designate only a start time and an end time. You follow these times with a spanning time interval, delta.

If you specify only one date for newdates, specifying a start and end time generates only times for that specific date.
newfts $=$ fillts (... sortmode) additionally denotes whether you want the order of the dates in the output object to stay the same as in the input object or to be sorted chronologically.
sortmode $=0$ (unsorted) appends any new dates to the end. The interpolation and zero-order processes that calculate the values for the new dates work on a sorted object. Upon completion, the existing dates are reordered as they were originally, and the new dates are appended to the end.
sortmode $=1$ sorts the output. After interpolation, no reordering of the date sequence occurs.

## Examples

Example 1. Create a financial time series object with missing data in the fourth and fifth rows.

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001';...
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
    times]);
OpenFts = fints(dates_times,[(1:3)'; nan; nan; 6],{'Data1'},1,...
'Open Financial Time Series');
```

OpenFts looks like this:

```
OpenFts =
```

```
desc: Open Financial Time Series
freq: Daily (1)
'dates: (6)' 'times: (6)' 'Data1: (6)
'01-Jan-2001' '11:00' [ 1]
' " ' '12:00' [ 2]
'02-Jan-2001' '11:00' [ 3]
' " ' '12:00' [ NaN]
'03-Jan-2001' '11:00' [ NaN]
    '12:00' [ 6]
```

Example 2. Fill the missing data in OpenFts using cubic interpolation.

```
FilledFts = fillts(OpenFts,'cubic')
FilledFts =
    desc: Filled Open Financial Time Series
    freq: Unknown (0)
\begin{tabular}{|c|c|c|c|}
\hline dates: (6)' & 'times: & (6) \({ }^{\prime}\) & 'Data1: (6)' \\
\hline 01-Jan-2001' & '11:00' & & \(1]\) \\
\hline " & '12:00' & & \(2]\) \\
\hline 02-Jan-2001' & '11:00' & & \(3]\) \\
\hline " & '12:00' & & \(3.0663]\) \\
\hline '03-Jan-2001' & '11:00' & & \(5.8411]\) \\
\hline " ' & '12:00' & & \(6.0000]\) \\
\hline
\end{tabular}
```

Example 3. Fill the missing data in OpenFts with a constant value.

```
FilledFts = fillts(OpenFts,0.3)
FilledFts =
    desc: Filled Open Financial Time Series
    freq: Unknown (0)
```

| 'dates: (6)' | 'times: | (6) ${ }^{\prime}$ | 'Data1: (6)' |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | $1]$ |
| ' " ' | '12:00' |  | 2] |
| '02-Jan-2001' | '11:00' |  | $3]$ |
| ' " ' | '12:00' |  | $0.3000]$ |
| '03-Jan-2001' | '11:00' |  | $0.3000]$ |
| ' " | '12:00' |  | $6]$ |

Example 4. You can use fillts to identify a specific time on a specific day for the replacement of missing data. This example shows how to replace missing data at 12:00 on January 2 and 11:00 on January 3.

```
FilltimeFts = fillts(OpenFts,'c',...
{'02-Jan-2001';'03-Jan-2001'}, {'12:00';'11:00'},0)
FilltimeFts =
    desc: Filled Open Financial Time Series
    freq: Unknown (0)
\begin{tabular}{|c|c|c|c|}
\hline 'dates: (6)' & 'times: & (6) ' & 'Data1: (6)' \\
\hline '01-Jan-2001' & '11:00' & & [ 1] \\
\hline " & '12:00' & & [ 2] \\
\hline '02-Jan-2001' & '11:00' & & [ 3] \\
\hline " & '12:00' & & [ 3.0663] \\
\hline '03-Jan-2001' & '11:00' & & [ 5.8411] \\
\hline " ' & '12:00' & & [ 6.0000] \\
\hline
\end{tabular}
```

Example 5. Use a spanning time interval to add an additional day to OpenFts.

```
SpanFts = fillts(OpenFts,'c','04-Jan-2001','span',...
    {'11:00';'12:00'},60,0)
SpanFts =
```

    desc: Filled Open Financial Time Series
    freq: Unknown (0)

| 'dates: (8)' | 'times: | (8) ${ }^{\prime}$ | 'Data1: (8)' |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | [ 1] |
| ' " ' | '12:00' |  | [ 2] |
| '02-Jan-2001' | '11:00' |  | [ 3] |
| ' | '12:00' |  | [ 3.0663] |
| '03-Jan-2001' | '11:00' |  | [ 5.8411] |
| " | '12:00' |  | [ 6.0000] |
| '04-Jan-2001' | '11:00' |  | [ 9.8404] |
| ' " ' | '12:00' |  | $9.9994]$ |

See Also
interp1

## Purpose Linear filtering

```
Syntax
newfts = filter(B, A, oldfts)
```

Description
filter filters an entire financial time series object with certain filter specifications. The filter is specified in a transfer function expression.
newfts $=$ filter ( $B, A$, oldfts) filters the data in the financial time series object oldfts with the filter described by vectors A and B to create the new financial time series object newfts. The filter is a "Direct Form II Transposed" implementation of the standard difference equation. newfts is a financial time series object containing the same data series (names) as the input oldfts.

## See Also

filter | filter2

## fints

## Purpose Construct financial time series object

```
Syntax tsobj = fints(dates_and_data)
tsobj = fints(dates, data)
tsobj = fints(dates, data, datanames)
tsobj = fints(dates, data, datanames, freq)
tsobj = fints(dates, data, datanames, freq, desc)
```


## Arguments

dates_and_data Column-oriented matrix containing one column of dates and a single column for each series of data. In this format, dates must be entered in serial date number format. If the input serial date numbers encode time-of-day information, the output object contains a column labeled 'dates ' containing the date information and another labeled 'times ' containing the time information.

You can use the function today to enter date information or the function now to enter date with time information.
dates Column vector of dates. Dates can be date strings or serial date numbers and can include time of day information. When entering time-of-day information as serial date numbers, the entry must be a column-oriented matrix when multiple entries are present. If the time-of-day information is in string format, the entry must be a column-oriented cell array of dates and times when multiple entries are present.

Valid date and time string formats are:

- 'ddmmmy hh:mm' or 'ddmmmyyyy hh:mm'
- 'mm/dd/yy hh:mm' or 'mm/dd/yyyy hh:mm'
- 'dd-mmm-yy hh:mm' or 'dd-mmm-yyyy hh:mm'
- 'mmm.dd,yy hh:mm' or 'mmm.dd,yyyy hh:mm'

Dates and times can initially be separate column-oriented vectors, but they must be concatenated into a single column-oriented matrix before being passed to fints. You can use the MATLAB functions today and now to assist in entering date and time information.
data
datanames

Column-oriented matrix containing a column for each series of data. The number of values in each data series must match the number of dates. If a mismatch occurs, MATLAB does not generate the financial time series object, and you receive an error message.
Cell array of data series names. Overrides the default data series names. Default data series names are series1, series2, and so on.

## fints

Note Not all strings are accepted as datanames parameters. Supported data series names cannot start with a number and must contain only these characters:

- Lowercase Latin alphabet, a to z
- Uppercase Latin alphabet, A to Z
- Underscore,

| freq | Frequency indicator. Allowed values are: |
| :---: | :---: |
|  | UNKNOWN, Unknown, unknown, $\mathrm{U}, \mathrm{u}, 0$ |
|  | DAILY, Daily, daily, D, d, 1 |
|  | WEEKLY, Weekly, weekly, W, w, 2 |
|  | MONTHLY, Monthly, monthly, M, m, 3 |
|  | QUARTERLY, Quarterly, quarterly, Q, q, 4 |
|  | SEMIANNUAL, Semiannual, semiannual, S, s, 5 |
|  | ANNUAL, Annual, annual, A, a, 6 |
|  | Default = Unknown. |
| desc | String providing descriptive name for financial time series object. Default = ' . |

Note The toolbox supports only hourly and minute time series. Seconds are disregarded when the object is created (for example, 01-jan-2001 12:00:01 is considered to be 01-jan-2001 12:00). If there are duplicate dates and times, fints sorts the dates and times and chooses the first instance of the duplicate dates and times. The other duplicate dates and times are removed from the object along with their corresponding data.

## Description

fints constructs a financial time series object. A financial time series object is a MATLAB object that contains a series of dates and one or more series of data. Before you perform an operation on the data, you must set the frequency indicator (freq). You can optionally provide a description (desc) for the time series.
tsobj = fints(dates_and_data) creates a financial time series object containing the dates and data from the matrix dates_and_data. If the dates contain time-of-day information, the object contains an additional series of times. The date series and each data series must each be a column in the input matrix. The names of the data series default to series1, ..., seriesn. The desc and freq fields are set to their defaults.
tsobj = fints(dates, data) generates a financial time series object containing dates from the dates column vector of dates and data from the matrix data. If the dates contain time-of-day information, the object contains an additional series of times. The data matrix must be column-oriented, that is, each column in the matrix is a data series. The names of the series default to series1, ..., seriesn, where $n$ is the total number of columns in data. The desc and freq fields are set to their defaults.
tsobj = fints(dates, data, datanames) additionally allows you to rename the data series. The names are specified in the datanames cell array. The number of strings in datanames must correspond to the number of columns in data. The desc and freq fields are set to their defaults.

## fints

tsobj = fints(dates, data, datanames, freq) additionally sets the frequency when you create the object. The desc field is set to its default ' '.
tsobj $=$ fints(dates, data, datanames, freq, desc) provides a description string for the financial time series object.

Examples
Example 1. Create a financial time series containing days and data only.

```
data = [1:6]'
data =
1
2
3
4
5
6
dates = [today:today+5]'
dates =
    731132
    731133
    731134
    731135
    731136
    731137
    tsobjkt = fints(dates, data)
    tsobjkt =
        desc: (none)
```

| freq: Unknown |  |  |
| :---: | :---: | :---: |
| 'dates: (6)' | 'series1: | (6) ${ }^{\prime}$ |
| '08-0ct-2001' | [ | $1]$ |
| '09-0ct-2001' | [ | $2]$ |
| '10-Oct-2001' | [ | $3]$ |
| '11-Oct-2001' | [ | 4] |
| '12-0ct-2001' | [ | 5] |
| '13-0ct-2001' | [ | $6]$ |

Example 2. Expand Example 1 to include time-of-day information:

```
dates = [now:now+5]';
tsobjkt = fints(dates, data)
tsobjkt =
    desc: (none)
    freq: Unknown (0)
\begin{tabular}{|c|c|c|}
\hline dates: (6)' & 'times: (6)' & 'series1: (6)' \\
\hline '08-0ct-2001' & '14:51' & 1] \\
\hline '09-0ct-2001' & '14:51' & 2] \\
\hline '10-0ct-2001' & '14:51' & \(3]\) \\
\hline '11-0ct-2001' & '14:51' & 4] \\
\hline '12-0ct-2001' & '14:51' & 5] \\
\hline '13-0ct-2001' & '14:51' & \(6]\) \\
\hline
\end{tabular}
```

Example 3. Create a financial time series object when dates and times are located in separate vectors.

Step 1. Create a column vector of times in date number format:

```
times = datenum(datestr(now:1/24+1/24/60:now+6/24+1/24/60,15))
times =
```


## fints

0.43750000000000
0.47986111111111
0.52222222222222
0.56458333333333
0.60694444444444
0.64930555555556

Step 2. Create a column vector of dates:

```
dates = [today:today+5]'
dates =
    731133
    7 3 1 1 3 4
    7 3 1 1 3 5
    731136
    7 3 1 1 3 7
    731138
```

Step 3. Concatenate dates and times into a single matrix:

```
dates_times = [dates, times]
dates times =
    1.0e+005
    7.31133000000000 0.00000437500000
    7.31134000000000 0.00000479861111
    7.31135000000000 0.00000522222222
    7.31136000000000 0.00000564583333
    7.31137000000000 0.00000606944444
    7.31138000000000 0.00000649305556
```

Step 4. Create column vector of data:

```
data = [1:6]'
```

Step 5. Create the financial time series object:

```
tsobj = fints(dates_times, data)
    tsobj =
```

        desc: (none)
        freq: Unknown (0)
    'dates: (6)' 'times: (6)' 'series1: (6)'
'09-0ct-2001' '10:30' [ 1]
'10-Oct-2001' '11:31' [ 2]
'11-Oct-2001' '12:32' [ 3]
'12-0ct-2001' '13:33' [ 4]
'13-0ct-2001' '14:34' [ 5]
'14-Oct-2001' '15:35' [ 6]
See Also datenum | datestr

## fpctkd

Purpose Fast stochastics

```
Syntax
```

```
[pctk, pctd] = fpctkd(highp, lowp, closep)
```

[pctk, pctd] = fpctkd(highp, lowp, closep)
[pctk, pctd] = fpctkd([highp lowp closep])
[pctk, pctd] = fpctkd([highp lowp closep])
[pctk, pctd] = fpctkd(highp, lowp, closep, kperiods, dperiods,
[pctk, pctd] = fpctkd(highp, lowp, closep, kperiods, dperiods,
dmamethod)
dmamethod)
[pctk, pctd] = fpctkd([highp lowp closep], kperiods, dperiods,
[pctk, pctd] = fpctkd([highp lowp closep], kperiods, dperiods,
dmamethod)
dmamethod)
pkdts = fpctkd(tsobj, kperiods, dperiods, dmamethod)
pkdts = fpctkd(tsobj, kperiods, dperiods, dmamethod)
pkdts = fpctkd(tsobj, kperiods, dperiods, dmamethod, ParameterName,
pkdts = fpctkd(tsobj, kperiods, dperiods, dmamethod, ParameterName,
ParameterValue, ...)

```
ParameterValue, ...)
```


## Arguments

| highp | High price (vector). |
| :--- | :--- |
| lowp |  |
| closep | Low price (vector). |
| Closing price (vector). |  |

fpctkd calculates the stochastic oscillator.
[pctk, pctd] = fpctkd(highp, lowp, closep) calculates the fast stochastics $\mathrm{F} \% \mathrm{~K}$ and $\mathrm{F} \% \mathrm{D}$ from the stock price data highp (high prices), lowp (low prices), and closep (closing prices).
[pctk, pctd] = fpctkd([highp lowp closep]) accepts a three-column matrix of high (highp), low (lowp), and closing prices (closep), in that order.
[pctk, pctd] = fpctkd(highp, lowp, closep, kperiods, dperiods, dmamethod) calculates the fast stochastics F\%K and F\%D from the stock price data highp (high prices), lowp (low prices), and closep (closing prices). kperiods sets the \%K period. dperiods sets the \%D period.
damethod specifies the $\% \mathrm{D}$ moving average method. Valid moving average methods for \%D are Exponential ('e') and Triangular ('t'). See tsmovavg for explanations of these methods.
[pctk, pctd]= fpctkd([highp lowp closep], kperiods, dperiods, dmamethod) accepts a three-column matrix of high (highp), low (lowp), and closing prices (closep), in that order.
pkdts = fpctkd(tsobj, kperiods, dperiods, dmamethod) calculates the fast stochastics F\%K and F\%D from the stock price data in the financial time series object tsobj. tsobj must minimally contain the series High (high prices), Low (low prices), and Close (closing prices). pkdts is a financial time series object with similar dates to tsobj and two data series named PercentK and PercentD.
pkdts = fpctkd(tsobj, kperiods, dperiods, dmamethod, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Parameter values are the strings that represent the valid parameter names.

## fpctkd

Examples

## References

## See Also

Compute the stochastic oscillator for Disney stock and plot the results:

```
load disney.mat
dis_FastStoc = fpctkd(dis)
plot(dis_FastStoc)
title('Stochastic Oscillator for Disney')
```



Achelis, Steven B., Technical Analysis from A to Z, Second Edition, McGraw-Hill, 1995, pp. 268-271.

spctkd | stochosc \| tsmovavg

Purpose Fractional currency value to decimal value
Syntax Decimal = frac2cur(Fraction, Denominator)
Description Decimal = frac2cur(Fraction, Denominator) converts a fractional currency value to a decimal value. Fraction is the fractional currency value input as a string, and Denominator is the denominator of the fraction.
Examples Decimal = frac2cur('12.1', 8)
returns
Decimal =
12.1250
See Also cur2frac | cur2str

## freqnum

Purpose
Convert string frequency indicator to numeric frequency indicator
Syntax

## Arguments

```
sfreq UNKNOWN, Unknown, unknown, U, u
    DAILY, Daily, daily, D, d
    WEEKLY, Weekly, weekly, W, w
    MONTHLY, Monthly, monthly, M, m
    QUARTERLY, Quarterly, quarterly, Q, q
    SEMIANNUAL, Semiannual, semiannual, S, s
    ANNUAL, Annual, annual, A, a
```


## Description

nfreq = freqnum(sfreq) converts a string frequency indicator into a numeric value.

```
String Frequency Indicator Numeric Representation
UNKNOWN, Unknown, unknown, U, u 0
DAILY, Daily, daily, D, d 1
WEEKLY, Weekly, weekly, W, w 2
MONTHLY, Monthly, monthly, M, m 3
QUARTERLY, Quarterly, 4
quarterly, Q, q
SEMIANNUAL, Semiannual, 5
semiannual, S, s
ANNUAL, Annual, annual, A, a 6
```


## See Also

freqstr

## freqstr

Purpose Convert numeric frequency indicator to string representation
Syntax $\quad$ sfreq $=$ freqstr $(n f r e q)$

Arguments
nfreq 0
1

2

3

4
5
6

## Description

sfreq = freqstr(nfreq) converts a numeric frequency indicator into a string representation.

| Numeric Frequency <br> Indicator | String Representation |
| :--- | :--- |
| 0 | Unknown |
| 1 | Daily |
| 2 | Weekly |
| 3 | Monthly |
| 4 | Quarterly |
| 5 | Semiannual |
| 6 | Annual |

See Also
freqnum

## frontcon

Purpose Mean-variance efficient frontier

Syntax<br>[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn, ExpCovariance, NumPorts, PortReturn, AssetBounds, Groups, GroupBounds, varargin)

## Arguments

| ExpReturn | 1 by number of assets (NASSETS) vector <br> specifying the expected (mean) return of each <br> asset. |
| :--- | :--- |
| ExpCovariance | NASSETS-by-NASSETS matrix specifying the <br> covariance of asset returns. |
| NumPorts |  |
|  | (Optional) Number of portfolios generated <br> along the efficient frontier. Returns are equally <br> spaced between the maximum possible return <br> and the minimum risk point. If NumPorts is <br> empty (entered as [ ], frontcon computes 10 <br> equally spaced points. When entering a target <br> rate of return (PortReturn), enter NumPorts |
| as an empty matrix []. |  |
| (Optional) Vector of length equal to the number |  |


| Groups | (Optional) Number of groups (NGROUPS) -by-NASSETS matrix specifying NGROUPS asset groups or classes. Each row specifies a group. Groups $(i, j)=1$ ( $j$ th asset belongs in the $i$ th group). Groups ( $i, j$ ) $=0$ ( $j$ th asset not a member of the $i$ th group). |
| :---: | :---: |
| GroupBounds | (Optional) NGROUPS-by-2 matrix specifying, for each group, the lower and upper bounds of the total weights of all assets in that group. Default lower bound $=$ all 0s. Default upper bound = all 1s. |
| varargin | (Optional) varargin supports the following parameter-value pairs: |
|  | - 'algorithm' - Defines which algorithm to use with frontcon. Use either a value of 'lcprog' or 'quadprog' to indicate the algorithm to use. The default is 'lcprog'. <br> - 'maxiter' - Maximum number of iterations before termination of algorithm. The default is 100000 . |
|  | - 'tiebreak' - Method to break ties for pivot selection. This value pair applies only to 'lcprog' algorithm. The default is 'first'. Options are: |
|  | - 'first' - Selects pivot with lowest index. <br> - 'last' - Selects pivot with highest index. |
|  | - 'random' - Selects pivot at random. |
|  | - 'tolcon' - Tolerance for constraint violations. This value pair applies only to 'lcprog' algorithm. The default is 1.0e-6. |

## frontcon

- 'tolpiv' - Pivot value below which a number is considered to be zero. This value pair applies only to 'lcprog' algorithm. The default is $1.0 \mathrm{e}-9$.


## Description

## Examples

[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn, ExpCovariance, NumPorts, PortReturn, AssetBounds, Groups, GroupBounds, varargin) returns the mean-variance efficient frontier with user-specified asset constraints, covariance, and returns. For a collection of NASSETS risky assets, computes a portfolio of asset investment weights that minimize the risk for given values of the expected return. The portfolio risk is minimized subject to constraints on the asset weights or on groups of asset weights.

PortRisk is an NPORTS-by-1 vector of the standard deviation of each portfolio.

PortReturn is a NPORTS-by-1 vector of the expected return of each portfolio.

PortWts is an NPORTS-by-NASSETS matrix of weights allocated to each asset. Each row represents a portfolio. The total of all weights in a portfolio is 1 .
frontcon generates a plot of the efficient frontier if you invoke it without output arguments.

The asset returns are assumed to be jointly normal, with expected mean returns of ExpReturn and return covariance ExpCovariance. The variance of a portfolio with 1-by-NASSETS weights PortWts is given by PortVar $=$ PortWts*ExpCovariance*PortWts'. The portfolio expected return is PortReturn $=\operatorname{dot}(E x p R e t u r n$, PortWts).

Given three assets with expected returns of

```
ExpReturn = [0.1 0.2 0.15];
```

and expected covariance of

$$
\left.\left.\begin{array}{rrr}
\text { ExpCovariance }=\left[\begin{array}{rrr}
0.0100 & -0.0061 & 0.0042 \\
& -0.0061 & 0.0400
\end{array}\right. & -0.0252 \\
& 0.0042 & -0.0252
\end{array}\right] 0.0225\right] ;
$$

compute the mean-variance efficient frontier for four points.

```
NumPorts = 4;
[PortRisk, PortReturn, PortWts] = frontcon(ExpReturn,...
ExpCovariance, NumPorts)
PortRisk =
            0.0426
            0.0483
            0.1089
            0.2000
PortReturn =
            0.1569
            0.1713
            0.1856
            0.2000
PortWts =
\begin{tabular}{rrr}
0.2134 & 0.3518 & 0.4348 \\
0.0096 & 0.4352 & 0.5552 \\
0 & 0.7128 & 0.2872 \\
0 & 1.0000 & 0
\end{tabular}
```


## See Also

ewstats | frontier | portopt | portstats

## frontier

Purpose Rolling efficient frontier

Syntax [PortWts, AllMean, AllCovariance] = frontier(Universe, Window, Offset, NumPorts, ActiveMap, ConSet, NumNonNan)

## Arguments

| Universe | Number of observations (NUMOBS) by number of assets plus one (NASSETS + 1) time series array containing total return data for a group of securities Each row represents an observation. Column 1 contains MATLAB serial date numbers. The remaining columns contain the total return data for each security. |
| :---: | :---: |
| Window | Number of data periods used to calculate each frontier. |
| Offset | Increment in number of periods between each frontier. |
| NumPorts | Number of portfolios to calculate on each frontier. |
| ActiveMap | (Optional) Number of observations (NUMOBS) by number of assets (NASSETS) matrix with Boolean elements corresponding to the Universe. Each element indicates if the asset is part of the Universe on the corresponding date. Default = NUMOBS-by-NASSETS matrix of 1's (all assets active on all dates). |


| Conset | (Optional) Constraint matrix for a portfolio of asset <br> investments, created using portcons with the <br> 'Default ' constraint type. This single constraint <br> matrix is applied to each frontier. |
| :--- | :--- |
| NumNonNan | (Optional) Minimum number of nonNaN points for <br> each active asset in each window of data needed <br> to perform the optimization. The default value is <br> Window - NASSETS. |

## Description

See Also
[PortWts, AllMean, AllCovariance] = frontier(Universe, Window, Offset, NumPorts, ActiveMap, ConSet, NumNonNan) generates a surface of efficient frontiers showing how asset allocation influences risk and return over time.

PortWts is a number of curves (NCURVES)-by-1 cell array, where each element is a NPORTS-by-NASSETS matrix of weights allocated to each asset.

AllMean is a NCURVES-by- 1 cell array, where each element is a 1-by-NASSETS vector of the expected asset returns used to generate each curve on the surface.

AllCovariance is a NCURVES-by- 1 cell array, where each element is a NASSETS-by-NASSETS vector of the covariance matrix used to generate each curve on the surface.

```
portcons | portopt
```

Purpose
Write elements of time-series data into ASCII file
Syntax
stat $=$ fts2ascii(filename, tsobj, exttext)
stat $=$ fts2ascii(filename, dates, data, colheads, desc, exttext)

## Arguments

| filename | Name of an ASCII file |
| :--- | :--- |
| tsobj | Financial time series object <br> exttext <br> (Optional) Extra text. A string written after the <br> description line (line 2 in the file). |
| dates | Column vector containing dates. Dates must be in <br> serial date number format and can specify time of <br> day. |
| data | Column-oriented matrix. Each column is a series. <br> (Optional) Cell array of column headers (names); <br> colheads <br> first cell must always be the one for the dates <br> column. colheads will be written to the file just <br> before the data. <br> (Optional) Description string, which will be the first <br> line in the file. |
| desc |  |

## Description

stat $=$ fts2ascii(filename, tsobj, exttext) writes the financial time series object tsobj into an ASCII file filename. The data in the file is tab delimited.
stat $=$ fts2ascii(filename, dates, data, colheads, desc, exttext) writes into an ASCII file filename the dates, times, and data contained in the column vector dates and the column-oriented matrix data. The first column in filename contains the dates, followed by times (if specified). Subsequent columns contain the data. The data in the file is tab delimited.
stat indicates whether file creation is successful (1) or not (0).

## See Also <br> ascii2fts

## fts2mat

## Purpose Convert to matrix

```
Syntax tsmat = fts2mat(tsobj)
tsmat = fts2mat(tsobj, datesflag)
tsmat = fts2mat(tsobj, seriesnames)
tsmat = fts2mat(tsobj, datesflag, seriesnames)
```


## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| datesflag | (Optional) Specifies inclusion of dates vector: |
|  | datesflag $=0$ (default) excludes dates. |
|  | datesflag $=1$ includes dates vector. |
| seriesnames | (Optional) Specifies the data series to be included in <br> the matrix. Can be a cell array of strings. |

## Description

tsmat $=$ fts2mat(tsobj) takes the data series in the financial time series object tsobj and puts them into the matrix tsmat as columns. The order of the columns is the same as the order of the data series in the object tsobj.
tsmat $=$ fts2mat(tsobj, datesflag) specifies whether or not you want the dates vector included. The dates vector will be the first column. The dates are represented as serial date numbers. Dates can include time-of-day information.
tsmat = fts2mat(tsobj, seriesnames) extracts the data series named in seriesnames and puts its values into tsmat. The seriesnames argument can be a cell array of strings.
tsmat = fts2mat(tsobj, datesflag, seriesnames) puts into tsmat the specific data series named in seriesnames. The datesflag argument must be specified. If datesflag is set to 1 , the dates vector
is included. If you specify an empty matrix ([ ]) for datesflag, the default behavior is adopted.

See Also subsref

## ftsbound

Purpose Start and end dates

```
Syntax
datesbound = ftsbound(tsobj)
datesbound = ftsbound(tsobj, dateform)
```

Arguments
tsobj Financial time series object
dateform dateform is an integer representing the format of a date string. See datestr for a description of these formats.

Description

See Also
ftsbound returns the start and end dates of a financial time series object. If the object contains time-of-day data, ftsbound additionally returns the starting time on the first date and the ending time on the last date.
datesbound $=$ ftsbound(tsobj) returns the start and end dates contained in tsobj as serial dates in the column matrix datesbound. The first row in datesbound corresponds to the start date, and the second corresponds to the end date.
datesbound $=$ ftsbound(tsobj, dateform) returns the starting and ending dates contained in the object, tsobj, as date strings in the column matrix, datesbound. The first row in datesbound corresponds to the start date, and the second corresponds to the end date. The dateform argument controls the format of the output dates.
datestr

## Purpose <br> Financial time series GUI

## Syntax <br> ftsgui

Description ftsgui displays the financial time series graphical user interface (GUI) main window.

The use of the financial time series GUI is described in Chapter 12, "Financial Time Series Graphical User Interface".

## Examples <br> ftsgui



## See Also

ftstool

## ftsinfo

Purpose Financial time series object information

```
Syntax
ftsinfo(tsobj)
infofts = ftsinfo(tsobj)
```

Arguments
tsobj Financial time series object.

## Description

ftsinfo(tsobj) displays information about the financial time series object tsobj.
infofts = ftsinfo(tsobj) stores information about the financial time series object tsobj in the structure infofts.
infofts has these fields.

| Field | Contents |
| :--- | :--- |
| version | Financial time series object version. |
| desc | Description of the time series object (tsobj. desc). <br> freq |
| Numeric representation of the time series data <br> frequency (tsobj. freq). See freqstr for list of <br> numeric frequencies and what they represent. |  |
| startdate | Earliest date in the time series. |
| enddate | Latest date in the time series. |
| seriesnames | Cell array containing the time series data column <br> names. |
| ndata | Number of data points in the time series. |
| nseries | Number of columns of time series data. |

Examples

Convert the supplied file disney . dat into a financial time series object named dis:

```
dis = ascii2fts('disney.dat', 1, 3);
```

Now use ftsinfo to obtain information about dis:

```
ftsinfo(dis)
```

    FINTS version: 2.0
        Description: Walt Disney Company (DIS)
            Frequency: Unknown
            Start date: 29-Mar-1996
            End date: 29-Mar-1999
    Series names: OPEN
                                    HIGH
                                    LOW
                                    CLOSE
                                    VOLUME
            \# of data: 782
        \# of series: 5
    Then, executing

```
infodis = ftsinfo(dis)
```

creates the structure infodis containing the values

```
infodis =
```

            ver: '2.0'
            desc: 'Walt Disney Company (DIS)'
            freq: 0
        startdate: '29-Mar-1996'
        enddate: '29-Mar-1999'
        seriesnames: \{5x1 cell\}
            ndata: 782
        nseries: 5
    
## ftsinfo

## See Also fints | freqnum | freqstr | ftsbound

## Purpose Financial time series tool

## Syntax ftstool

Description ftstool creates and manages Financial Time Series objects. ftstool allows the creation and management of Financial Time Series objects via a graphical user interface. ftstool can interface with ftsgui, meaning Line Plots generated with ftstool can be analyzed with ftsguiFTSGUI. However, ftsgui must be running prior to the generation of any Line Plots.

The use of the financial time series tool is described in Chapter 11, "Financial Time Series Tool (FTSTool)".

## Examples <br> ftstool

## ftstool



## See Also

ftsgui

## Purpose Determine uniqueness

```
Syntax
uniq = ftsuniq(dates_and_times)
[uniq, dup] = ftsuniq(dates_and_times)
```


## Arguments

$$
\begin{array}{ll}
\text { dates_and_times } & \text { A single column vector of serial date } \\
& \text { numbers. The serial date numbers can include } \\
& \text { time-of-day information. }
\end{array}
$$

Description uniq = ftsuniq(dates_and_times) returns 1 if the dates and times within the financial time series object are unique and 0 if duplicates exist.
[uniq, dup] = ftsuniq(dates_and_times) additionally returns a structure dup. In the structure

- dup.DT contains the strings of the duplicate dates and times and their locations in the object.
- dup.intIdx contains the integer indices of duplicate dates and times in the object.


## See Also

fints

Purpose Future value of discounted security
Syntax FutureVal = fvdisc(Settle, Maturity, Price, Discount, Basis)

## Arguments

| Settle | Settlement date. Enter as serial date number or date <br> string. Settle must be earlier than Maturity. |
| :--- | :--- |
| Maturity | Maturity date. Enter as serial date number or date <br> string. |
| Price | Price (present value) of the security. |
| Discount | Bank discount rate of the security. Enter as decimal <br> fraction. |
| Basis | (Optional) Day-count basis of the instrument. A <br> vector of integers. |

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- $13=\mathrm{BUS} / 252$

For more information, seebasis on page Glossary-1.

```
Description FutureVal = fvdisc(Settle, Maturity, Price, Discount,
Basis) finds the amount received at maturity for a fully vested
security.
Examples Using this data
Settle = '02/15/2001';
Maturity = '05/15/2001';
Price = 100;
Discount = 0.0575;
Basis = 2;
FutureVal = fvdisc(Settle, Maturity, Price, Discount, Basis)
returns
```

```
FutureVal =
```

FutureVal =
101.44
101.44
References Mayle, Standard Securities Calculation Methods, Volumes I-II, 3rd
edition.

```

\section*{See Also}
```

acrudisc | discrate | prdisc | ylddisc

```
```

acrudisc | discrate | prdisc | ylddisc

```

Purpose Future value with fixed periodic payments
Synfax FutureVal = fvfix(Rate, NumPeriods, Payment, PresentVal, Due)

\section*{Arguments}

\section*{Description}

FutureVal = fvfix(Rate, NumPeriods, Payment, PresentVal, Due) returns the future value of a series of equal payments.

Examples \(\begin{aligned} & \text { A savings account has a starting balance of } \$ 1500 . \$ 200 \text { is added at } \\ & \text { the end of each month for } 10 \text { years and the account pays } 9 \% \text { interest } \\ & \text { compounded monthly. Using this data }\end{aligned}\)
\[
\text { FutureVal }=\text { fvfix }(0.09 / 12,12 * 10,200,1500,0)
\]
returns

> FutureVal =
42379.89

See Also fvvar | pvfix | pvvar
Purpose Future value of varying cash flow
Syntax FutureVal = fvar(CashFlow, Rate, IrrCFDates)

\section*{Arguments}

\section*{Description FutureVal = fvvar(CashFlow, Rate, IrrCFDates) returns the future value of a varying cash flow. \\ Examples This cash flow represents the yearly income from an initial investment of \(\$ 10,000\). The annual interest rate is \(8 \%\).}

Year \(1 \quad \$ 2000\)
Year \(2 \quad \$ 1500\)
Year 3 \$3000
Year 4 \$3800
Year \(5 \quad \$ 5000\)

For the future value of this regular (periodic) cash flow
```

    FutureVal = fvvar([-10000 2000 1500 3000 3800 5000], 0.08)
    returns
FutureVal =
2520.47

```

An investment of \(\$ 10,000\) returns this irregular cash flow. The original investment and its date are included. The periodic interest rate is \(9 \%\).

\section*{Cash Flow}
\begin{tabular}{ll}
\((\$ 10000)\) & January 12, 2000 \\
\(\$ 2500\) & February 14, 2001 \\
\(\$ 2000\) & March 3, 2001 \\
\(\$ 3000\) & June 14, 2001 \\
\(\$ 4000\) & December 1, 2001
\end{tabular}

To calculate the future value of this irregular (nonperiodic) cash flow
```

    CashFlow = [-10000, 2500, 2000, 3000, 4000];
    ```
    IrrCFDates = ['01/12/2000'
            '02/14/2001'
            '03/03/2001'
            '06/14/2001'
            '12/01/2001'];
FutureVal = fvvar(CashFlow, 0.09, IrrCFDates)
returns
FutureVal =

\author{
See Also fvfix | irr | payuni | pvfix | pvvar
}

Purpose Zero curve given forward curve
Syntax [ZeroRates, CurveDates] = fwd2zero(ForwardRates, CurveDates,

\section*{Arguments}

ForwardRates A number of bonds (NUMBONDS)-by-1 vector of annualized implied forward rates, as decimal fractions. In aggregate, the rates in ForwardRates constitute an implied forward curve for the investment horizon represented by CurveDates. The first element pertains to forward rates from the settlement date to the first curve date.

CurveDates A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the forward rates.

Settle A serial date number that is the common settlement date for the forward rates.

Compounding (Optional) Output compounding. A scalar that sets the compounding frequency per year for annualizing the output zero rates. Allowed values are:
1 Annual compounding
2 Semiannual compounding (default)
3 Compounding three times per year
4 Quarterly compounding
6 Bimonthly compounding
12 Monthly compounding
365 Daily compounding
```

-1 Continuous compounding
Basis (Optional) Output day-count basis for annualizing the output zero rates.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (PSA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ISMA)
- $9=$ actual/360 (ISMA)
- $10=\mathrm{actual} / 365$ (ISMA)
- $11=30 / 360 \mathrm{E}$ (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

```

For more information, seebasis on page Glossary-1.

\section*{Description}
[ZeroRates, CurveDates] = fwd2zero(ForwardRates, CurveDates, Settle, Compounding, Basis) returns a zero curve given an implied forward rate curve and its maturity dates.

ZeroRates A NUMBONDS-by-1 vector of decimal fractions. In aggregate, the rates in ZeroRates constitute a zero curve for the investment horizon represented by CurveDates.

CurveDates A NUMBONDS-by-1 vector of maturity dates (as serial date numbers) that correspond to the zero rates in ZeroRates. This vector is the same as the input vector CurveDates.

\section*{Examples}

Given an implied forward rate curve over a set of maturity dates, a settlement date, and a compounding rate, compute the zero curve.
```

ForwardRates = [0.0469
0.0519
0.0549
0.0535
0.0558
0.0508
0.0560
0.0545
0.0615
0.0486];
CurveDates = [datenum('06-Nov-2000')
datenum('11-Dec-2000')
datenum('15-Jan-2001')
datenum('05-Feb-2001')
datenum('04-Mar-2001')
datenum('02-Apr-2001')
datenum('30-Apr-2001')
datenum('25-Jun-2001')
datenum('04-Sep-2001')
datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');

```
```

Compounding = 1;

```

Execute the function
```

[ZeroRates, CurveDates] = fwd2zero(ForwardRates, CurveDates,...
Settle, Compounding)

```
which returns the zero curve ZeroRates at the maturity dates CurveDates.
```

ZeroRates =
0.0469
0.0515
0.0531
0.0532
0.0538
0.0532
0.0536
0.0539
0.0556
0.0543
CurveDates =
730796
7 3 0 8 3 1
730866
730887
730914
730943
7 3 0 9 7 1
731027
7 3 1 0 9 8
731167

```

For readability, ForwardRates and ZeroRates are shown here only to the basis point. However, MATLAB software computed them at full precision. If you enter ForwardRates as shown, ZeroRates may differ due to rounding.

\section*{See Also zero2fwd}

How To
- "Term Structure of Interest Rates" on page 2-36

\section*{Purpose \\ Geometric to arithmetic moments of asset returns}

\section*{Syntax}
```

[ma, Ca] = geom2arith(mg, Cg);
[ma, Ca] = geom2arith(mg, Cg, t);

```

Arguments
\begin{tabular}{ll}
mg & \begin{tabular}{l} 
Continuously-compounded or "geometric" mean of \\
asset returns (positive n-vector).
\end{tabular} \\
Cg & \begin{tabular}{l} 
Continuously-compounded or "geometric" \\
covariance of asset returns (n-by-n symmetric, \\
positive-semidefinite matrix).
\end{tabular} \\
t & \begin{tabular}{l} 
(Optional) Target period of arithmetic moments \\
in terms of periodicity of geometric moments with \\
default value 1 (positive scalar).
\end{tabular}
\end{tabular}

\section*{Description}
geom2arith transforms moments associated with a continuously-compounded geometric Brownian motion into equivalent moments associated with a simple Brownian motion with a possible change in periodicity.
[ma, Ca] = geom2arith(mg, Cg, t) returns ma, arithmetic mean of asset returns over the target period ( \(n\)-vector), andCa, which is an arithmetric covariance of asset returns over the target period (n-by-n matrix).

Geometric returns over period \(t_{G}\) are modeled as multivariate lognormal random variables with moments
\[
E[\mathrm{Y}]=1+\mathrm{m}_{G}
\]
and
\[
\operatorname{cov}(\mathrm{Y})=\mathrm{C}_{G}
\]

Arithmetic returns over period \(t_{A}\) are modeled as multivariate normal random variables with moments
\[
\begin{gathered}
E[\mathrm{X}]=\mathrm{m}_{A} \\
\operatorname{cov}(\mathrm{X})=\mathrm{C}_{A}
\end{gathered}
\]

Given \(t=t_{A} / t_{G}\), the transformation from geometric to arithmetic moments is
\[
\begin{aligned}
\mathrm{C}_{A_{i j}} & =t \log \left(1+\frac{\mathrm{C}_{G_{i j}}}{\left(1+\mathrm{m}_{G_{i}}\right)\left(1+\mathrm{m}_{G_{j}}\right)}\right) \\
\mathrm{m}_{A_{i}} & =t \log \left(1+\mathrm{m}_{G_{i}}\right)-\frac{1}{2} \mathrm{C}_{A_{i i}} \\
\text { For } i, j & =1, \ldots, n .
\end{aligned}
\]

Note If \(t=1\), then \(\mathbf{X}=\log (\mathbf{Y})\).

This function requires that the input mean must satisfy \(1+\mathrm{mg}>\) 0 and that the input covariance Cg must be a symmetric, positive, semidefinite matrix.

The functions geom2arith and arith2geom are complementary so that, given \(m, C\), and \(t\), the sequence
```

[ma, Ca] = geom2arith(m, C, t);
[mg, Cg] = arith2geom(ma, Ca, 1/t);

```
yields \(\mathrm{mg}=\mathrm{m}\) and \(\mathrm{Cg}=\mathrm{C}\).
Examples Example 1. Given geometric mean \(m\) and covariance \(C\) of monthly total returns, obtain annual arithmetic mean ma and covariance Ca. In this
case, the output period (1 year) is 12 times the input period ( 1 month) so that \(t=12\) with
```

[ma, Ca] = geom2arith(m, C, 12);

```

Example 2. Given annual geometric mean \(m\) and covariance \(C\) of asset returns, obtain monthly arithmetic mean ma and covariance Ca . In this case, the output period ( 1 month) is \(1 / 12\) times the input period ( 1 year) so that \(\mathrm{t}=1 / 12\) with
```

[ma, Ca] = geom2arith(m, C, 1/12);

```

Example 3. Given geometric means \(m\) and standard deviations \(s\) of daily total returns (derived from 260 business days per year), obtain annualized arithmetic mean ma and standard deviations sa with
```

[ma, Ca] = geom2arith(m, diag(s .^2), 260);
sa = sqrt(diag(Ca));

```

Example 4. Given geometric mean \(m\) and covariance C of monthly total returns, obtain quarterly arithmetic return moments. In this case, the output is 3 of the input periods so that \(t=3\) with
```

[ma, Ca] = geom2arith(m, C, 3);

```

Example 5. Given geometric mean m and covariance C of 1254 observations of daily total returns over a 5 -year period, obtain annualized arithmetic return moments. Since the periodicity of the geometric data is based on 1254 observations for a 5 -year period, a 1 -year period for arithmetic returns implies a target period of \(t=\) \(1254 / 5\) so that
```

[ma, Ca] = geom2arith(m, C, 1254/5);

```

See Also arith2geom

\section*{Portfolio.getAssetMoments}

Purpose
Obtain mean and covariance of asset returns from portfolio object

\section*{Syntax}

Description

Tips

Input
Arguments
Output
Arguments
[AssetMean, AssetCovar] = getAssetMoments(obj)
[AssetMean, AssetCovar] = getAssetMoments(obj) to obtain the mean and covariance of asset returns from a portfolio object.

Use dot notation to obtain the mean and covariance of asset returns from a portfolio object:
```

[AssetMean, AssetCovar] = obj.getAssetMoments;

```
obj
A portfolio object [Portfolio].

AssetMean
Mean of asset returns [vector].

AssetCovar
Covariance of asset returns [matrix].

\section*{Attributes}

\section*{Examples}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Given the mean and covariance of asset returns in the variables \(m\) and C , the asset moment properties can be set and then obtained using getAssetMoments:
```

m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;

```

\section*{Portfolio.getAssetMoments}
```

            0.00408 0.0289 0.0204 0.0119;
            0.00192 0.0204 0.0576 0.0336;
            0 0.0119 0.0336 0.1225 ];
    m = m/12;
C = C/12;
p = Portfolio;
p = p.setAssetMoments(m, C);
[assetmean, assetcovar] = p.getAssetMoments
assetmean =
0.0042
0.0083
0.0100
0.0150
assetcovar =

| 0.0005 | 0.0003 | 0.0002 | 0 |
| ---: | ---: | ---: | ---: |
| 0.0003 | 0.0024 | 0.0017 | 0.0010 |
| 0.0002 | 0.0017 | 0.0048 | 0.0028 |
| 0 | 0.0010 | 0.0028 | 0.0102 |

```

\section*{See Also setAssetMoments}

Tutorials . "Working with Asset Returns and Moments of Asset Returns" on page 4-36

\section*{Portfolio.getBounds}

Superclasses AbstractPortfolio
Purpose Obtain bounds for portfolio weights from portfolio object
Syntax [LowerBound, UpperBound] = getBounds(obj)
Description [LowerBound, UpperBound] = getBounds(obj) to obtain bounds for portfolio weights from a portfolio object.

Tips
Use dot notation to obtain bounds for portfolio weights from the portfolio object:
[LowerBound, UpperBound] = obj.getBounds;
Input
Arguments
Output
Arguments
```

obj

```

A portfolio object [Portfolio].
LowerBound
Lower-bound weight for each asset [vector].
UpperBound
Upper-bound weight each asset [vector].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Examples Given portfolio p with the default constraints set, obtain the values for LowerBound and UpperBound:
p = Portfolio;
\(\mathrm{p}=\mathrm{p}\).setDefaultConstraints(5);
[LowerBound, UpperBound] = p.getBounds
LowerBound =
    0
    0
    0
    0
    0
    UpperBound =
    []

\section*{Portfolio.getBudget}

Superclasses AbstractPortfolio
Purpose Obtain budget constraint bounds from portfolio object
Syntax [LowerBudget, UpperBudget] = getBudget(obj)
Description [LowerBudget, UpperBudget] = getBudget(obj) to obtain budget constraint bounds from a portfolio object.

Tips
Use dot notation to obtain the budget constraint bounds from the portfolio object:
[LowerBudget, UpperBudget] = obj.getBudget;
Input
Arguments
Output
Arguments
obj
A portfolio object [Portfolio].
LowerBudget
Lower-bound for budget constraint [scalar].
UpperBudget
Upper-bound for budget constraint [scalar].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Examples Given portfolio p with the default constraints set, obtain the values for LowerBudget and UpperBudget:
p = Portfolio;
\(\mathrm{p}=\mathrm{p}\).setDefaultConstraints(5);
[LowerBudget, UpperBudget] = p.getBudget
LowerBudget =
    1
    UpperBudget =
    1

See Also setBudget

\section*{Portfolio.getCosts}

Superclasses AbstractPortfolio
Purpose Obtain buy and sell transaction costs from portfolio object
Syntax [BuyCost, SellCost] = getCosts(obj)
Description [BuyCost, SellCost] = getCosts(obj) to obtain buy and sell transaction costs from the portfolio object.

Tips
Use dot notation to obtain the buy and sell transaction costs from the portfolio object:
```

[BuyCost, SellCost] = obj.getCosts;

```

Input
Arguments
obj
A portfolio object [Portfolio].
Output Buycost
Arguments
Proportional transaction cost to purchase each asset [vector]. SellCost

Proportional transaction cost to sell each asset [vector].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Examples Given portfolio p with the costs set, obtain the values for BuyCost and SellCost:
```

p = Portfolio;
p = p.setCosts(0.001, 0.001, 5);
[BuyCost, SellCost] = p.getCosts
BuyCost =
1.0000e-003
SellCost =
1.0000e-003

```
See Also setCosts |

\section*{Portfolio.getEquality}

Superclasses AbstractPortfolio
Purpose Obtain equality constraint arrays from portfolio object
Syntax [AEquality, bEquality] = getEquality(obj)
Description

Tips
[AEquality, bEquality] = getEquality(obj) to obtain the equality constraint arrays from a portfolio object.

Use dot notation to obtain the equality constraint arrays from the portfolio object:
[AEquality, bEquality] = obj.getEquality;
Input
Arguments
obj
A portfolio object [Portfolio].
Output
Arguments
AEquality
Matrix to form linear equality constraints [matrix]. bEquality

Vector to form linear equality constraints [vector].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Examples Suppose you have a portfolio of five assets and you want to ensure that the first three assets are exactly \(50 \%\) of your portfolio. Given a portfolio
object \(p\), set the linear equality constraints and obtain the values for AEquality and bEquality:
```

A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = p.setEquality(A, b);
[AEquality, bEquality] = p.getEqualityAEquality =
1 1 1 1 1 0
bEquality =
0.5000

```

See Also setEquality

\section*{Portfolio.getGroupRatio}

Superclasses AbstractPortfolio
Purpose Obtain group ratio constraint arrays from portfolio object
Syntax [GroupA, GroupB, LowerRatio, UpperRatio] = getGroupRatio(obj)
Description [GroupA, GroupB, LowerRatio, UpperRatio] = getGroupRatio(obj) to obtain the group ratio constraint arrays from a portfolio object.

Tips
Use dot notation to obtain the equality constraint arrays from the portfolio object:
```

[GroupA, GroupB, LowerRatio, UpperRatio] = obj.getGroupRatio;

```

Input
obj
Arguments
Output
Arguments

\section*{GroupA}

Matrix that forms base groups for comparison [matrix].

\section*{GroupB}

Matrix that forms comparison groups [matrix].

\section*{LowerGroup}

Lower-bound for ratio of GroupB groups to GroupA groups [vector].
UpperRatio
Upper-bound for ratio of GroupB groups to GroupA groups [vector].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

\section*{Portfolio.getGroupRatio}
To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Examples}
Suppose you want to make sure that the ratio of financial to nonfinancial companies in your portfolios never goes above \(50 \%\). Assume you have 12 assets with 6 financial companies (assets 1-6) and 6 nonfinanical companies (assets 7-12). After setting group ratio constraints, obtain the values for GroupA, GroupB, LowerRatio, and UpperRatio:
```

GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % non-financial companies
p = Portfolio;
p = p.setGroupRatio(GA, GB, [], 0.5);
[GroupA, GroupB, LowerRatio, UpperRatio] = p.getGroupRatio
GroupA =
1
GroupB =
0
LowerRatio =
[]
UpperRatio =
0.5000

```
See Also
setGroupRatio

\section*{Portfolio.getGroups}

Superclasses AbstractPortfolio

\section*{Purpose Obtain group constraint arrays from portfolio object}

Syntax [GroupMatrix, LowerGroup, UpperGroup] = getGroups(obj)
Description [GroupMatrix, LowerGroup, UpperGroup] = getGroups(obj) to obtain the group constraint arrays from a portfolio object.

Tips

Input
Arguments
Output
Arguments

Use dot notation to obtain the group constraint arrays from the portfolio object:
[GroupMatrix, LowerGroup, UpperGroup] = obj.getGroups;
obj
A portfolio object [Portfolio].
GroupMatrix
Group constraint matrix [matrix].
LowerGroup
Lower-bound for group constraints [vector].
UpperGroup
Upper-bound for group constraints [vector].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Portfolio.getGroups}
```

Examples Suppose you have a portfolio of five assets and you want to ensure that
the first three assets constitute no more than $30 \%$ of your portfolio.
Given a portfolio object $p$ with the group constraints set, obtain the
values for GroupMatrix, LowerGroup, and UpperGroup:
$G=[$ true true true false false ];
p = Portfolio;
p = p.setGroups(G, [], 0.3);
[GroupMatrix, LowerGroup, UpperGroup] = p.getGroups
GroupMatrix =
$\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}$
LowerGroup =
[]
UpperGroup =
0.3000

```

See Also setGroups

\section*{Purpose Content of specific field}
```

Syntax fieldval = getfield(tsobj, field)
fieldval = getfield(tsobj, field, {dates})

```

\section*{Arguments}
\begin{tabular}{ll} 
tsobj & Financial time series object. \\
field & Field name within tsobj. \\
dates & \begin{tabular}{l} 
Date range. Dates can be expanded to include \\
time-of-day information.
\end{tabular}
\end{tabular}

\section*{Description}

\section*{Examples}
getfield treats the contents of a financial times series object tsobj as fields in a structure.
fieldval = getfield(tsobj, field) returns the contents of the specified field. This is equivalent to the syntax fieldval = tsobj field.
fieldval = getfield(tsobj, field, \{dates\}) returns the contents of the specified field for the specified dates. dates can be individual cells of date strings or a cell of a date string range using the : : operator, such as '03/01/99::03/31/99'.

Create a financial time series object containing both date and time-of-day information:
```

dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),···.
times]);
AnFts = fints(dates_times,[(1:4)'; nan; 6],{'Data1'},1,...
'Yet Another Financial Time Series')

```
```

AnFts =
desc: Yet Another Financial Time Series
freq: Daily (1)
'dates: (6)' 'times: (6)' 'Data1: (6)
'01-Jan-2001' '11:00' [ 1]
' " ' '12:00' [ 2]
'02-Jan-2001' '11:00' [ 3]
' " ' '12:00' [ 4]
'03-Jan-2001' '11:00' [ NaN]
" ' '12:00' [ 6]

```

Example 1. Get the contents of the times field in AnFts:
```

F = datestr(getfield(AnFts, 'times'))
F =
11:00 AM
12:00 PM
11:00 AM
12:00 PM
11:00 AM
12:00 PM

```

Example 2. Extract the contents of specific data fields within AnFts:
```

FF = getfield(AnFts,'Data1',...
01-Jan-2001 12:00::02-Jan-2001 12:00')
FF =
2
3
4

```

\author{
See Also chfield | fieldnames | isfield | rmfield | setfield
}

\section*{Superclasses AbstractPortfolio}

Purpose
Syntax
Description

Tips

Input
Arguments

\section*{Output \\ Arguments}

Obtain inequality constraint arrays from portfolio object
[AInequality, bInequality] = getInequality(obj)
[AInequality, bInequality] = getInequality(obj) to obtain the inequality constraint arrays from a portfolio object.

Use dot notation to obtain the inequality constraint arrays from the portfolio object:
[AInequality, bInequality] = obj.getInequality;
obj
A portfolio object [Portfolio].
AInequality
Matrix to form linear inequality constraints [matrix].

\section*{bInequality}

Vector to form linear inequality constraints [vector].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Examples Suppose you have a portfolio of five assets and you want to ensure that the first three assets are no more than \(50 \%\) of your portfolio. Given a

\section*{Portfolio.getInequality}
portfolio object p , set the linear inequality constraints and then obtain values for AInequality and bInequality:
```

A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = p.setInequality(A, b);
[AInequality, bInequality] = p.getInequalityAInequality =
1 1 1 1 1 0
bInequality =
0.5000

```
See Also setInequality

\section*{Purpose Find name in list}

Syntax nameidx = getnameidx(list, name)

\section*{Arguments}

\section*{Description}
nameidx = getnameidx(list, name) finds the occurrence of a name or set of names in a list. It returns an index (order number) indicating where the specified names are located within the list. If name is not found, nameidx returns 0 .

If name is a cell array of names, getnameidx returns a vector containing the indices (order number) of the name strings within list. If none of the names in the name cell array is in list, it returns zero. If some of the names in name are not found, the indices for these names will be zeros.
getnameidx finds only the first occurrence of the name in the list of names. This function is meant to be used on a list of unique names (strings) only. It does not find multiple occurrences of a name or a list of names within list.

\section*{Examples Given}
```

poultry = {'duck', 'chicken'}
animals = {'duck', 'cow', 'sheep', 'horse', 'chicken'}
nameidx = getnameidx(animals, poultry)
ans =
5

```

Given
```

poultry = {'duck', 'goose', 'chicken'}

```

\section*{getnameidx}
```

animals = {'duck', 'cow', 'sheep', 'horse', 'chicken'}
nameidx = getnameidx(animals, poultry)
ans =
105

```

See Also strcmp | strfind

\section*{Purpose Highest high}

Syntax \(\quad\) hhv \(=\) hhigh (data)
hhv = hhigh(data, nperiods, dim)
hhvts = hhigh(tsobj, nperiods)
hhvts = hhigh(tsobj, nperiods, ParameterName, ParameterValue)

\section*{Arguments}
\begin{tabular}{ll} 
data & Data series matrix. \\
nperiods & (Optional) Number of periods. Default =14. \\
dim & (Optional) Dimension. \\
tsobj & Financial time series object. \\
ParameterName & The valid parameter name is: \\
& - HighName: high prices series name \\
ParameterValue & \begin{tabular}{l} 
The parameter value is a string that represents \\
the valid parameter name.
\end{tabular}
\end{tabular}

\section*{Description}
hhv = hhigh(data) generates a vector of highest high values the past 14 periods from the matrix data.
hhv = hhigh(data, nperiods, dim) generates a vector of highest high values the past nperiods periods. dim indicates the direction in which the highest high is to be searched. If you input [ ] for nperiods, the default is 14 .
hhvts = hhigh(tsobj, nperiods) generates a vector of highest high values from tsobj, a financial time series object. tsobj must include at least the series High. The output hhvts is a financial time series object with the same dates as tsobj and data series named HighestHigh. If nperiods is specified, hhigh generates a financial time series object of highest high values for the past nperiods periods.
hhvts \(=\) hhigh(tsobj, nperiods, ParameterName, ParameterValue) specifies the name for the required data series when it is different from the default name. The valid parameter name isHighName. The parameter value is a string that represents the valid parameter name.

\section*{Examples Compute the highest high prices for Disney stock and plot the results:}
```

load disney.mat
dis_HHigh = hhigh(dis)
plot(dis_HHigh)
title('Highest High for Disney')

```


See Also llow

Purpose Time series High-Low plot

Syntax
highlow(tsobj) highlow(tsobj, color) highlow(tsobj, color, dateform) highlow(tsobj, color, dateform, ParameterName, ParameterValue, ...) hhll = highlow(tsobj, color, dateform, ParameterName, ParameterValue, ...)

\section*{Arguments}

Description

\section*{Examples}
highlow(tsobj) generates a High-Low plot of the data in the financial time series object tsobj. tsobj must contain at least four data series representing the high, low, open, and closing prices. These series must have the names High, Low, Open, and Close (case-insensitive).
highlow(tsobj, color) additionally specifies the color of the plot.
highlow(tsobj, color, dateform) additionally specifies the date string format used as the \(x\)-axis tick labels. See datestr for a list of date string formats.
highlow(tsobj, color, dateform, ParameterName, ParameterValue, ...) indicates the actual name(s) of the required data series if the data series do not have the default names.

You can specify open prices as optional by providing the parameter name 'OpenName ' and the parameter value ' ' (empty string).
```

highlow(tsobj, color, dateform, 'OpenName', '')

```
hhll = highlow(tsobj, color, dateform, ParameterName, ParameterValue, ...) returns the handle to the line object that makes up the High-Low plot.

Generate a High-Low plot for Disney stock for the dates from May 28 to June 18, 1998:
```

load disney.mat
highlow(dis('28-May-1998::18-Jun-1998'))
title('High-Low Plot for Disney')

```


See Also
candle

Purpose High, low, open, close chart
```

Syntax
highlow(High, Low, Close, Open, Color)
Handles = highlow(High, Low, Close, Open, Color)

```

\section*{Arguments}
\begin{tabular}{ll} 
High & High prices for a security. A column vector. \\
Low & Low prices for a security. A column vector. \\
Close & Closing prices for a security. A column vector. \\
Open & \begin{tabular}{l} 
(Optional) Opening prices for a security. A column \\
vector. To specify Color when Open is unknown, \\
enter Open as an empty matrix [ ]. \\
Color
\end{tabular} \\
& \begin{tabular}{l} 
(Optional) Vertical line color. A string. MATLAB \\
software supplies a default color if none is \\
specified. The default color differs depending on \\
the background color of the figure window. See \\
ColorSpec in the MATLAB documentation for color \\
names.
\end{tabular}
\end{tabular}

\section*{Description}
highlow(High, Low, Close, Open, Color) plots the high, low, opening, and closing prices of an asset. Plots are vertical lines whose top is the high, bottom is the low, open is a short horizontal tick to the left, and close is a short horizontal tick to the right.
Handles \(=\) highlow(High, Low, Close, Open, Color) plots the figure and returns the handles of the lines.

Examples The high, low, and closing prices for an asset are stored in equal-length vectors AssetHi, AssetLo, and AssetCl respectively.
```

highlow(AssetHi, AssetLo, AssetCl, [], 'cyan')

```
plots the price data using cyan lines.

\section*{See Also \\ bolling | candle | dateaxis | highlow | movavg | pointfig}

\section*{Purpose Histogram}
```

Syntax
hist(tsobj, numbins)
ftshist = hist(tsobj, numbins)
[ftshist, binpos] = hist(tsobj, numbins)

```

\section*{Arguments}

\section*{Description}
hist(tsobj, numbins) calculates and displays the histogram of the data series contained in the financial time series object tsobj.
ftshist = hist(tsobj, numbins) calculates, but does not display, the histogram of the data series contained in the financial time series object tsobj. The output ftshist is a structure with field names similar to the data series names of tsobj.
[ftshist, binpos] = hist(tsobj, numbins) additionally returns the bin positions binpos. The positions are the centers of each bin. binpos is a column vector.

\section*{Examples Create a histogram of Disney open, high, low, and close prices:}
```

load disney.mat
dis = rmfield(dis,'VOLUME') % Remove VOLUME field
hist(dis)
title('Disney Histogram')

```


See Also
mean | std | hist

\section*{holdings2weights}

Purpose Portfolio holdings into weights
```

Syntax Weights = holdings2weights(Holdings, Prices, Budget)

```

\section*{Arguments}

\section*{Description Weights = holdings2weights(Holdings, Prices, Budget) converts portfolio holdings into portfolio weights. The weights must satisfy a budget constraint such that the weights sum to Budget for each portfolio. \\ Weights is a NPORTS by NASSETS matrix containing the normalized weights of NPORTS portfolios containing NASSETS assets.}

\section*{Notes}
- Holdings may be negative to indicate a short position, but the overall portfolio weights must satisfy a nonzero budget constraint.
- The weights in each portfolio sum to the Budget value (which is 1 if Budget is unspecified.)

\footnotetext{
See Also
weights2holdings
}

Purpose Holidays and nontrading days
Syntax
\(\mathrm{H}=\) holidays
H = holidays(StartDate, EndDate)
H = holidays(AltHolidays)

Description

Input
Arguments

\section*{Output Arguments}

\section*{Definitions}
\(H=\) holidays returns a vector of serial date numbers corresponding to all holidays and nontrading days.

H = holidays(StartDate, EndDate) returns a vector of serial date numbers corresponding to the holidays and nontrading days between StartDate and EndDate, inclusive.
\(\mathrm{H}=\) holidays(AltHolidays) returns a vector of serial date numbers corresponding to the alternate list of holidays and nontrading days.

StartDate
Start date. Enter as a serial date number or date string.

\section*{EndDate}

End date. Enter as a serial date number or date string.

\section*{AltHolidays}

Alternate list of holidays and nontrading days stored as serial date numbers.

\section*{H}

Returns a vector of serial date numbers corresponding to all holidays and nontrading days.
holidays is based on a modern 5-day workweek. This function contains all holidays and special nontrading days for the New York Stock Exchange from January 1, 1885 to December 31, 2050. Since the New York Stock Exchange was open on Saturdays before September 29, 1952, exact closures from 1885 to 2050 should include Saturday trading days. To capture these dates, use the function nyseclosures.

\section*{holidays}

The results from holidays and nyseclosures are identical if the WorkWeekFormat in nyseclosures is 'Modern'.

\section*{Examples}

730852
730866
730901
730954
730999
The serial date numbers for these values are:
\begin{tabular}{ll} 
01-Jan-2001 & (New Year's Day) \\
15-Jan-2001 & (Martin Luther King Day) \\
19-Feb-2001 & (President's Day) \\
13-Apr-2001 & (Good Friday) \\
\(28-\) May- 2001 & (Memorial Day)
\end{tabular}

See Also
busdate | createholidays | fbusdate | isbusday | lbusdate | nyseclosures

\section*{Purpose \\ Description}

\section*{Examples}

Concatenate financial time series objects horizontally
horzcat implements horizontal concatenation of financial time series objects. horzcat essentially merges the data columns of the financial time series objects. The time series objects must contain the exact same dates and times.

When multiple instances of a data series name occur, concatenation adds a suffix to the current names of the data series. The suffix has the format _objectname \(\mathrm{n}>\), where n is a number indicating the position of the time series, from left to right, in the concatenation command. The n part of the suffix appears only when there is more than one instance of a particular data series name.

The description fields are concatenated as well. They are separated by two forward slashes (//).

Construct three financial time series, each containing a data series named DataSeries:
```

firstfts = fints((today:today+4)', (1:5)','DataSeries','d');
secondfts = fints((today:today+4)', (11:15)','DataSeries','d');
thirdfts = fints((today:today+4)', (21:25)','DataSeries','d');

```

Concatenate the time series horizontally into a new financial time series newfts:
```

newfts = [firstfts secondfts thirdfts secondfts];

```

The resulting object newfts has data series names DataSeries_firstfts, DataSeries_secondfts2, DataSeries_thirdfts, and DataSeries_secondfts4.
Verify this with the command
```

fieldnames(newfts)

```
ans =

\section*{horzcat}
```

'desc'
'freq'
'dates'
'DataSeries_firstfts'
'DataSeries_secondfts2'
'DataSeries_thirdfts'
'DataSeries_secondfts4'
'times'

```

Use chfield to change the data series names.

> Note If all input objects have the same frequency, the new object has that frequency as well. However, if one of the objects concatenated has a different frequency from the others, the frequency indicator of the resulting object is set to Unknown (0).

\section*{See Also} vertcat
Purpose Hour of date or time
Syntax Hour = hour(Date)
Description Hour = hour(Date) returns the hour of the day given a serial date number or a date string.
Examples Hour \(=\) hour (730473.5584278936)
or
Hour \(=\) hour('19-dec-1999, 13:24:08.17')
returns
Hour =13
See Also datevec | minute | second

\section*{inforatio}

Purpose Calculate information ratio for one or more assets

\author{
Syntax \\ inforatio(Asset, Benchmark) \\ Ratio = inforatio(Asset, Benchmark) \\ [Ratio, TE] = inforatio(Asset, Benchmark)
}

\section*{Arguments}

\section*{Description}

Asset NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES observations of asset returns for NUMSERIES asset return series.

Benchmark NUMSAMPLES vector of returns for a benchmark asset. The periodicity must be the same as the periodicity of Asset. For example, if Asset is monthly data, then Benchmark must be monthly returns.

Given NUMSERIES assets with NUMSAMPLES returns for each asset in a NUMSAMPLES x NUMSERIES matrix Asset and given a NUMSAMPLES vector of benchmark returns in Benchmark, inforatio computes the information ratio and tracking error for each asset relative to the Benchmark.

To summarize the outputs of inforatio:
- Ratio is a \(1 \times\) NUMSERIES row vector of information ratios for each series in Asset. Any series in Asset with a tracking error of 0 will have a NaN value for its information ratio.
- TE is a \(1 \times\) NUMSERIES row vector of tracking errors, that is, the standard deviation of Asset relative to Benchmark returns, for each series.

\title{
Note NaN values in the data are ignored. If the Asset and Benchmark series are identical, the information ratio will be NaN since the tracking error is 0 . The information ratio and the Sharpe ratio of an Asset versus a riskless Benchmark (a Benchmark with standard deviation of returns equal to 0 ) are equivalent. This equivalence is not necessarily true if the Benchmark is risky.
}

\section*{Examples See "Information Ratio Example" on page 5-8.}
\(\begin{array}{ll}\text { References } & \begin{array}{l}\text { Richard C. Grinold and Ronald N. Kahn, Active Portfolio Management, } \\ \text { 2nd. ed., McGraw-Hill, 2000. }\end{array} \\ & \text { Jack Treynor and Fischer Black, "How to Use Security Analysis to } \\ & \text { Improve Portfolio Selection," Journal of Business, Vol. 46, No. 1, } \\ & \text { January 1973, pp. 66-86. }\end{array}\)
See Also portalpha | sharpe

Purpose Internal rate of return
\(\begin{array}{ll}\text { Syntax } & \left.\begin{array}{l}\text { Return }=\operatorname{irr}(\text { CashFlow }) \\ \\ \end{array} \text { Return, AllRates }\right]=\operatorname{irr}(\text { CashFlow })\end{array}\)
Description Return = irr(CashFlow) calculates the internal rate of return for a series of periodic cash flows.
[Return, AllRates] = irr(CashFlow) calculates the internal rate of return and a vector of all internal rates for a series of periodic cash flows.

\section*{Input \\ Arguments}

\section*{Output Arguments}

\section*{Definitions}

CashFlow
A vector containing a stream of periodic cash flows. The first entry in CashFlow is the initial investment. If CashFlow is a matrix, irr handles each column of CashFlow as a separate cash-flow stream.

Return
An internal rate of return associated to CashFlow. If CashFlow is a matrix, then Return is a vector whose entry \(j\) is an internal rate of return for column \(j\) in CashFlow.

\section*{AllRates}

A vector containing all the internal rates of return associated with CashFlow. If CashFlow is a matrix, then AllRates is also a matrix, with the same number of columns as CashFlow and one less row. Also, column j in AllRates contains all the rates of return associated to column j in CashFlow (including complex-valued rates).
irr uses the following conventions:
- If one or more internal rate of returns (warning if multiple) are strictly positive rates, Return sets to the minimum.
- If no strictly positive rate of returns, but one or multiple (warning if multiple) returns are nonpositive rates, Return sets to the maximum.
- If no real-valued rates exist, Return sets to NaN (no warnings).

Examples
Find the internal rate of return for a simple investment with a unique positive rate of return. The initial investment is \(\$ 100,000\) and the following cash flows represent the yearly income from the investment.
- Year 1 - \$10,000
- Year 2 - \$20,000
- Year 3 - \$30,000
- Year 4 - \$40,000
- Year 5 - \$50,000

Calculate the internal rate of return on the investment:
```

Return = irr([-100000 10000 20000 30000 40000 50000])

```

This returns:
```

Return =

```
0.1201

If the cash flow payments were monthly, then the resulting rate of return is multiplied by 12 for the annual rate of return.

Find the internal rate of return for multiple rates of return. The project has the following cash flows and a market rate of \(10 \%\).
```

CashFlow = [-1000 6000 -10900 5800]

```

Use irr with a single output argument:
```

Return = irr(CashFlow)

```

A warning appears and irr returns a \(100 \%\) rate of return. The \(100 \%\) rate on the project looks attractive:
```

Warning: Multiple rates of return
> In irr at 166
Return =
1.0000

```

Use irr with two output arguments:
```

[Return, AllRates] = irr(CashFlow)

```

This returns:
```

>> [Return, AllRates] = irr(CashFlow)
Return =

```
    1.0000
AllRates =
        -0.0488
        1.0000
        2.0488

The rates of return in AllRates are \(-4.88 \%, 100 \%\), and \(204.88 \%\). Though some rates are lower and some higher than the market rate, based on the work of Hazen, any rate gives a consistent recommendation on the project. However, you can use a present value analysis in these kinds of situations. To check the present value of the project, use pvvar:
```

PV = pvvar(CashFlow,0.10)

```

This returns:
PV =
\(-196.0932\)

The second argument is the \(10 \%\) market rate. The present value is -196.0932, negative, so the project is undesirable.

\section*{References Brealey and Myers, Principles of Corporate Finance, McGraw-Hill Higher Education, Chapter 5, 2003. \\ Hazen G., "A New Perspective on Multiple Internal Rates of Return," The Engineering Economist, Vol. 48-1, 2003, pp. 31-51.}

See Also effrr | mirr | nomrr | xirr | pvvar
How To . "Interest Rates/Rates of Return" on page 2-17

Purpose True for dates that are business days
Syntax Busday = isbusday (Date, Holiday, Weekend)

\section*{Arguments}

\section*{Description Busday = isbusday (Date, Holiday, Weekend) returns logical true} (1) if Date is a business day and logical false (0) otherwise.

\section*{Examples Example 1:}
```

Busday = isbusday('16 jun 2001')
Busday =

```
```

Date = ['15 feb 2001'; '16 feb 2001'; '17 feb 2001'];
Busday = isbusday(Date)
Busday =

```

Example 2: Set June 21, 2003 (a Saturday) as a business day.
```

Weekend = [1 0 0 0 0 0 0];
isbusday('June 21, 2003', [], Weekend)
ans =

```
    1

Note If the second argument, Holiday, is empty ([ ]), the default Holidays vector (generated with holidays and then associated to the NYSE calendar) is used. To consider a calendar without holidays (except for weekends as controlled by third input), use a serial (or text date) out of your range. For example \(0-\) Jan- 0 or the value 0 are with high probabilities out of the range. This value overwrites the default calendar allowing you to remove any specific country holiday:
```

isbusday(datenum('06092010','ddmmyyyy'),0)
ans =

```

1

\section*{See Also}
busdate | fbusdate | holidays | lbusdate

\section*{iscompatible}
Purpose Structural equality
```

Syntax iscomp = iscompatible(tsobj_1, tsobj_2)

```

\section*{Arguments}
tsobj_1, tsobj_2 A pair of financial time series objects.

Description
iscomp = iscompatible(tsobj_1, tsobj_2) returns 1 if both financial time series objects tsobj_1 and tsobj_2 have the same dates and data series names. It returns 0 if any component is different.
iscomp \(=1\) indicates that the two objects contain the same number of data points and equal number of data series. However, the values contained in the data series can be different.

Note Data series names are case-sensitive.

\section*{See Also \\ isequal}
\(\begin{array}{ll}\text { Purpose } & \text { Multiple object equality } \\ \text { Syntax } & \text { iseq }=\text { isequal }\left(t s o b j \_1, \text { tsobj_2, } \ldots \text { ) }\right.\end{array}\)

Arguments
tsobj_1... A list of financial time series objects.

\section*{Description iseq = isequal(tsobj_1, tsobj_2, ...) returns 1 if all listed financial time series objects have the same dates, data series names, and values contained in the data series. It returns 0 if any of those components is different.}

Note Data series names are case-sensitive.
iseq \(=1\) implies that each object contains the same number of dates and the same data. Only the descriptions can differ.

See Also
eq | iscompatible

\section*{isempty}

Purpose True for empty financial time series objects

\section*{Syntax \(\quad\) tf \(=\) isempty (fts)}

Arguments
\[
\text { fts } \quad \text { Financial time series object. }
\]

Description isempty for financial times series objects is based on the MATLAB isempty function. See isempty in the MATLAB documentation.
\(\mathrm{tf}=\) isempty ( fts ) returns true (1) if fts is an empty financial time series object and false ( 0 ) otherwise. An empty financial times series object has no elements, that is, length \((f t s)=0\).

See Also nanmax | nanmean | nanmedian | nanmin | nanstd | nanvar
Purpose Check whether string is field name
Syntax ..... F = isfield(tsobj, name)
Description \(F=\) isfield(tsobj, name) returns true (1) if name is the name of a data series in tsobj. Otherwise, isfield returns false (0).
See Also ..... fieldnames | getfield | setfield

Purpose Check whether dates and times are monotonically increasing
Syntax monod = issorted(tsobj)

Arguments
tsobj Financial time series object

Description monod \(=\) issorted(tsobj) returns 1 if the dates and times in tsobj are monotonically increasing or 0 if they are not.
\begin{tabular}{ll} 
Purpose & Kagi chart \\
Syntax & kagi \((X)\) \\
Arguments &
\end{tabular}

\section*{Description kagi(X) plots asset price with respect to dates.}

\section*{Examples \\ If asset \(X\) is an M-by-2 matrix of date numbers and asset price:}
\[
x=[\ldots
\]
\begin{tabular}{ll}
733299.00 & \(41.99 ; \ldots\) \\
733300.00 & \(42.14 ; \ldots\) \\
733303.00 & \(41.93 ; \ldots\) \\
733304.00 & \(41.98 ; \ldots\) \\
733305.00 & \(41.75 ; \ldots\) \\
733306.00 & \(41.61 ; \ldots\) \\
733307.00 & \(42.29 ; \ldots\) \\
733310.00 & \(42.19 ; \ldots\) \\
733311.00 & \(41.82 ; \ldots\) \\
733312.00 & \(41.93 ; \ldots\) \\
733313.00 & \(41.81 ; \ldots\) \\
733314.00 & \(41.37 ; \ldots\) \\
733317.00 & \(41.17 ; \ldots\) \\
733318.00 & \(42.02]\)
\end{tabular}
then the Kagi chart is
kagi (X)
which plots the asset prices with respect to dates as follows.


See Also
bolling | candle | highlow | linebreak | movavg | pointfig | priceandvol | renko | volarea

\section*{Purpose Lag time series object}
```

Syntax newfts = lagts(oldfts)
newfts = lagts(oldfts, lagperiod)
newfts = lagts(oldfts, lagperiod, padmode)

```

\section*{Arguments}

Description

See Also
oldfts Financial time series object
lagperiod Number of lag periods expressed in the frequency of the time series object
padmode Data padding value
lagts delays a financial time series object by a specified time step. newfts \(=\) lagts(oldfts) delays the data series in oldfts by one time series date entry and returns the result in the object newfts. The end will be padded with zeros, by default.
newfts = lagts(oldfts, lagperiod) shifts time series values to the right on an increasing time scale. lagts delays the data series to happen at a later time. lagperiod is the number of lag periods expressed in the frequency of the time series object oldfts. For example, if oldfts is a daily time series, lagperiod is specified in days. lagts pads the data with zeros (default).
newfts = lagts(oldfts, lagperiod, padmode) lets you pad the data with an arbitrary value, NaN , or Inf rather than zeros by setting padmode to the desired value.

\section*{Ibusdate}

Purpose Last business date of month
```

Syntax Date = lbusdate(Year, Month, Holiday, Weekend)

```

\section*{Arguments}
\begin{tabular}{ll} 
Year & Enter as four-digit integer. \\
Month & Enter as integer from 1 through 12. \\
Holiday & \begin{tabular}{l} 
(Optional) Vector of holidays and nontrading-day \\
dates. All dates in Holiday must be the same format: \\
either serial date numbers or date strings. (Using \\
date numbers improves performance.) The holidays \\
function supplies the default vector.
\end{tabular} \\
Weekend & \(\left.\begin{array}{l}\text { (Optional) Vector of length 7, containing } 0 \text { and 1, the } \\
\text { value 1 indicating weekend days. The first element } \\
\text { of this vector corresponds to Sunday. Thus, when } \\
\text { Saturday and Sunday form the weekend (default), } \\
\text { then Weekend }=\left[\begin{array}{llll}1 & 0 & 0 & 0\end{array}\right. \\
\end{array} \quad \begin{array}{l}1\end{array}\right]\).
\end{tabular}

Description Date \(=\) lbusdate (Year, Month, Holiday, Weekend) returns the serial date number for the last business date of the given year and month. Holiday specifies nontrading days.

Year and Month can contain multiple values. If one contains multiple values, the other must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1-by-n vector of integers or a single integer. Date is then a 1-by-n vector of date numbers.

Use the function datestr to convert serial date numbers to formatted date strings.
```

Examples Example 1.
Date = lbusdate(2001, 5)
Date =
7 3 1 0 0 2
datestr(Date)
ans =
31-May-2001
c
ans =
31-May-2001
31-May-2002
30-May-2003

```

Example 2. You can indicate that Saturday is a business day by appropriately setting the Weekend argument.
```

Weekend = [1 0 0 0 0 0 0];

```

May 31, 2003, is a Saturday. Use lbusdate to check that this Saturday is actually the last business day of the month.
```

Date = datestr(lbusdate(2003, 5, [], Weekend))
Date =

```

31-May-2003
See Also busdate | eomdate | fbusdate | holidays | isbusday

\section*{leadts}

\section*{Purpose Lead time series object}
```

Syntax newfts = leadts(oldfts)
newfts = leadts(oldfts, leadperiod)
newfts = leadts(oldfts, leadperiod, padmode)

```

\section*{Arguments}
\begin{tabular}{ll} 
oldfts & Financial time series object. \\
leadperiod & \begin{tabular}{l} 
Number of lead periods expressed in the frequency \\
of the time series object.
\end{tabular} \\
padmode & Data padding value.
\end{tabular}

Description

See Also
lagts
Purpose Get number of dates (rows)

Syntax lenfts = length(tsobj)

Description lenfts = length(tsobj) returns the number of dates (rows) in the financial time series object tsobj. This is the same as issuing lenfts \(=\) size(tsobj, 1).

See Also size | length

\section*{linebreak}

Purpose Line break chart
Syntax linebreak( X )
Arguments
X M -by-2 matrix where the first column contains date numbers and the second column is the asset price.

Description linebreak \((X)\) plots asset price with respect to dates.

\section*{Examples \\ If asset X is an M -by- 2 matrix of date numbers and asset price:}
X = [...
\begin{tabular}{ll}
733299.00 & \(41.99 ; \ldots\) \\
733300.00 & \(42.14 ; \ldots\) \\
733303.00 & \(41.93 ; \ldots\) \\
733304.00 & \(41.98 ; \ldots\) \\
733305.00 & \(41.75 ; \ldots\) \\
733306.00 & \(41.61 ; \ldots\) \\
733307.00 & \(42.29 ; \ldots\) \\
733310.00 & \(42.19 ; \ldots\) \\
733311.00 & \(41.82 ; \ldots\) \\
733312.00 & \(41.93 ; \ldots\) \\
733313.00 & \(41.81 ; \ldots\) \\
733314.00 & \(41.37 ; \ldots\) \\
733317.00 & \(41.17 ; \ldots\) \\
733318.00 & \(42.02]\)
\end{tabular}
then the Line break chart is
```

linebreak(X)

```
which plots the asset prices with respect to dates as follows.


See Also
bolling | candle | highlow | kagi | movavg | pointfig | priceandvol | renko | volarea

\section*{llow}

Purpose Lowest low
```

Syntax llv = llow(data)
llv = llow(data, nperiods, dim)
llvts = llow(tsobj, nperiods)
llvts = llow(tsobj, nperiods, ParameterName, ParameterValue)

```

\section*{Arguments}
\begin{tabular}{ll} 
data & Data series matrix. \\
nperiods & (Optional) Number of periods. Default =14. \\
dim & Dimension. \\
tsobj & Financial time series object. \\
ParameterName & The valid parameter name is: \\
& - LowName: low prices series name \\
Parametervalue & \begin{tabular}{l} 
The parameter value is a string that represents \\
the valid parameter name.
\end{tabular}
\end{tabular}

\section*{Description}
llv = llow(data) generates a vector of lowest low values for the past 14 periods from the matrix data.
llv = llow(data, nperiods, dim) generates a vector of lowest low values for the past nperiods periods. dim indicates the direction in which the lowest low is to be searched. If you input [] for nperiods, the default is 14 .
llvts = llow(tsobj, nperiods) generates a vector of lowest low values from tsobj, a financial time series object. tsobj must include at least the series Low. The output llvts is a financial time series object with the same dates as tsobj and data series named LowestLow. If nperiods is specified, llow generates a financial time series object of lowest low values for the past nperiods periods.
llvts = llow(tsobj, nperiods, ParameterName, ParameterValue) specifies the name for the required data series when it is different from the default name. The parameter value is a string that represents the valid parameter name.

\section*{Examples}

Compute the lowest low prices for Disney stock and plot the results.
```

load disney.mat
dis_LLow = llow(dis)
plot(dis_LLow)
title('Lowest Low for Disney')

```


See Also hhigh
```

Purpose Natural logarithm
Syntax newfts = log(tsobj)
Description newfts = log(tsobj) calculates the natural logarithm (log base e)
of the data series in a financial time series object tsobj. It returns
another time series object newfts containing the natural logarithms.

```

See Also \(\exp |\log 2| \log 10\)
Purpose Common logarithm
Syntax newfts \(=\log 10(t s o b j)\)
Description newfts \(=\log 10(\) tsobj \()\) calculates the common logarithm (base 10) of all the data in the data series of the financial time series object tsobj and returns the result in the object newfts.
See Also ..... \(\exp |\log | \log 2\)

Purpose Base 2 logarithm
Syntax newfts = log2(tsobj)

Description newfts \(=\log 2(t s o b j)\) calculates the base 2 logarithm of the data series in a financial time series object tsobj. It returns another time series object newfts containing the logarithms.

See Also exp \(|\log | \log 10\)

\section*{Purpose}

Compute sample lower partial moments of data

\section*{Syntax}
```

lpm(Data)
lpm(Data, MAR)
lpm(Data, MAR, Order)
Moment = lpm(Data, MAR, Order)

```

\section*{Arguments}
\begin{tabular}{ll} 
Data & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES \\
observations of NUMSERIES asset returns.
\end{tabular} \\
MAR & \begin{tabular}{l} 
(Optional) Scalar minimum acceptable return \\
(default MAR = 0). This is a cutoff level of return such \\
that all returns above MAR contribute nothing to the
\end{tabular} \\
Order & \begin{tabular}{l} 
lower partial moment. \\
(Optional) Either a scalar or a NUMORDERS-vector \\
of nonnegative integer moment orders. If no order \\
specified, default Order = 0, which is the shortfall \\
probability. Although this function will work for \\
noninteger orders and, in some cases, for negative \\
orders, this falls outside customary usage.
\end{tabular}
\end{tabular}

Given NUMSERIES assets with NUMSAMPLES returns in a NUMSAMPLES-by-NUMSERIES matrix Data, a scalar minimum acceptable return MAR, and one or more nonnegative moment orders in a NUMORDERS vector Order, lpm computes lower partial moments relative to MAR for each asset in a NUMORDERS x NUMSERIES matrix Moment.

The output Moment is a NUMORDERS x NUMSERIES matrix of lower partial moments with NUMORDERS Orders and NUMSERIES series, that is, each row contains lower partial moments for a given order.

Note To compute upper partial moments, just reverse the signs of both Data and MAR (do not reverse the sign of the output). This function computes sample lower partial moments from data. To compute expected lower partial moments for multivariate normal asset returns with a specified mean and covariance, use elpm. With lpm, you can compute various investment ratios such as Omega ratio, Sortino ratio, and Upside Potential ratio, where:
- Omega = lpm(-Data, -MAR, 1) / lpm(Data, MAR, 1)
- Sortino \(=(\) mean (Data) - MAR) \(/ \operatorname{sqrt}(1 p m(D a t a, ~ M A R, 2))\)
- Upside \(=1 p m(-\) Data, - MAR, 1\() / \operatorname{sqrt(lpm(Data,~MAR,~2))~}\)

\footnotetext{
Examples See "Sample Lower Partial Moments Example" on page 5-14.
References Vijay S. Bawa, "Safety-First, Stochastic Dominance, and Optimal Portfolio Choice," Journal of Financial and Quantitative Analysis, Vol. 13, No. 2, June 1978, pp. 255-271.
W. V. Harlow, "Asset Allocation in a Downside-Risk Framework," Financial Analysts Journal, Vol. 47, No. 5, September/October 1991, pp. 28-40.
W. V. Harlow and K. S. Rao, "Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence," Journal of Financial and Quantitative Analysis, Vol. 24, No. 3, September 1989, pp. 285-311.

Frank A. Sortino and Robert van der Meer, "Downside Risk," Journal of Portfolio Management, Vol. 17, No. 5, Spring 1991, pp. 27-31.

\section*{See Also elpm}
}

\section*{Purpose \\ Date of last occurrence of weekday in month \\ Syntax \\ LastDate = lweekdate(Weekday, Year, Month, NextDay)}

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline Weekday & Weekday whose date you seek. Enter as an integer from 1 through 7: \\
\hline & 1 Sunday \\
\hline & 2 Monday \\
\hline & 3 Tuesday \\
\hline & 4 Wednesday \\
\hline & 5 Thursday \\
\hline & 6 Friday \\
\hline & 7 Saturday \\
\hline Year & Year. Enter as a four-digit integer. \\
\hline Month & Month. Enter as an integer from 1 through 12. \\
\hline NextDay & (Optional) Weekday that must occur after Weekday in the same week. Enter as an integer from 0 through 7 , where \(0=\) ignore (default) and 1 through 7 are the same as for Weekday. \\
\hline
\end{tabular}

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1 -by-n vector of integers or a single integer. LastDate is then a 1-by-n vector of date numbers.

Description LastDate = lweekdate(Weekday, Year, Month, NextDay) returns the serial date number for the last occurrence of Weekday in the given year and month and in a week that also contains NextDay.
Use the function datestr to convert serial date numbers to formatted date strings.

Examples Example 1. To find the last Monday in June 2001
```

LastDate = lweekdate(2, 2001, 6); datestr(LastDate)
ans =
25-Jun-2001

```

Example 2. To find the last Monday in a week that also contains a Friday in June 2001
```

LastDate = lweekdate(2, 2001, 6, 6); datestr(LastDate)
ans =
25-Jun-2001

```

Example 3. To find the last Monday in May for 2001, 2002, and 2003
```

Year = [2001:2003];
LastDate = lweekdate(2, Year, 5)
LastDate =
730999 731363 731727
datestr(LastDate)
ans =

```
28-May-2001

27-May-2002
26-May-2003
See Also eomdate | lbusdate | nweekdate

\section*{m2xdate}

Purpose MATLAB serial date number to Excel serial date number
Syntax DateNum = m2xdate (MATLABDateNumber, Convention)

\section*{Arguments}
\begin{tabular}{ll} 
MATLABDateNumber & \begin{tabular}{l} 
A vector or scalar of MATLAB serial date \\
numbers. \\
(Optional) Excel date system. A vector or \\
scalar. When Convention \(=0\) (default), the \\
Excel 1900 date system is in effect. When
\end{tabular} \\
Convention 1 , the Excel 1904 date system \\
in used.
\end{tabular}\(\quad\)\begin{tabular}{l} 
In the Excel 1900 date system, the Excel serial \\
date number 1 corresponds to January 1, \\
\\
1900 A.D. In the Excel 1904 date system, date \\
number 0 is January 1, 1904 A.D. \\
\\
\begin{tabular}{l} 
Due to a software limitation in Excel software, \\
the year 1900 is considered a leap year. As a \\
result, all DATEVALUE's reported by Excel \\
software between Jan. 1, 1900 and Feb. 28, \\
1900 (inclusive) differs from the values reported \\
by 1. For example:
\end{tabular} \\
\hline
\end{tabular}
- In Excel software, Jan. 1, \(1900=1\)
- In MATLAB, Jan. 1, \(1900=2\)

Vector arguments must have consistent dimensions.

\section*{Description}

DateNum = m2xdate(MATLABDateNumber, Convention) converts MATLAB serial date numbers to Excel serial date numbers. MATLAB date numbers start with \(1=\) January 1, 0000 A.D., hence there is a
difference of 693960 relative to the 1900 date system, or 695422 relative to the 1904 date system. This function is useful with Spreadsheet Link \({ }^{\text {TM }}\) EX software.

Examples Given MATLAB date numbers for Christmas 2001 through 2004
DateNum = datenum(2001:2004, 12, 25)
DateNum =
\(\begin{array}{llll}731210 & 731575 & 731940 & 732306\end{array}\)
convert them to Excel date numbers in the 1904 system
ExDate = m2xdate(DateNum, 1)
ExDate =
\(\begin{array}{llll}35788 & 36153 & 36518 & 36884\end{array}\)
or the 1900 system
ExDate = m2xdate(DateNum)
ExDate =
\(\begin{array}{llll}37250 & 37615 & 37980 & 38346\end{array}\)

\section*{See Also}
datenum | datestr | x2mdate

\section*{Purpose Moving Average Convergence/Divergence (MACD)}
```

Syntax [macdvec, nineperma] = macd(data)
[macdvec, nineperma] = macd(data, dim)
macdts = macd(tsobj, series_name)

```

\section*{Arguments}
\begin{tabular}{ll} 
data & Data matrix \\
dim & Dimension. Default = 1 (column orientation). \\
tsobj & Financial time series object \\
series_name & Data series name
\end{tabular}
[macdvec, nineperma] \(=\) macd(data) calculates the Moving Average Convergence/Divergence (MACD) line, macdvec, from the data matrix, data, as well as the nine-period exponential moving average, nineperma, from the MACD line.

When the two lines are plotted, they can give you an indication of whether to buy or sell a stock, when an overbought or oversold condition is occurring, and when the end of a trend might occur.

The MACD is calculated by subtracting the 26-period (7.5\%) exponential moving average from the 12 -period ( \(15 \%\) ) moving average. The 9 -day ( \(20 \%\) ) exponential moving average of the MACD line is used as the signal line. For example, when the MACD and the \(20 \%\) moving average line have just crossed and the MACD line falls below the other line, it is time to sell.
[macdvec, nineperma] = macd(data, dim) lets you specify the orientation direction for the input. If the input data is a matrix, you need to indicate whether each row is a set of observations (dim \(=2\) ) or each column is a set of observations (dim = 1, the default).
macdts = macd(tsobj, series_name) calculates the MACD line from the financial time series tsobj, as well as the nine-period exponential moving average from the MACD line. The MACD is calculated for the closing price series in tsobj, presumed to have been named Close. The result is stored in the financial time series object macdts. The macdts object has the same dates as the input object tsobj and contains only two series, named MACDLine and NinePerMA. The first series contains the values representing the MACD line and the second is the nine-period exponential moving average of the MACD line.

\section*{Examples Compute the MACD for Disney stock and plot the results:}
```

load disney.mat
dis_CloseMACD = macd(dis);
dis_OpenMACD = macd(dis, 'OPEN');
plot(dis_CloseMACD);
plot(dis_OpenMACD);
title('MACD for Disney')

```


See Also
adline | willad

\section*{Purpose Maximum value}

Syntax tsmax \(=\max (\mathrm{tsobj})\)

Description tsmax \(=\max (\mathrm{tsobj})\) finds the maximum value in each data series in the financial time series object tsobj and returns it in a structure tsmax. The tsmax structure contains field name(s) identical to the data series name(s).

Note tsmax returns only the values and does not return the dates associated with the values. The maximum values are not necessarily from the same date.

See Also min

\section*{maxdrawdown}

\section*{Purpose Compute maximum drawdown for one or more price series}
```

Syntax MaxDD = maxdrawdown(Data)
MaxDD = maxdrawdown(Data, Format)
[MaxDD, MaxDDIndex] = maxdrawdown(Data, Format)

```

\section*{Arguments}
\[
\text { Data } \begin{gathered}
\text { Format } \quad \begin{array}{l}
\text { T-by-N matrix with } T \text { samples of } N \text { total return price } \\
\text { series (also known as total equity). } \\
\text { (Optional) MATLAB string indicating format of data. } \\
\text { Possible values are: } \\
\text { 'return ' (default): Maximum drawdown in terms of } \\
\text { maximum percentage drop from a peak. } \\
\text { 'arithmetic ': Maximum drawdown of an } \\
\text { arithmetic Brownian motion with drift (differences } \\
\text { of data from peak to trough) using the equation } \\
\\
\quad d X(t)=\mu d t+\sigma d W(t) \text {. } \\
\text { 'geometric ': Maximum drawdown of a geometric } \\
\text { Brownian motion with drift (differences of log of data } \\
\text { from peak to trough) using the equation }
\end{array}
\end{gathered}
\]
\[
d S(t)=\mu_{0} S(t) d t+\sigma_{0} S(t) d W(t)
\]

\section*{Description MaxDD = maxdrawdown(Data, Format) computes maximum drawdown} for each series in an N -vector MaxDD and identifies start and end indexes of maximum drawdown periods for each series in a 2 x N matrix MaxDDIndex.

To summarize the outputs of maxdrawdown:
- MaxDD is a 1 -by- N vector with maximum drawdown for each of N time series.
- MaxDDIndex is a 2-by-N vector of start and end indexes for each maximum drawdown period for each total equity time series, where the first row contains the start indexes and the second row contains the end indexes of each maximum drawdown period.

\section*{Notes}
- Drawdown is the percentage drop in total returns from the start to the end of a period. If the total equity time series is increasing over an entire period, drawdown is 0 . Otherwise, it is a positive number. Maximum drawdown is an ex-ante proxy for downside risk that computes the largest drawdown over all intervals of time that can be formed within a specified interval of time.
- Maximum drawdown is sensitive to quantization error.

\section*{Examples}

\section*{References Christian S. Pederson and Ted Rudholm-Alfvin, "Selecting a Risk-Adjusted Shareholder Performance Measure," Journal of Asset Management, Vol. 4, No. 3, 2003, pp. 152-172.}

Purpose Arithmetic average
Syntax tsmean \(=\) mean \((t s o b j)\)

Description tsmean \(=\) mean(tsobj) computes the arithmetic mean of all data in all series in tsobj and returns it in a structure tsmean. The tsmean structure contains field name(s) identical to the data series name(s).

See Also
peravg \| tsmovavg

\section*{Purpose Median price}
```

Syntax
mprc = medprice(highp, lowp)
mprc = medprice([highp lowp])
mprcts = medprice(tsobj)
mprcts = medprice(tsobj, ParameterName, ParameterValue, ...)

```

\section*{Arguments}
\begin{tabular}{ll} 
highp & High price (vector) \\
lowp & Low price (vector) \\
tsobj & Financial time series object \\
ParameterName & Valid parameter names are: \\
ParameterValue & \begin{tabular}{l} 
- HighName: high prices series name \\
Parameter values are the strings that \\
represent the valid parameter names.
\end{tabular}
\end{tabular}

\section*{Description}
mprc \(=\) medprice(highp, lowp) calculates the median prices mprc from the high (highp) and low (lowp) prices. The median price is the average of the high and low price for each period.
mprc = medprice([highp lowp]) accepts a two-column matrix as the input rather than two individual vectors. The columns of the matrix represent the high and low prices, in that order.
mprcts \(=\) medprice(tsobj) calculates the median prices of a financial time series object tsobj. The object must minimally contain the series High and Low. The median price is the average of the high and low price each period. mprcts is a financial time series object with the same dates as tsobj and the data series MedPrice.
mprcts \(=\) medprice(tsobj, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Parameter values are the strings that represent the valid parameter names.

\section*{Examples Compute the median price for Disney stock and plot the results:}
```

load disney.mat
dis_MedPrice = medprice(dis)
plot(dis_MedPrice)
title('Median Price for Disney')

```


References
Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 177-178.
```

Purpose
Merge multiple financial time series objects
Syntax newfts = merge(fts1, fts2)
newfts = merge(fts1, fts2, ..., ftsx)
newfts = merge(fts1, fts2, ..., ftsx, 'PARAM1', VALUE1, 'PARAM2',
VALUE2, ...)

```

\section*{Arguments}
fts1,
fts2,.. \begin{tabular}{l} 
Comma-separated list of financial time series \\
objects to merge.
\end{tabular}

> Note Multiple Financial Time Series objects can be merged at once. The merged objects must appear in a comma separated list before the optional inputs. The order of the inputs is significant.

\footnotetext{
' DateSetMethod ' (Optional) Merge method. Valid merge values are: 'union ': (Default) Returns the combined values of all merged objects.
'intersection': Returns the values common to all merged objects.
Ref0bf: Maps all values to a reference time contained in a Financial Time Series object (RefObj) or vector of date numbers.
}
\begin{tabular}{ll} 
'DataSetMethod ' & \begin{tabular}{l} 
(Optional) Merge method. Valid merge values are: \\
'closest': (Default) Returns data based on the
\end{tabular} \\
order of the inputs. However, the first missing \\
data point (NaN value) of a date will be replaced by \\
the closest non-NaN data point that appears on the \\
same date of subsequent merged objects. \\
'order': Returns data based strictly on the order \\
of the inputs.
\end{tabular}

\section*{Description}
newfts = merge(fts1, fts2, ..., ftsx, 'PARAM1', VALUE1, 'PARAM2', VALUE2', ...) merges multiple financial time series objects. The optional parameter and value pair argument specifies the values contained in the output financial time series object ftsout.

Examples Example 1. Create three financial time series objects and merge them into a single object.
```

dates = {'jan-01-2001'; 'jan-02-2001'; 'jan-03-2001'; ...
'jan-04-2001'; 'jan-06-2001'};
data = [1; 1; 1; 1; 1];
t1 = fints(dates, data);
dates = {'jan-02-2001'; 'jan-03-2001'; 'jan-04-2001';
'jan-05-2001'};
data = [2; 2; 2; 2];
t2 = fints(dates, data);
dates = {'jan-03-2001'; 'jan-04-2001'; 'jan-05-2001';
'jan-06-2001'};

```
```

data = [3; 3; 3; 3];
t3 = fints(dates, data);
t123 = merge(t1, t2, t3)
ans =
desc: || ||
freq: Unknown (0)
'dates: (6)' 'series1: (6)'
'01-Jan-2001' [ 1]
'02-Jan-2001' [ 1]
'03-Jan-2001' [ 1]
'04-Jan-2001' [ 1]
'05-Jan-2001' [ 2]
'06-Jan-2001' [ 1]

```

If you change the order of input time series, the output may contain different data when duplicate dates exist. Here, for example, is the result of using the same three time series defined above but with the order changed.
```

merge(t3, t2, t1)
ans =
desc: || ||
freq: Unknown (0)
'dates: (6)' 'series1: (6)'
'01-Jan-2001' [ 1]
'02-Jan-2001' [ 2]
'03-Jan-2001' [ 3]
'04-Jan-2001' [ 3]
'05-Jan-2001' [ 3]
'06-Jan-2001' [ 3]%

```

Note t123 contains all 1s except on '05-Jan-2001' because t1 appears first in the list of inputs and takes precedence. The same logic can be applied tot321.

By changing the order of inputs, you can overwrite old financial time series data with new data by placing the new time series ahead of the old one in the list of inputs to the merge function.

Example 2. Merging time series objects with different headers (series names).
```

dates = {'jan-01-2001'; 'jan-02-2001'; 'jan-03-2001'; ...
'jan-04-2001'; 'jan-06-2001'};
data = [1; 1; 1; 1; 1];
t4 = fints(dates, data, 'ts4');
dates = {'jan-02-2001'; 'jan-03-2001'; 'jan-04-2001'; 'jan-05-2001'};
data = [2; 2; 2; 2];
t5 = fints(dates, data, 'ts5');
t45 = merge(t4, t5)
t45 =
desc: ||
freq: Unknown (0)
'dates: (6)' 'ts4: (6)' 'ts5: (6)'
'01-Jan-2001' [ 1] [ NaN]
'02-Jan-2001' [ 1] [ 2]
'03-Jan-2001' [ 1] [ 2]
'04-Jan-2001' [ 1] [ 2]
'05-Jan-2001' [ NaN] [ 2]
'06-Jan-2001' [ 1] [ NaN]

```

\section*{See Also \\ horzcat | vertcat}

\section*{Purpose Minimum value}

Syntax \(\quad\) tsmin \(=\min (t s o b j)\)

Description tsmin \(=\min (\mathrm{tsobj})\) finds the minimum value in each data series in the financial time series object tsobj and returns it in the structure tsmin. The tsmin structure contains field name(s) identical to the data series name(s).

Note tsmin returns only the values and does not return the dates associated with the values. The minimum values are not necessarily from the same date.

See Also max

\section*{minus}

Purpose Financial time series subtraction
```

Syntax newfts = tsobj_1 - tsobj_2
newfts = tsobj - array
newfts = array - tsobj

```

\section*{Arguments}

\section*{Description}

See Also
minus is an element-by-element subtraction of the components.
newfts = tsobj_1 - tsobj_2 subtracts financial time series objects. If an object is to be subtracted from another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when one financial time series object is subtracted from another, follows the order of the first object.
newfts = tsobj - array subtracts an array element by element from a financial time series object.
newfts = array - tsobj subtracts a financial time series object element by element from an array.
rdivide | plus | times
Purpose Minute of date or time
Syntax Minute = minute(Date)
Description Minute \(=\) minute(Date) returns the minute given a serial date number or a date string.
Examples Minute = minute(731204.5591223380)
or
Minute \(=\) minute('19-dec-2001, 13:25:08.17')
returns
Minute =
25
See Also ..... datevec | hour | second

Purpose Modified internal rate of return
```

Syntax Return = mirr(CashFlow, FinRate, Reinvest)

```

\section*{Arguments}

CashFlow Vector of cash flows. The first entry is the initial investment.

FinRate Finance rate for negative cash flow values. Enter as a decimal fraction.

Reinvest Reinvestment rate for positive cash flow values, as a decimal fraction.

Description Return = mirr(CashFlow, FinRate, Reinvest) calculates the modified internal rate of return for a series of periodic cash flows. This function calculates only positive rates of return; for nonpositive rates of return, Return \(=0\).

Examples \(\begin{aligned} & \text { This cash flow represents the yearly income from an initial investment } \\ & \text { of } \$ 100,000 \text {. The finance rate is } 9 \% \text { and the reinvestment rate is } 12 \% \text {. }\end{aligned}\)

Year \(1 \quad \$ 20,000\)
Year \(2 \quad(\$ 10,000)\)
Year \(3 \quad \$ 30,000\)
Year \(4 \quad \$ 38,000\)
Year \(5 \quad \$ 50,000\)

To calculate the modified internal rate of return on the investment
```

    Return = mirr([-100000 20000 -10000 30000 38000 50000], 0.09,...
    0.12)
    returns
Return =
0.0832 (8.32%)

```

References Brealey and Myers, Principles of Corporate Finance, Chapter 5
See Also annurate | effrr | irr | nomrr | pvvar | xirr

\section*{Purpose Month of date}
```

Syntax [MonthNum, MonthString] = month(Date)
[MonthNum, MonthString] = month(Date, F)

```

Description [MonthNum, MonthString] = month(Date) returns the month in numeric and string form given a serial date number or a date string.
[MonthNum, MonthString] = month(Date, F) returns the day of the of the month, given a serial date number or date string, in a specified date format.

Examples [MonthNum, MonthString] = month(730368)
or
[MonthNum, MonthString] = month('05-Sep-1999')
returns
MonthNum =

9

MonthString =
Sep
You can also use the \(F\) argument to designate a country-specific date format:
[MonthNum, MonthString] = month('1999/05/09','yyyy/dd/mm')
returns
hmiMonthNum =

\section*{9}

MonthString =
Sep
See Also datevec | day | year

Purpose Number of whole months between dates
Syntax MyMonths = months(StartDate, EndDate, EndMonthFlag)

\section*{Arguments}
\begin{tabular}{ll} 
StartDate & Enter as serial date numbers or date strings. \\
EndDate & Enter as serial date numbers or date strings. \\
EndMonthFlag & (Optional) end-of-month flag. If StartDate and \\
& EndDate are end-of-month dates and EndDate \\
& \begin{tabular}{l} 
has fewer days than StartDate, EndMonthFlag \\
\\
\(=1\) (default) treats EndDate as the end of a \\
whole month, while EndMonthFlag \(=0\) does
\end{tabular} \\
& not.
\end{tabular}

\section*{Description}

\section*{Examples}

MyMonths = months(StartDate, EndDate, EndMonthFlag) returns the number of whole months between StartDate and EndDate. If EndDate is earlier than StartDate, MyMonths is negative. Enter dates as serial date numbers or date strings.

Any input argument can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if StartDate is an \(n\)-row character array of date strings, then EndDate must be an \(n\)-row character array of date strings or a single date. MyMonths is then an \(n\)-by- 1 vector of numbers.
```

MyMonths = months('may 31 2000', 'jun 30 2000', 1)
MyMonths =
1
MyMonths = months('may 31 2000','jun 30 2000', 0)
MyMonths =
0

```
```

Dates = ['mar 31 2002'; 'apr 30 2002'; 'may 31 2002'];
MyMonths = months(Dates, 'jun 30 2002')
MyMonths =
3
2
1

```

See Also yearfrac

Purpose Leading and lagging moving averages chart
```

Syntax
movavg(Asset, Lead, Lag, Alpha)
[Short, Long] = movavg(Asset, Lead, Lag, Alpha)

```

\section*{Arguments}

\section*{Description}

\section*{Examples}
\begin{tabular}{ll} 
Asset & \begin{tabular}{l} 
Security data, a vector of time-series prices. \\
Number of samples to use in leading average \\
calculation. A positive integer. Lead must be less \\
than or equal to Lag.
\end{tabular} \\
Lag & \begin{tabular}{l} 
Number of samples to use in the lagging average \\
calculation. A positive integer.
\end{tabular} \\
Alpha & \begin{tabular}{l} 
(Optional) Control parameter that determines \\
the type of moving averages. \(0=\) simple moving \\
average (default), \(0.5=\) square root weighted moving \\
average, \(1=\) linear moving average, \(2=\) square \\
weighted moving average, and so on. To calculate \\
the exponential moving average, set Alpha \(=\) ' \(\mathrm{e}^{\prime}\).
\end{tabular}
\end{tabular}
movavg(Asset, Lead, lag, Alpha) plots leading and lagging moving averages.
[Short, Long] = movavg(Asset, Lead, lag, Alpha) returns the leading Short and lagging Long moving average data without plotting it.

If asset A is a vector of stock price data from 01/01/2006 to 02/01/2006
```

>> A(:,2)
ans =
8.6500

```
9.0000
8.8500
9.3500
9.5000
9.3500
9.2500
9.7000
9.9500
10.5000
10.1000
9.9000
10.0000
9.9000
9.6000
9.7000
9.8000
9.7000
9.9500
10.1500
9.8500
9.9000
10.2000
10.0000
9.9500
9.8500
9.9500
10.0000
10.0000
10.5400
10.5900
11.1900
11.0400
11.0900
10.7400
10.3500
10.2500
10.4500

17-535
10.7400
10.5900
10.3000
10.0500
9.8500
9.9000
9.9000
10.3000
10.4500
10.7400
10.9400
10.6900
10.9400
10.7900
10.2000
11.4900
11.9400
11.9400
12.4800
13.1800
12.9800
12.9800
13.6800
13.4800
13.9300
14.9200
13.9300
14.4200
14.0300
14.9700
15.0700
15.0200
15.0700
14.4200
13.7800
14.2800
14.8700
15.5700
15.5200
15.5200
15.5200
15.5200
15.7200
15.8200
15.9700
15.2700
14.9500
14.8000
15.1500
14.5000
15.0500
14.5500
14.6000
14.5500
17.5500
16.7000
16.8000
17.9500
17.3000
17.6000
17.5500
16.5000
15.8500
16.3000
then the moving average is
```

[Short,Long]= movavg(A(:,2),3,20,1);
movavg(A(:,2),3,20,1);
ylabel('Price')
legend('Asset Price','Lagging Long','Leading Short')

```
this plots linear three-sample leading and 20-sample lagging moving averages


See Also
bolling | candle | dateaxis | highlow | pointfig

\section*{Purpose Financial time series matrix division}
```

Syntax

```
newfts = array / tsobj
```

```
newfts = tsobj_1 / tsobj_2
```

newfts = tsobj_1 / tsobj_2
newfts = tsobj / array

```
newfts = tsobj / array
```


## Arguments

$$
\begin{array}{ll}
\text { tsobj_1, tsobj_2 } & \text { A pair of financial time series objects. } \\
\text { array } & \text { A scalar value or array with number of rows } \\
& \text { equal to the number of dates in tsobj and } \\
& \text { number of columns equal to the number of data } \\
\text { series in tsobj. }
\end{array}
$$

## Description

The mrdivide method divides element by element the components of one financial time series object by the components of the other. You can also divide the whole object by an array or divide a financial time series object into an array.

If an object is to be divided by another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when an object is divided by another object, follows the order of the first object.
newfts = tsobj_1 / tsobj_2 divides financial time series objects element by element.
newfts = tsobj / array divides a financial time series object element by element by an array.
newfts = array / tsobj divides an array element by element by a financial time series object.

For financial time series objects, the mrdivide operation is identical to the rdivide operation.

## mrdivide

See Also minus | plus | rdivide | times

## Purpose Financial time series matrix multiplication

## Syntax

```
newfts = tsobj_1 * tsobj_2
newfts = tsobj * array
newfts = array * tsobj
```


## Arguments

## Description

The mtimes method multiplies element by element the components of one financial time series object by the components of the other. You can also multiply the entire object by an array.

If an object is to be multiplied by another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when an object is multiplied by another object, follows the order of the first object.
newfts = tsobj_1 * tsobj_2 multiplies financial time series objects element by element.
newfts = tsobj * array multiplies a financial time series object element by element by an array.
newfts = array * tsobj newfts = array / tsobj multiplies an array element by element by a financial time series object.

For financial time series objects, the mtimes operation is identical to the times operation.

[^6]```
Purpose
Fisher information matrix for multivariate normal or least-squares regression
```

```
Syntax
```

Syntax
Fisher = mvnrfish(Data, Design, Covariance, MatrixFormat,
Fisher = mvnrfish(Data, Design, Covariance, MatrixFormat,
CovarFormat)

```
CovarFormat)
```


## Arguments

Data |  | NUMSAMPLES-by-NUMSERIES matrix |
| :--- | :--- |
| with NUMSAMPLES samples of a |  |
| NUMSERIES-dimensional random vector. |  |
| If a data sample has missing values, |  |
| represented as NaNs, the sample is ignored. |  |
| Design | A matrix or a cell array that handles two model |
| structures: |  |
|  | - If NUMSERIES $=1$, Design is a |
|  | NUMSAMPLES-by-NUMPARAMS matrix with |
|  | known values. This structure is the standard |
|  | form for regression on a single series. |
|  | - If NUMSERIES $\geq$ 1, Design is a cell array. |
|  | The cell array contains either one or |
|  | NUMSAMPLES cells. Each cell contains a |
|  | NUMSERIES-by-NUMPARAMS matrix of known |
|  | values. |
|  | If Design has a single cell, it is assumed |
|  | to have the same Design matrix for each |
|  | sample. If Design has more than one cell, |
|  | each cell contains a Design matrix for each |
|  | sample. |

| Covariance | NUMSERIES-by-NUMSERIES matrix of estimates <br> for the covariance of the residuals of the <br> regression. |
| :--- | :--- |
| MatrixFormat | (Optional) String that identifies parameters to <br> be included in the Fisher information matrix: |

- full - Default format. Compute the full Fisher information matrix for both model and covariance parameter estimates.
- paramonly - Compute only components of the Fisher information matrix associated with the model parameter estimates.

CovarFormat
(Optional) String that specifies the format for the covariance matrix. The choices are:

- 'full' - Default method. The covariance matrix is a full matrix.
- 'diagonal' - The covariance matrix is a diagonal matrix.


## Description

Fisher = mvnrfish(Data, Design, Covariance, MatrixFormat, CovarFormat) computes a Fisher information matrix based on current maximum likelihood or least-squares parameter estimates.
Fisher is a TOTALPARAMS-by-TOTALPARAMS Fisher information matrix. The size of TOTALPARAMS depends on MatrixFormat and on current parameter estimates. If MatrixFormat = 'full',

TOTALPARAMS = NUMPARAMS + NUMSERIES * (NUMSERIES + 1)/2

If MatrixFormat = 'paramonly',

```
TOTALPARAMS = NUMPARAMS
```

Note mvnrfish operates slowly if you calculate the full Fisher information matrix.

Examples See "Multivariate Normal Linear Regression" on page 7-3.<br>See Also mvnrstd | mvnrmle

## Purpose

Multivariate normal regression (ignore missing data)
Syntax
[Parameters, Covariance, Resid, Info] = mvnrmle(Data, Design, MaxIterations, TolParam, TolObj, CovarO, CovarFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix <br> with NUMSAMPLES samples of a <br>  <br> NUMSERIES-dimensional random vector. <br> If a data sample has missing values, <br> represented as NaNs, the sample is ignored. <br> (Use ecmmvnrmle to handle missing data.) |
| :--- | :--- |
| DesignMatrix or a cell array that handles two model <br> structures: |  |
|  | - If NUMSERIES = 1, Design is a |
|  | NUMSAMPLES-by-NUMPARAMS matrix with |
|  | known values. This structure is the standard |
|  | form for regression on a single series. |

- If NUMSERIES $\geq 1$, Design is a cell array. The cell array contains either one or NUMSAMPLES cells. Each cell contains a NUMSERIES-by-NUMPARAMS matrix of known values.

If Design has a single cell, it is assumed to have the same Design matrix for each sample. If Design has more than one cell, each cell contains a Design matrix for each sample.

MaxIterations (Optional) Maximum number of iterations for the estimation algorithm. Default value is 100 .
(Optional) Convergence tolerance for estimation algorithm based on changes in model parameter estimates. Default value is sqrt(eps) which is about $1.0 \mathrm{e}-8$ for double precision. The convergence test for changes in model parameters is

$$
\| \text { Param }_{k}-\text { Param }_{k-1} \|<\text { TolParam } \times\left(1+\| \text { Param }_{k} \|\right)
$$

where Param represents the output Parameters, and iteration $k=2,3, \ldots$. Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam $\leq$ 0 and TolObj $\leq 0$, do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests.
(Optional) Convergence tolerance for estimation algorithm based on changes in the objective function. Default value is eps $\wedge 3 / 4$ which is about $1.0 \mathrm{e}-12$ for double precision. The convergence test for changes in the objective function is

$$
\left|\mathrm{Obj}_{k}-\mathrm{Obj}_{k-1}\right|<\operatorname{TolObj} \times\left(1+\left|\mathrm{Obj}_{k}\right|\right)
$$

for iteration $k=2,3, \ldots$. Convergence is assumed when both the TolParam and TolObj conditions are satisfied. If both TolParam $\leq$ 0 and TolObj $\leq 0$, do the maximum number of iterations (MaxIterations), whatever the results of the convergence tests.

| Covaro | (Optional) NUMSERIES-by-NUMSERIES matrix <br> that contains a user-supplied initial or known <br> estimate for the covariance matrix of the <br> regression residuals. |
| :--- | :--- |
| CovarFormat | (Optional) String that specifies the format for <br> the covariance matrix. The choices are: |
| - 'full' - Default method. Compute the full |  |
| covariance matrix. |  |
| - 'diagonal' - Force the covariance matrix to |  |
| be a diagonal matrix. |  |

## Description

[Parameters, Covariance, Resid, Info] = mvnrmle(Data, Design, MaxIterations, TolParam, TolObj, Covaro, CovarFormat) estimates a multivariate normal regression model without missing data. The model has the form

$$
\text { Data }_{k} \square N\left(\text { Design }_{k} \times \text { Parameters, Covariance }\right)
$$

for samples $k=1, \ldots$, NUMSAMPLES.
mvnrmle estimates a NUMPARAMS-by-1 column vector of model parameters called Parameters, and a NUMSERIES-by-NUMSERIES matrix of covariance parameters called Covariance.
mvnrmle(Data, Design) with no output arguments plots the log-likelihood function for each iteration of the algorithm.

To summarize the outputs of mvnrmle:

- Parameters is a NUMPARAMS-by- 1 column vector of estimates for the parameters of the regression model.
- Covariance is a NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the regression model's residuals.
- Resid is a NUMSAMPLES-by-NUMSERIES matrix of residuals from the regression. For any row with missing values in Data, the corresponding row of residuals is represented as all NaN missing values, since this routine ignores rows with NaN values.

Another output, Info, is a structure that contains additional information from the regression. The structure has these fields:

- Info. Obj - A variable-extent column vector, with no more than MaxIterations elements, that contains each value of the objective function at each iteration of the estimation algorithm. The last value in this vector, Obj (end), is the terminal estimate of the objective function. If you do maximum likelihood estimation, the objective function is the log-likelihood function.
- Info.PrevParameters - NUMPARAMS-by-1 column vector of estimates for the model parameters from the iteration just before the terminal iteration.
- Info.PrevCovariance - NUMSERIES-by-NUMSERIES matrix of estimates for the covariance parameters from the iteration just before the terminal iteration.

Notes
mvnrmle does not accept an initial parameter vector, because the parameters are estimated directly from the first iteration onward.

You can configure Design as a matrix if NUMSERIES $=1$ or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES = 1 , each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-by-NUMPARAMS matrix.

These points concern how Design handles missing data:

- Although Design should not have NaN values, ignored samples due to NaN values in Data are also ignored in the corresponding Design array.
- If Design is a 1-by-1 cell array, which has a single Design matrix for each sample, no NaN values are permitted in the array. A model with this structure must have NUMSERIES $\geq$ NUMPARAMS with rank(Design\{1\}) = NUMPARAMS.
- Two functions for handling missing data, ecmmvnrmle and ecmlsrmle, are stricter about the presence of NaN values in Design.

Use the estimates in the optional output structure Info for diagnostic purposes.

## Examples

References
See "Multivariate Normal Regression" on page 7-17, "Least-Squares Regression" on page 7-18, "Covariance-Weighted Least Squares" on page 7-19, "Feasible Generalized Least Squares" on page 7-20, and "Seemingly Unrelated Regression" on page 7-21.

Roderick J. A. Little and Donald B. Rubin, Statistical Analysis with Missing Data, 2nd ed., John Wiley \& Sons, Inc., 2002.

Xiao-Li Meng and Donald B. Rubin, "Maximum Likelihood Estimation via the ECM Algorithm," Biometrika, Vol. 80, No. 2, 1993, pp. 267-278.

See Also

| Purpose | Log-likelihood function for multivariate normal regression without <br> missing data |
| :--- | :--- |
| Syntax | Objective $=$ mvnrobj (Data, Design, Parameters, Covariance, <br> CovarFormat) |

## Arguments

$\left.\begin{array}{ll}\text { Data } & \begin{array}{l}\text { NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES } \\ \text { samples of a NUMSERIES-dimensional random vector. } \\ \text { If a data sample has missing values, represented } \\ \text { as NaNs, the sample is ignored. (Use ecmmvnrmle to } \\ \text { handle missing data.) }\end{array} \\ \text { Design } & \begin{array}{l}\text { A matrix or a cell array that handles two model } \\ \text { structures: }\end{array} \\ & \text { - If NUMSERIES = 1, Design is a } \\ & \text { NUMSAMPLES-by-NUMPARAMS matrix with } \\ & \text { known values. This structure is the standard form } \\ & \text { for regression on a single series. }\end{array}\right\}$

Covariance NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the residuals of the regression.

CovarFormat (Optional) String that specifies the format for the covariance matrix. The choices are:

- 'full' - Default method. The covariance matrix is a full matrix.
- 'diagonal' - The covariance matrix is a diagonal matrix.


## Description

Notes

Examples

See Also

Objective = mvnrobj(Data, Design, Parameters, Covariance, CovarFormat) computes the log-likelihood function based on current maximum likelihood parameter estimates without missing data. Objective is a scalar that contains the log-likelihood function.

You can configure Design as a matrix if NUMSERIES = 1 or as a cell array if NUMSERIES $\geq 1$.

- If Design is a cell array and NUMSERIES $=1$, each cell contains a NUMPARAMS row vector.
- If Design is a cell array and NUMSERIES $>1$, each cell contains a NUMSERIES-by-NUMPARAMS matrix.

Although Design should not have NaN values, ignored samples due to NaN values in Data are also ignored in the corresponding Design array.

See "Multivariate Normal Regression" on page 7-17, "Least-Squares Regression" on page 7-18, "Covariance-Weighted Least Squares" on page 7-19, "Feasible Generalized Least Squares" on page 7-20, and "Seemingly Unrelated Regression" on page 7-21.
ecmmvnrmle | ecmmvnrobj | mvnrmle

Purpose Evaluate standard errors for multivariate normal regression model

Syntax [StdParameters, StdCovariance] = mvnrstd(Data, Design, Covariance, CovarFormat)

## Arguments

| Data | NUMSAMPLES-by-NUMSERIES matrix with NUMSAMPLES <br> samples of a NUMSERIES-dimensional random vector. |
| :--- | :--- |
|  | If a data sample has missing values, represented as <br> NaNs, the sample is ignored. (Use ecmmvnrmle to |
| Design | handle missing data.) |
| A matrix or a cell array that handles two model <br> structures: |  |

- If NUMSERIES = 1, Design is a NUMSAMPLES-by-NUMPARAMS matrix with known values. This structure is the standard form for regression on a single series.
- If NUMSERIES $\geq 1$, Design is a cell array. The cell array contains either one or NUMSAMPLES cells. Each cell contains a NUMSERIES-by-NUMPARAMS matrix of known values.

If Design has a single cell, it is assumed to have the same Design matrix for each sample. If Design has more than one cell, each cell contains a Design matrix for each sample.

Covariance NUMSERIES-by-NUMSERIES matrix of estimates for the covariance of the regression residuals.

CovarFormat (Optional) String that specifies the format for the covariance matrix. The choices are:

- 'full' - Default method. The covariance matrix is a full matrix.
- 'diagonal' - The covariance matrix is a diagonal matrix.


## Description

[StdParameters, StdCovariance] = mvnrstd(Data, Design, Covariance, CovarFormat) evaluates standard errors for a multivariate normal regression model without missing data. The model has the form

Data $_{k} \square N\left(\right.$ Design $_{k} \times$ Parameters, Covariance $)$
for samples $k=1, \ldots$, NUMSAMPLES.
mvnrstd computes two outputs:

- StdParameters is a NUMPARAMS-by-1 column vector of standard errors for each element of Parameters, the vector of estimated model parameters.
- StdCovariance is a NUMSERIES-by-NUMSERIES matrix of standard errors for each element of Covariance, the matrix of estimated covariance parameters.

Note mvnrstd operates slowly when you calculate the standard errors associated with the covariance matrix Covariance.

[^7]
## Purpose Covariance ignoring NaNs

```
Syntax
c = nancov(X)
c = nancov(..., 'pairwise')
```


## Arguments

## Description

$$
\begin{array}{ll}
X & \text { Financial times series object. } \\
Y & \text { Financial times series object. }
\end{array}
$$

nancov for financial times series objects is based on the Statistics Toolbox function nancov. See nancov in the Statistics Toolbox documentation.
$c=\operatorname{nancov}(X)$, if $X$ is a financial time series object with one series and returns the sample variance of the values in X , treating NaNs as missing values. For a financial time series object containing more than one series, where each row is an observation and each series a variable, nancov $(X)$ is the covariance matrix computing using rows of $X$ that do not contain any $\operatorname{NaN}$ values. nancov $(X, Y)$, where $X$ and $Y$ are financial time series objects with the same number of elements, is equivalent to nancov([X(:) Y(:)]).
nancov $(X)$ or nancov $(X, Y)$ normalizes by $(N-1)$ if $N>1$, where $N$ is the number of observations after removing missing values. This makes nancov the best unbiased estimate of the covariance matrix if the observations are from a normal distribution. For $\mathrm{N}=1$, cov normalizes by N.
nancov( $\mathrm{X}, 1$ ) or nancov( $\mathrm{X}, \mathrm{Y}, 1$ ) normalizes by N and produces the second moment matrix of the observations about their mean. nancov $(X, Y, 0)$ is the same as $\operatorname{nancov}(X, Y)$, and $\operatorname{nancov}(X, 0)$ is the same as nancov ( X ).
c = nancov(..., 'pairwise') computes c(i,j) using rows with no NaN values in columns ior $j$. The result may not be a positive definite
matrix. c $=$ nancov(..., 'complete') is the default, and it omits rows with any NaN values, even if they are not in column $i$ or $j$. The mean is removed from each column before calculating the result.

## Examples

To generate random data having nonzero covariance between column 4 and the other columns:

```
x = randn(30, 4); % uncorrelated data
x(:, 4) = sum(x, 2); % introduce correlation
x(2, 3) = NaN; % introduce one missing value
f = fints((today:today+29)', x); % create a fints object using x
c = nancov(f) % compute sample covariance
```

See Also
cov | nanvar | var

## Purpose Maximum ignoring NaNs

```
Syntax m = nanmax (X)
[m,ndx] = nanmax(X)
m = nanmax(X,Y)
[m,ndx] = nanmax(X,[],DIM)
```


## Arguments

## Description

nanmax for financial times series objects is based on the Statistics Toolbox function nanmax. See nanmax in the Statistics Toolbox documentation.
$m=\operatorname{nanmax}(X)$ returns the maximum of a financial time series object $X$ with NaNs treated as missing. $m$ is the largest non-NaN element in X .
[ $m, n d x$ ] $=$ nanmax $(X)$ returns the indices of the maximum values in $X$. If the values along the first nonsingleton dimension contain multiple maximal elements, the index of the first one is returned.
$m=\operatorname{nanmax}(X, Y)$ returns an array the same size as $X$ and $Y$ with the largest elements taken from $X$ or $Y$. Only $Y$ can be a scalar double.
[m,ndx] = nanmax(X, [],DIM) operates along the dimension DIM.

## Examples To compute nanmax for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
f.series1(1) = nan;
f.series2(3) = nan;
```

```
f.series3(2) = nan;
[nmax, maxidx] = nanmax(f)
nmax =
    4 5 6
maxidx =
    3 2 1
```


## See Also max | nanmean | nanmedian | nanmin | nanstd | nanvar

## Purpose Mean ignoring NaNs

Syntax
$\mathrm{m}=$ nanmean $(\mathrm{X})$
m = nanmean(X,DIM)

## Arguments

## Description

X Financial times series object.
DIM Dimension along which the operation is conducted.
nanmean for financial times series objects is based on the Statistics Toolbox function nanmean. See nanmean in the Statistics Toolbox documentation.
$m=$ nanmean $(X)$ returns the sample mean of a financial time series object $X$, treating NaNs as missing values. $m$ is a row vector containing the mean value of the non- NaN elements in each series.
$m=$ nanmean(X,DIM) takes the mean along dimension DIM of $X$.

## Examples

To compute nanmean for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
f.series1(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
nmean = nanmean(f)
nmean =
```

$$
\begin{array}{lll}
3.5000 & 3.0000 & 4.0000
\end{array}
$$

See Also mean | nanmax | nanmin | nanstd | nansum | nanvar

## Purpose Median ignoring NaNs

```
Syntax m = nanmedian(X)
m = nanmedian(X,DIM)
```

Arguments

## Description

## Examples

To compute nanmedian for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007';'04-Jan-2007'};
f = fints(dates, magic(4));
f.series1(1) = nan;
f.series2(2) = nan;
f.series3([1 3]) = nan;
nmedian = nanmedian(f)
nmedian =
    5.0000 7.0000 12.5000 10.0000
```

See Also mean | nanmax | nanmin | nanstd | nansum | nanvar

## Purpose Minimum ignoring NaNs

Syntax

```
m = nanmin(X)
[m,ndx] = nanmin(X)
\(\mathrm{m}=\) nanmin \((\mathrm{X}, \mathrm{Y})\)
[m,ndx] = nanmin(X,[],DIM)
```


## Arguments

| $X$ | Financial times series object. |
| :--- | :--- |
| $Y$ | Financial times series object or scalar. |
| DIM | Dimension along which the operation is conducted. |

## Description

nanmin for financial times series objects is based on the Statistics Toolbox function nanmin. See nanmin in the Statistics Toolbox documentation.
$\mathrm{m}=$ nanmin $(X)$ returns the minimum of a financial time series object $X$ with NaNs treated as missing. $m$ is the smallest non-NaN element in $X$.
[ $m, n d x]=$ nanmin( $X)$ returns the indices of the minimum values in $X$. If the values along the first nonsingleton dimension contain multiple elements, the index of the first one is returned.
$m=n a n m i n(X, Y)$ returns an array the same size as $X$ and $Y$ with the smallest elements taken from $X$ or $Y$. Only $Y$ can be a scalar double.
[m,ndx] = nanmin(X, [], DIM) operates along the dimension DIM.

## Examples

To compute nanmin for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
f.series1(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
```

```
[nmin, minidx] = nanmin(f)
nmin =
    3 1 1 2
minidx =
    2 1 3
```

See Also
mean | nanmax | nanstd | nanvar

## Purpose Standard deviation ignoring NaNs

Syntax

```
y = nanstd(X)
y = nanstd(X,1)
y = nanstd(X,FLAG,DIM)
```


## Arguments

| X | Financial times series object. |
| :--- | :--- |
| FLAG | Normalization flag. |
| DIM | Dimension along which the operation is conducted. |

## Description

nanstd for financial times series objects is based on the Statistics Toolbox function nanstd. See nanstd in the Statistics Toolbox documentation.
$y=$ nanstd $(X)$ returns the sample standard deviation of the values in a financial time series object $X$, treating NaNs as missing values. $y$ is the standard deviation of the non-NaN elements of $X$.
nanstd normalizes $y$ by $(N-1)$, where $N$ is the sample size. This is the square root of an unbiased estimator of the variance of the population from which $X$ is drawn, as long as $X$ consists of independent, identically distributed samples and data are missing at random.
$y=$ nanstd $(X, 1)$ normalizes by $N$ and produces the square root of the second moment of the sample about its mean. nanstd $(X, 0)$ is the same as nanstd ( X ).
$y=n a n s t d(X, f l a g, d i m)$ takes the standard deviation along the dimension dim of $X$. Set the value of flag to 0 to normalize the result by $n-1$; set the value of flag to 1 to normalize the result by $n$.

## Examples

To compute nanstd for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
```

```
f = fints(dates, magic(3));
f.series1(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
nstd = nanstd(f)
```

See Also nanmax | nanmean | nanmedian | nanmin | nanvar | std

## Purpose Sum ignoring NaNs

```
Syntax
y = nansum(X)
y = nansum(X,DIM)
```

Arguments

| X | Financial time series object. |
| :--- | :--- |
| DIM | Dimension along which the operation is conducted. |

## Description

nansum for financial times series objects is based on the Statistics Toolbox function nansum. See nansum in the Statistics Toolbox documentation.
$\mathrm{y}=\operatorname{nansum}(\mathrm{X})$ returns the sum of a financial time series object X , treating NaNs as missing values. y is the sum of the non-NaN elements in $X$.
$\mathrm{y}=$ nansum(X,DIM) takes the sum along dimension DIM of X .

## Examples

To compute nansum for the following dates:

```
dates = {'01-Jan-2007';'02-Jan-2007';'03-Jan-2007'};
f = fints(dates, magic(3));
f.series1(1) = nan;
f.series2(3) = nan;
f.series3(2) = nan;
nsum = nansum(f)
nsum =
    7 6 8
```

See Also nanmax | nanmean | nanmedian | nanmin | nanstd | nanvar

## Purpose Variance ignoring NaNs

## Syntax

```
y = nanvar(X)
y = nanvar(X,1)
y = nanvar(X,W)
y = nanvar(X,W,DIM)
```


## Arguments

| X | Financial times series object. |
| :--- | :--- |
| W | Weight vector. |
| DIM | Dimension along which the operation is conducted. |

## Description

nanvar for financial times series objects is based on the Statistics Toolbox function nanvar. See nanvar in the Statistics Toolbox documentation.
$y=$ nanvar $(X)$ returns the sample variance of the values in a financial time series object X , treating NaNs as missing values. y is the variance of the non-NaN elements of each series in X .
nanvar normalizes y by $N-1$ if $N>1$, where $N$ is the sample size of the non-NaN elements. This is an unbiased estimator of the variance of the population from which X is drawn, as long as X consists of independent, identically distributed samples, and data are missing at random. For $N$ $=1$, y is normalized by N .
$\mathrm{y}=$ nanvar( $\mathrm{X}, 1)$ normalizes by N and produces the second moment of the sample about its mean. nanvar $(X, 0)$ is the same as nanvar $(X)$.
$y=$ nanvar( $X, W$ ) computes the variance using the weight vector $W$. The length of $W$ must equal the length of the dimension over which nanvar operates, and its non-NaN elements must be nonnegative. Elements of X corresponding to NaN elements of Ware ignored.
$\mathrm{y}=$ nanvar(X,W,DIM) takes the variance along dimension DIM of X .

## Examples To compute nanvar:

```
f = fints((today:today+1)', [4 -2 1; 9 5 7])
f.series1(1) = nan;
f.series3(2) = nan;
nvar = nanvar(f)
nvar =
    0 24.5000 0
```

See Also nanmax | nanmean | nanmedian | nanmin | nanstd | var

Purpose Negative volume index

```
Syntax
nvi = negvolidx(closep, tvolume, initnvi)
nvi = negvolidx([closep tvolume], initnvi)
nvits = negvolidx(tsobj)
nvits = negvolidx(tsobj, initnvi, ParameterName, ParameterValue,
...)
```


## Arguments

| closep | Closing price (vector). |
| :--- | :--- |
| tvolume | Volume traded (vector). |
| initnvi | (Optional) Initial value for negative volume <br> index (Default = 100). |
| tsobj | Financial time series object. |
| ParameterName | Valid parameter names are: |
| ParameterValue | - CloseName: closing prices series name <br> - VolumeName: volume traded series name <br> represent values are the strings that |
|  |  |

## Description

nvi = negvolidx(closep, tvolume, initnvi) calculates the negative volume index from a set of stock closing prices (closep) and volume traded (tvolume) data. nvi is a vector representing the negative volume index. If initnvi is specified, negvolidx uses that value instead of the default (100).
nvi $=$ negvolidx([closep tvolume], initnvi) accepts a two-column matrix, the first column representing the closing prices (closep) and the second representing the volume traded (tvolume). If initnvi is specified, negvolidx uses that value instead of the default (100).
nvits $=$ negvolidx(tsobj) calculates the negative volume index from the financial time series object tsobj. The object must contain, at least, the series Close and Volume. The nvits output is a financial time series object with dates similar to tsobj and a data series named NVI. The initial value for the negative volume index is arbitrarily set to 100 .
nvits $=$ negvolidx(tsobj, initnvi, ParameterName, ParameterValue, ...) accepts parameter name/ parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Parameter values are the strings that represent the valid parameter names.

Examples Compute the negative volume index for Disney stock and plot the results:

```
load disney.mat
dis_NegVol = negvolidx(dis)
plot(dis_NegVol)
title('Negative Volume Index for Disney')
```



References

See Also

Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 193-194.
Purpose Nominal rate of return
Syntax

Return = nomrr(Rate, NumPeriods)

## Arguments

Description Return $=$ nomrr(Rate, NumPeriods) calculates the nominal rate of return.
Examples To find the nominal annual rate of return based on an effective annual percentage rate of $9.38 \%$ compounded monthly
Return $=$ nomrr(0.0938, ..... 12)
returns
Return =

$$
0.0900 \text { (9.0\%) }
$$

See Also ..... effrr | irr | mirr | taxedrr | xirr
Purpose Current date and time
Syntax t = now
Description $\mathrm{t}=$ now returns the current date and time as a serial date number. Toreturn the time only, use rem(now,1). To return the date only, usefloor(now).
Examples ..... t1 = now, t2 = rem(now,1)

$$
\mathrm{t} 1=
$$

$$
7.2908 e+05
$$

t2 =

$$
0.4013
$$

See Also date | datenum | today

## Purpose Date of specific occurrence of weekday in month

```
Syntax
Date = nweekdate(n, Weekday, Year, Month, Same)
```


## Arguments

| n | Nth occurrence of the weekday in a month. Enter as integer from 1 through 5. |
| :---: | :---: |
| Weekday | Weekday whose date you seek. Enter as integer from 1 through 7. |
|  | 1 Sunday |
|  | 2 Monday |
|  | 3 Tuesday |
|  | 4 Wednesday |
|  | 5 Thursday |
|  | 6 Friday |
|  | 7 Saturday |
| Year | Year. Enter as a four-digit integer. |
| Month | Month. Enter as an integer from 1 through 12. |
| Same | (Optional) Weekday that must occur in the same week with Weekday. Enter as an integer from 0 through 7 , where $0=$ ignore (default) and 1 through 7 are as for Weekday. |

Description
Date $=$ nweekdate ( n , Weekday, Year, Month, Same) returns the serial date number for the specific occurrence of the weekday in the given year and month, and in a week that also contains the weekday Same.

If n is larger than the last occurrence of Weekday, Date $=0$.

Any input can contain multiple values, but if so, all other inputs must contain the same number of values or a single value that applies to all. For example, if Year is a 1-by-n vector of integers, then Month must be a 1-by-n vector of integers or a single integer. Date is then a 1 -by-n vector of date numbers.
Use the function datestr to convert serial date numbers to formatted date strings.

## Examples

To find the first Thursday in May 2001

```
Date = nweekdate(1, 5, 2001, 5); datestr(Date)
ans =
03-May-2001
```

To find the first Thursday in a week that also contains a Wednesday in May 2001

```
Date = nweekdate(2, 5, 2001, 5, 4); datestr(Date)
ans =
10-May-2001
```

To find the third Monday in February for 2001, 2002, and 2003

```
Year = [2001:2003];
Date = nweekdate(3, 2, Year, 2)
Date =
    730901 731265 731629
datestr(Date)
ans =
```

19-Feb-2001
18-Feb-2002
17-Feb-2003
See Also
fbusdate | lbusdate | lweekdate
Purpose New York Stock Exchange closures from 1885 to 2050

Syntax $\quad$| Closures $=$ nyseclosures(StartDate, EndDate, WorkWeekFormat) |
| :--- |
| SatTransition $=$ nyseclosures(StartDate, EndDate, |
| WorkWeekFormat) |

## Description

## Input <br> Arguments

Closures = nyseclosures(StartDate, EndDate, WorkWeekFormat) returns a vector of serial date numbers corresponding to market closures between StartDate and EndDate, inclusive. If you do not specify StartDate and EndDate, Closures contains all known or anticipated closures from January 1, 1885 to December 31, 2050. By default, WorkWeekFormat argument uses the 'Implicit' value.

SatTransition = nyseclosures(StartDate, EndDate, WorkWeekFormat) returns the date of transition for the New York Stock Exchange from a 6 -day workweek to a 5 -day workweek. The date for this transition is September 29, 1952 and this date returns the serial date number 713226.

Since the New York Stock Exchange was open on Saturdays before September 29, 1952, exact closures from 1885 to 1952 are based on a 6 -day workweek. nyseclosures contains all holiday and special non-trading days for the New York Stock Exchange from 1885 through 2050 based on a six-day work week (always closed on Sundays). Use WorkWeekFormat to modify the list of dates.

## StartDate

First date of a specified date range that is a date string or a serial date number.

Default: January 1, 1885 (start of the default date range)
EndDate
Last date of a specified date range that is a date string or a serial date number. If specified, the EndDate must be a date after the StartDate.

Default: December 31, 2050 (end of the default date range)

## WorkWeekFormat

Specifies method to handle the workweek. The default is 'Implicit'. This function accepts the first letter for each method as input and is not case sensitive. Acceptable values are:

- 'Modern' - 5-day workweek with all Saturday trading days removed
- 'Implicit' - 6-day workweek until 1952 and 5-day week afterward (no need to exclude Saturdays)
- 'Archaic' - 6-day workweek throughout and Saturdays treated as closures after 1952


## Output Arguments

## Definitions

Examples

## Closures

A vector of serial date numbers corresponding to market closures between the dates StartDate and EndDate, inclusive

## SatTransition

The date of transition for the New York Stock Exchange from a 6 -day workweek to a 5 -day workweek.
holidays is based on a modern 5-day workweek and contains all holidays and special nontrading days for the New York Stock Exchange from January 1, 1885 to December 31, 2050. Since the New York Stock Exchange was open on Saturdays before September 29, 1952, exact closures for the period from 1885 to 2050 should include Saturday trading days. To capture these dates, use the function nyseclosures. The results from holidays and nyseclosures are identical if the WorkWeekFormat in nyseclosures is 'modern'.

Find the NYSE closures for 1899:

[^8]This returns:

```
ans =
02-Jan-1899 Mon
11-Feb-1899 Sat
13-Feb-1899 Mon
22-Feb-1899 Wed
31-Mar-1899 Fri
29-May-1899 Mon
30-May-1899 Tue
03-Jul-1899 Mon
04-Jul-1899 Tue
04-Sep-1899 Mon
29-Sep-1899 Fri
30-Sep-1899 Sat
07-Nov-1899 Tue
25-Nov-1899 Sat
30-Nov-1899 Thu
25-Dec-1899 Mon
```

Find the NYSE closure dates using the 'Archaic' value for WorkWeekFormat:

```
datestr(nyseclosures('1-sep-1952','31-oct-1952','a'),1)
```

This returns:

```
ans =
```

01 -Sep-1952
06-Sep-1952
13-Sep-1952
20-Sep-1952
27-Sep-1952
04-Oct-1952

$$
\begin{aligned}
& 11-0 c t-1952 \\
& 13-0 c t-1952 \\
& 18-0 c t-1952 \\
& 25-0 c t-1952
\end{aligned}
$$

The exchange was closed on Saturdays for much of 1952 before the official transition to a 5 -day workweek.

## See Also

busdate | createholidays | fbusdate | isbusday | lbusdate | holidays

Purpose On-Balance Volume (OBV)

```
Syntax obv = onbalvol(closep, tvolume)
obv = onbalvol([closep tvolume])
obvts = onbalvol(tsobj)
obvts = onbalvol(tsobj, ParameterName, ParameterValue, ...)
```


## Arguments

| closep | Closing price (vector) |
| :--- | :--- |
| tvolume | Volume traded |
| tsobj | Financial time series object |

## Description

obv = onbalvol(closep, tvolume) calculates the On-Balance Volume (OBV) from the stock closing price (closep) and volume traded (tvolume) data.
obv = onbalvol([closep tvolume]) accepts a two-column matrix representing the closing price (closep) and volume traded (tvolume), in that order.
obvts = onbalvol(tsobj) calculates the OBV from the stock data in the financial time series object tsobj. The object must minimally contain series names Close and Volume. The obvts output is a financial time series object with the same dates as tsobj and a series named OnBalVol.
obvts = onbalvol(tsobj, ParameterName, ParameterValue, ...) accepts parameter name/ parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- CloseName: closing prices series name
- VolumeName: volume traded series name

Parameter values are the strings that represent the valid parameter names.

## Examples

References

See Also

Compute the OBV for Disney stock and plot the results:

```
load disney.mat
dis_OnBalVol = onbalvol(dis)
plot(dis_OnBalVol)
title('On-Balance Volume for Disney')
```



Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 207-209.

Purpose Option profit
Syntax Profit = opprofit(AssetPrice, Strike, Cost, PosFlag, OptType)

## Arguments

## Description

## Examples

AssetPrice Asset price.
Strike Strike or exercise price.
Cost Cost of the option.

PosFlag Option position. $0=$ long, $1=$ short.
OptType Option type. $0=$ call option, $1=$ put option.

Profit = opprofit(AssetPrice, Strike, Cost, PosFlag, OptType) returns the profit of an option.

Buying (going long on) a call option with a strike price of $\$ 90$ on an underlying asset with a current price of $\$ 100$ for a cost of $\$ 4$

```
Profit = opprofit(100, 90, 4, 0, 0)
```

returns

$$
\text { Profit }=6.00
$$

a profit of $\$ 6$ if the option is exercised under these conditions.
See Also binprice | blsprice

## Purpose Periodic payment given number of advance payments <br> Syntax Payment = payadv(Rate, NumPeriods, PresentValue, FutureValue, Advance)

## Arguments

| Rate | Lending or borrowing rate per period. Enter <br> as a decimal fraction. Must be greater than or <br> equal to 0. |
| :--- | :--- |
| NumPeriods | Number of periods in the life of the instrument. <br> PresentValue <br> FutureValuePresent value of the instrument. |
| Advance | Future value or target value to be attained <br> after NumPeriods periods. |
|  | Number of advance payments. If the payments <br> are made at the beginning of the period, add <br> 1 to Advance. |

## Description

## Examples

Payment = payadv(Rate, NumPeriods, PresentValue, FutureValue, Advance) returns the periodic payment given a number of advance payments.

The present value of a loan is $\$ 1000.00$ and it will be paid in full in 12 months. The annual interest rate is $10 \%$ and three payments are made at closing time. Using this data

```
    Payment = payadv(0.1/12, 12, 1000, 0, 3)
```

returns
Payment =
for the periodic payment.
See Also amortize | payodd | payper
Purpose Payment of loan or annuity with odd first period

```
Syntax
Payment = payodd(Rate, NumPeriods, PresentValue, FutureValue, Days)
```


## Arguments

\(\left.$$
\begin{array}{ll}\text { rate } & \begin{array}{l}\text { Interest rate per period. Enter as a decimal } \\
\text { fraction. }\end{array} \\
\text { NumPeriods } & \begin{array}{l}\text { Number of periods in the life of the instrument. } \\
\text { PresentValue } \\
\text { FutureValue }\end{array} \\
\text { Present value of the instrument. }\end{array}
$$ \begin{array}{l}Future value or target value to be attained <br>

after NumPeriods periods.\end{array}\right]\)| Actual number of days until the first payment |
| :--- |
| is made. |

Description

Payment = payodd(Rate, NumPeriods, PresentValue,
FutureValue, Days) returns the payment for a loan or annuity with
an odd first period.

## Examples <br> A two-year loan for $\$ 4000$ has an annual interest rate of $11 \%$. The first payment will be made in 36 days. To find the monthly payment

$$
\text { Payment }=\text { payodd(0.11/12, 24, 4000, 0, 36) }
$$

returns

> Payment =
186.77

## See Also <br> amortize | payadv | payper

Purpose Periodic payment of loan or annuity
Syntax Payment = payper(Rate, NumPeriods, PresentValue, FutureValue, Due)
Arguments

| Rate | Interest rate per period. Enter as a decimal <br> fraction. |
| :--- | :--- |
| NumPeriods | Number of payment periods in the life of the <br> instrument. |
| PresentValue | Present value of the instrument. <br> (Optional) Future value or target value to be <br> attained after NumPeriods periods. Default = 0. |
| Due | (Optional) When payments are due: $0=$ end of <br> period (default), or $1=$ beginning of period. |

## Description

Payment = payper(Rate, NumPeriods, PresentValue, FutureValue, Due) returns the periodic payment of a loan or annuity.

## Examples

Find the monthly payment for a three-year loan of $\$ 9000$ with an annual interest rate of $11.75 \%$

```
Payment = payper(0.1175/12, 36, 9000, 0, 0)
```

returns

```
Payment =
```

297.86

## See Also

Purpose Uniform payment equal to varying cash flow
Syntax

Series = payuni(CashFlow, Rate)

## Arguments

DescriptionCashFlow A vector of varying cash flows. Include the initialinvestment as the initial cash flow value (a negativenumber).Rate Periodic interest rate. Enter as a decimal fraction.
Examples This cash flow represents the yearly income from an initial investmentof $\$ 10,000$. The annual interest rate is $8 \%$.
Year 1 ..... $\$ 2000$
Year 2 ..... \$1500
Year 3 ..... $\$ 3000$
Year 4 ..... $\$ 3800$
Year 5 ..... $\$ 5000$
To calculate the uniform series value

```
Series = payuni([-10000 2000 1500 3000 3800 5000], 0.08)
```

returns
Series =

## See Also <br> fvfix | fvvar | irr | pvfix | pvvar

## Purpose Linear inequalities for individual asset allocation

Syntax

## Arguments

## Description

## Examples

| AssetMin | Scalar or NASSETS vector of minimum allocations in <br> each asset. NaN indicates no constraint. |
| :--- | :--- |
| AssetMax | Scalar or NASSETS vector of maximum allocations in <br> each asset. NaN indicates no constraint. |
| NumAssets | (Optional) Number of assets. Default = length of |
|  | AssetMin or AssetMax. |

[A,b] = pcalims(AssetMin, AssetMax, NumAssets) specifies the lower and upper bounds of portfolio allocations in each of NumAssets available asset investments.

A is a matrix and $b$ is a vector such that $A * P o r t W t s{ }^{\prime}$ <= $b$, where PortWts is a 1 -by-NASSETS vector of asset allocations.

If pcalims is called with fewer than two output arguments, the function returns A concatenated with b [A, b].

Set the minimum weight in every asset to 0 (no short-selling), and set the maximum weight of IBM stock to 0.5 and CSCO to 0.8 , while letting the maximum weight in INTC float.

| Asset | IBM | INTC | CSCO |
| :--- | :--- | :--- | :--- |
| Min. Wt. | 0 | 0 | 0 |
| Max. Wt. | 0.5 |  | 0.8 |

```
AssetMin = 0
AssetMax = [0.5 NaN 0.8]
[A,b] = pcalims(AssetMin, AssetMax)
A =
\begin{tabular}{rrr}
1 & 0 & 0 \\
0 & 0 & 1 \\
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{tabular}
b =
    0.5000
    0.8000
        0
        0
        0
```

Portfolio weights of $50 \%$ in IBM and $50 \%$ in INTC satisfy the constraints.

Set the minimum weight in every asset to 0 and the maximum weight to 1 .

| Asset | IBM | INTC | CSCO |
| :--- | :--- | :--- | :--- |
| Min. Wt. | 0 | 0 | 0 |
| Max. Wt. | 1 | 1 | 1 |

```
AssetMin = 0
AssetMax = 1
NumAssets = 3
[A,b] = pcalims(AssetMin, AssetMax, NumAssets)
```

$A=$

| 1 | 0 | 0 |
| ---: | ---: | ---: |
| 0 | 1 | 0 |
| 0 | 0 | 1 |
| -1 | 0 | 0 |
| 0 | -1 | 0 |
| 0 | 0 | -1 |

b =
1
1
1
0
0
0

Portfolio weights of $50 \%$ in IBM and $50 \%$ in INTC satisfy the constraints.

## See Also

pcgcomp | pcglims | pcpval | portcons | portopt

Purpose Linear inequalities for asset group comparison constraints
Syntax $\quad[A, b]=\operatorname{pcgcomp}(G r o u p A$, AtoBmin, AtoBmax, GroupB)

## Arguments

| GroupA | Number of groups (NGROUPS) by number of assets <br> (NASSETS) specifications of groups to compare. <br> Each row specifies a group. For a specific group, <br> Group $(i, j)=1$ if the group contains asset $j ;$ <br> otherwise, Group $(i, j)=0$. |
| :--- | :--- |
| AtoBmin | Scalar or NGROUPS-long vectors of minimum <br> and maximum ratios of allocations in GroupA to <br> allocations in GroupB. NaN indicates no constraint <br> between the two groups. Scalar bounds are applied to <br> all group pairs. The total number of assets allocated <br> to GroupA divided by the total number of assets <br> allocated to GroupB is $>=$ AtoBmin and <= AtoBmax. |

## Description

[A,b] = pcgcomp(GroupA, AtoBmin, AtoBmax, GroupB) specifies that the ratio of allocations in one group to allocations in another group is at least AtoBmin to 1 and at most AtoBmax to 1 . Comparisons can be made between an arbitrary number of group pairs NGROUPS comprising subsets of NASSETS available investments.
 is a 1-by-NASSETS vector of asset allocations.

If pcgcomp is called with fewer than two output arguments, the function returns A concatenated with b [A, b].

## Examples

| Asset | INTC | XOM | RD |
| :--- | :--- | :--- | :--- |
| Region | North America | North America | Europe |
| Sector | Technology | Energy |  |
|  |  |  |  |
| Energy |  |  |  |
| Group | Min. Exposure | Max. Exposure |  |
| North America | 0.30 |  | 0.75 |
| Europe | 0.10 |  | 0.55 |
| Technology | 0.20 |  | 0.50 |
| Energy | 0.20 |  | 0.80 |

Make the North American energy sector compose exactly $20 \%$ of the North American investment.


Portfolio weights of $40 \%$ for INTC, $10 \%$ for XOM, and $50 \%$ for RD satisfy the constraints.

## See Also <br> pcalims | pcglims | pcpval | portcons | portopt

## Purpose

Linear inequalities for asset group minimum and maximum allocation
Syntax

## Arguments

## Description

[A,b] = pcglims(Groups, GroupMin, GroupMax) specifies minimum and maximum allocations to groups of assets. An arbitrary number of groups, NGROUPS, comprising subsets of NASSETS investments, is allowed.
$A$ is a matrix and $b$ a vector such that $A * P o r t W t s^{\prime}<=b$, where PortWts is a 1 -by-NASSETS vector of asset allocations.

If pcglims is called with fewer than two output arguments, the function returns A concatenated with b [A, b].

## Examples

| Asset | INTC | XOM | RD |
| :--- | :--- | :--- | :--- |
| Region | North America | North America | Europe |
| Sector | Technology | Energy | Energy |


| Group | Min. Exposure | Max. Exposure |
| :--- | :--- | :--- |
| North America | 0.30 | 0.75 |
| Europe | 0.10 | 0.55 |
| Technology | 0.20 | 0.50 |
| Energy | 0.50 | 0.50 |

Set the minimum and maximum investment in various groups.

```
% INTC XOM RD
Groups = [ 1 1 1 0 ; % North America
            0 0 1 ; % Europe
            1 0 0 ; % Technology
            0 1 1 ]; % Energy
GroupMin = [0.30
        0.10
        0.20
        0.50];
GroupMax = [0.75
        0.55
        0.50
        0.50];
[A,b] = pcglims(Groups, GroupMin, GroupMax)
A =
\begin{tabular}{rrr}
-1 & -1 & 0 \\
0 & 0 & -1 \\
-1 & 0 & 0 \\
0 & -1 & -1 \\
1 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{tabular}
```

```
            0 1 1
b =
    -0.3000
    -0.1000
    -0.2000
    -0.5000
    0.7500
    0.5500
    0.5000
0.5000
```

Portfolio weights of $50 \%$ in INTC, $25 \%$ in XOM, and $25 \%$ in RD satisfy the constraints.

See Also
pcalims | pcgcomp | pcpval | portcons | portopt

# Purpose Linear inequalities for fixing total portfolio value 

```
Syntax \(\quad[A, b]=\) pcpval (PortValue, NumAssets)
```


## Arguments

| PortValue | Scalar total value of asset portfolio (sum of <br> the allocations in all assets). PortValue $=1$ <br> specifies weights as fractions of the portfolio <br> and return and risk numbers as rates instead <br> of value. |
| :--- | :--- |
| NumAssets | Number of available asset investments. |

## Description

[A,b] = pcpval(PortValue, NumAssets) scales the total value of a portfolio of NumAssets assets to PortValue. All portfolio weights, bounds, return, and risk values except ExpReturn and ExpCovariance (see portopt) are in terms of PortValue.
$A$ is a matrix and $b$ a vector such that $A * P o r t W t s ~ ' ~<=~ b, ~ w h e r e ~ P o r t W t s ~$ is a 1-by-NASSETS vector of asset allocations.

If pcpval is called with fewer than two output arguments, the function returns A concatenated with b [A, b].

## Examples Scale the value of a portfolio of three assets $=1$, so all return values are rates and all weight values are in fractions of the portfolio.

```
PortValue = 1;
NumAssets = 3;
[A,b] = pcpval(PortValue, NumAssets)
A =
    1
    1
```

```
    -1 -1 -1
    b =
```

        1
    -1

Portfolio weights of $40 \%, 10 \%$, and $50 \%$ in the three assets satisfy the constraints.

## See Also <br> pcalims | pcgcomp | pcglims | portcons | portopt

Purpose Periodic average of FINTS object

```
Syntax
avgfts = peravg(tsobj)
avgfts = peravg(tsobj, numperiod)
avgfts = peravg(tsobj, daterange)
```


## Arguments

| tsobj | Financial time series object |
| :--- | :--- |
| numperiod | (Optional) Integer specifying the number of data <br> points over which each periodic average should be <br> averaged |
| daterange | (Optional) Time period over which the data is <br> averaged |

## Description

peravg calculates periodic averages of a financial time series object. Periodic averages are calculated from the values per period defined. If the period supplied is a string, it is assumed as a range of date string. If the period is entered as numeric, the number represents the number of data points (financial time series periods) to be included in a period for the calculation. For example, if you enter '01/01/98: :01/01/99' as the period input argument, peravg returns the average of the time series between those dates, inclusive. However, if you enter the number 5 as the period input, peravg returns a series of averages from the time series data taken 5 date points (financial time series periods) at a time.
avgfts = peravg(tsobj, numperiod) returns a structure avgfts that contains the periodic (per numperiod periods) average of the financial time series object. avgfts has field names identical to the data series names of tsobj.
avgfts = peravg(tsobj, daterange) returns a structure avgfts that contains the periodic (as specified by daterange) average of the
financial time series object. avgfts has field names identical to the data series names of tsobj.

Note peravg calculates periodic averages of a FINTS object. Periodic averages are calculated from the values per period defined. If the period supplied is a string, it is assumed as a range of date strings. If the period is entered as numeric, the number represents the number of data points to be included in a period for the calculation.

## Examples

If you enter 01-Jan-2001::03-Jan-2001 as the period input argument, peravg returns the average of the time series between those dates, inclusive. However, if you enter the number 5 as the period input, peravg returns a series of averages from the time series data, taken 5 date points at a time.

```
%% Create the FINTS object %%
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
'02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ', size(dates, 1), 1), times]);
data = [(1:6)', 2*(1:6)'];
myFts = fints(dates_times, data, {'Data1', 'Data2'}, 1, 'My first FINTS')
%% Create the FINTS object %%
[p, pFts] = peravg(myFts, 3)
p =
Data1: [2 5]
Data2: [4 10]
pFts =
    desc: My first FINTS
    freq: Daily (1)
```



## See Also

mean | tsmovavg | mean

| Purpose | Periodic total returns from total return prices |
| :--- | :--- |
| Syntax | TotalReturn $=$ periodicreturns(TotalReturnPrices) <br> TotalReturn $=$ <br> periodicreturns(TotalReturnPrices, Period) |

## Arguments

## Description

TotalReturn = periodicreturns(TotalReturnPrices) calculates the daily total returns from a daily total return price series.

TotalReturn = periodicreturns(TotalReturnPrices, Period) calculates the total returns for a periodicity you specify from a daily total return price series.

TotalReturn is a NUMOBS-by-NASSETS matrix containing month-end dates and return values. Each row represents an observation. Column 1 contains month-end dates in MATLAB serial date number format. The remaining columns contain monthly return values.

> Note Although input returns can have dates in either ascending or descending order, output total returns in TotalReturn have dates in ascending order, with the earliest date in the first row TotalReturn, and the most recent date in the last row of TotalReturn.

See Also totalreturnprice

## Purpose Plot data series

Syntax

```
plot(tsobj)
hp = plot(tsobj)
plot(tsobj, linefmt)
hp = plot(tsobj, linefmt)
plot(..., volumename, bar)
hp = plot(..., volumename, bar)
```


## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| linefmt | (Optional) Line format. <br> volumename <br> (Optional) Specifies which data series is the volume <br> series. volumename must be the exact data series <br> name for the volume column (case sensitive). <br> (Optional) |
| bar | - bar $=0$ (default). Plot volume as a line. |
| - bar $=1$. Plot volume as a bar chart. The width of |  |
| each bar is the same as the default in bar, barh. |  |

## Description

plot(tsobj) plots the data series contained in the object tsobj. Each data series will be a line. plot automatically generates a legend and dates on the $x$-axis. Grid is turned on by default. plot uses the default color order as if plotting a matrix.

The plot command automatically creates subplots when multiple time series are encountered, and they differ greatly on their decimal scales. For example, subplots are generated if one time series data set is in the 10 s and another is in the 10,000 s.
$\mathrm{hp}=$ plot(tsobj) additionally returns the handle(s) to the object(s) inside the plot figure. If there are multiple lines in the plot, hp is a vector of multiple handles.
plot(tsobj, linefmt) plots the data series in tsobj using the line format specified. For a list of possible line formats, see plot in the MATLAB documentation. The plot legend is not generated, but the dates on the $x$-axis and the plot grid are. The specified line format is applied to all data series; that is, all data series will have the same line type.
$\mathrm{hp}=\mathrm{plot}(\mathrm{tsobj}$, linefmt) plots the data series in tsobj using the format specified. The plot legend is not generated, but the dates on the $x$-axis and the plot grid are. The specified line format is applied to all data series, that is, all data series can have the same line type. If there are multiple lines in the plot, hp is a vector of multiple handles.
plot(..., volumename, bar) additionally specifies which data series is the volume. The volume is plotted in a subplot below the other data series. If bar $=1$, the volume is plotted as a bar chart. Otherwise, a line plot is used.
hp $=\operatorname{plot}(. .$. , volumename, bar) returns handles for each line. If bar $=1$, the handle to the patch for the bars is also returned.

Note To turn the legend off, enter legend off at the MATLAB command line. Once you turn it off, the legend is essentially deleted. To turn it back on, recreate it using the legend command as if you are creating it for the first time. To turn the grid off, enter grid off. To turn it back on, enter grid on.

## Portfolio.plotFrontier

Superclasses AbstractPortfolio
Purpose Plot efficient frontier
Syntax

[prsk, pret] = plotFrontier(obj)

[prsk, pret] = plotFrontier(obj, varargin)
Description[prsk, pret] = plotFrontier(obj) to plot the efficient frontier.
[prsk, pret] = plotFrontier(obj, varargin) to plot the efficientfrontier with multiple types of input methods. There are four ways touse plotFrontier:

- Method 1 - Given a portfolio object obj, estimate efficient frontier with default number of 10 portfolios on the frontier.
- Method 2 - Given a portfolio object obj, estimate efficient frontier with specified number of portfolios NumPorts on the frontier.
- Method 3 - Given a portfolio object obj with estimated efficient portfolios in PortWeights, plot the efficient frontier with those portfolios. This method assumes that you provide valid inputs with either efficient portfolios or efficient portfolio risks and returns.
- Method 4 - Given a portfolio object obj with estimated portfolio risks (PortRisk) and returns (PortReturn), plot the efficient frontier. This method assumes that you provide valid inputs with either efficient portfolios or efficient portfolio risks and returns.

[^9]
## Portfolio.plotFrontier

Tips

Input
Arguments

Use dot notation to plot the efficient frontier:

```
[prsk, pret] = obj.plotFrontier;
```

obj
A portfolio object [Portfolio].
varargin
(Optional) varargin can be NumPorts, PortRisk, PortReturn, or PortWeights depending on which of the four input methods you use:

- Method 1 - Given a portfolio object obj, estimate efficient frontier with default number of 10 portfolios on the frontier:

```
[prsk, pret, pwgt] = obj.plotFrontier
```

- Method 2 - Given a portfolio object obj, estimate efficient frontier with specified number of portfolios NumPorts on the frontier:

```
[prsk, pret, pwgt] = obj.plotFrontier(NumPorts)
```

- Method 3 - Given a portfolio object obj with estimated efficient portfolios in PortWeights, plot the efficient frontier with those portfolios:

```
[prsk, pret, pwgt] = obj.plotFrontier(PortWeights)
```

This method assumes that you provide valid inputs with either efficient portfolios or efficient portfolio risks and returns.

- Method 4 - Given a portfolio object obj with estimated portfolio risks (PortRisk) and returns (PortReturn), plot the efficient frontier:

```
[prsk, pret, pwgt] = obj.plotFrontier(PortRisk,PortReturn)
```


## Portfolio.plotFrontier

## Output prsk <br> Arguments

This method assumes that you provide valid inputs with either efficient portfolios or efficient portfolio risks and returns.

Estimated efficient portfolio returns.
pret
Estimated efficient portfolio risks (standard deviation of returns).

Note If the portfolio object has a name in the Name property, the name is displayed as the title of the plot. Otherwise, the plot is just labeled "Efficient Frontier".

If the Portfolio object has an initial portfolio in the InitPort property, the initial portfolio is plotted and labeled.

If portfolio risks and returns are inputs, make sure that risks come first in the calling sequence. In addition, if portfolio risks and returns are not sorted in ascending order, this method performs the sort. On output, the sorted moments are returned.

## Attributes

Access
public
Static
false
Hidden
false
To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

## Examples Given portfolio $p$, plot the efficient frontier:

load CAPMuniverse

## Portfolio.plotFrontier

```
p = Portfolio('AssetList',Assets(1:12));
p = p.estimateAssetMoments(Data(:,1:12),'missingdata',true);
p = p.setDefaultConstraints;
p.plotFrontier;
```



See Also
Portfolio

## Tutorials

- "Plotting the Efficient Frontier" on page 4-90


## Purpose Financial time series addition

```
Syntax
newfts = tsobj_1 + tsobj_2
newfts = tsobj + array
newfts = array + tsobj
```


## Arguments

## Description

plus is an element-by-element addition of the components.
newfts = tsobj_1 + tsobj_2 adds financial time series objects. If an object is to be added to another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when one financial time series object is added to another, follows the order of the first object.
newfts $=$ tsobj + array adds an array element by element to a financial time series object.
newfts = array + tsobj adds a financial time series object element by element to an array.

See Also minus | rdivide | times

## pointfig

Purpose Point and figure chart
Syntax pointfig(Asset)

Description
pointfig(Asset) plots a point and figure chart for a vector of price data Asset. Upward price movements are plotted as X's and downward price movements are plotted as 0's.

See Also
bolling | candle | dateaxis | highlow | movavg

## Purpose

Optimal capital allocation to efficient frontier portfolios

## Syntax

[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction, OverallRisk, OverallReturn] = portalloc(PortRisk, PortReturn, PortWts, RisklessRate, BorrowRate, RiskAversion)

## Arguments

| PortRisk | Standard deviation of each risky asset efficient <br> frontier portfolio. A number of portfolios <br> (NPORTS) by 1 vector. |
| :--- | :--- |
| PortReturn | Expected return of each risky asset efficient <br> frontier portfolio. An NPORTS-by-1 vector. |
| PortWts | Weights allocated to each asset. An NPORTS by <br> number of assets (NASSETS) matrix of weights <br> allocated to each asset. Each row represents an <br> efficient frontier portfolio of risky assets. Total <br> of all weights in a portfolio is 1. |
| RisklessRate | Risk-free lending rate. A decimal number. <br> BorrowRate <br> (Optional) Borrowing rate. A decimal number. |
| RiskAversion | If borrowing is not desired, or not an option, <br> set to NaN (default). <br> (Optional) Coefficient of investor's degree of <br> risk aversion. Higher numbers indicate greater <br> risk aversion. Typical coefficients range <br> between 2.0 and 4.0 (Default = 3). |

## portalloc

> Note Consider that a less risk-averse investor would be expected to accept much greater risk and, consequently, a more risk-averse investor would accept less risk for a given level of return. Therefore, making the RiskAversionargument higher reflects the risk-return tradeoff in the data.

## Description

[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction, OverallRisk, OverallReturn] = portalloc(PortRisk, PortReturn, PortWts, RisklessRate, BorrowRate, RiskAversion) computes the optimal risky portfolio, and the optimal allocation of funds between the risky portfolio and the risk-free asset.

RiskyRisk is the standard deviation of the optimal risky portfolio.
RiskyReturn is the expected return of the optimal risky portfolio.
RiskyWts is a 1-by-NASSETS vector of weights allocated to the optimal risky portfolio. The total of all weights in the portfolio is 1.

RiskyFraction is the fraction of the complete portfolio allocated to the risky portfolio.

OverallRisk is the standard deviation of the optimal overall portfolio.
OverallReturn is the expected rate of return of the optimal overall portfolio.
portalloc generates a plot of the optimal capital allocation if you invoke it without output arguments.

Examples Generate the efficient frontier from the asset data.

```
ExpReturn = [0.1 0.2 0.15];
```

```
ExpCovariance = [0.005 -0.010 0.004
    -0.010 0.040 -0.002
    0.004 -0.002 0.023];
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,...
ExpCovariance);
```

Find the optimal risky portfolio and allocate capital. The risk free investment return is $8 \%$, and the borrowing rate is $12 \%$.

```
RisklessRate = 0.08;
BorrowRate = 0.12;
RiskAversion = 3;
[RiskyRisk, RiskyReturn, RiskyWts, RiskyFraction, ...
OverallRisk, OverallReturn] = portalloc(PortRisk, PortReturn,...
PortWts, RisklessRate, BorrowRate, RiskAversion)
RiskyRisk =
    0.1283
RiskyReturn =
    0.1788
RiskyWts =
    0.0265 0.6023 0.3712
RiskyFraction =
    1.1898
OverallRisk =
    0.1527
OverallReturn =
```


## portalloc

### 0.1899

References Bodie, Kane, and Marcus, Investments, Second Edition, Chapters 6 and 7.

## See Also frontcon | portrand | portstats

## Purpose

Compute risk-adjusted alphas and returns for one or more assets

## Syntax

```
portalpha(Asset, Benchmark)
    portalpha(Asset, Benchmark, Cash)
    portalpha(Asset, Benchmark, Cash, Choice)
    Alpha = portalpha(Asset, Benchmark, Cash, Choice)
    [Alpha, RAReturn] = portalpha(Asset, Benchmark, Cash, Choice)
```


## Arguments

 NUMSAMPLES observations of asset returns for NUMSERIES asset return series.Benchmark NUMSAMPLES vector of returns for a benchmark asset. The periodicity must be the same as the periodicity of Asset. For example, if Asset is monthly data, then Benchmark should be monthly returns.
Cash (Optional) Either a scalar return for a riskless asset or a vector of asset returns to be a proxy for a "riskless" asset. In either case, the periodicity must be the same as the periodicity of Asset. For example, if Asset is monthly data, then Cash must be monthly returns. If no value is supplied, the default value for Cash returns is 0 .

Choice
(Optional) A number, string, or cell array of numbers or strings to indicate one or more measures to be computed from among a number of risk-adjusted alphas and return measures. The number of choices selected in Choice is NUMCHOICES. The current list of choices is given in the following table:

## portalpha

| Code | Description |
| :--- | :--- |
| 'xs' | Excess Return (no risk <br> adjustment) |
| 'sml' | Security Market Line |
| 'capm' | Jensen's Alpha |
| 'mm' | Modigliani \& Modigliani |
| 'gh1' | Graham-Harvey 1 |
| 'gh2' | Graham-Harvey 2 |
| 'all' | Compute all measures |

Choice is specified by using the code from the table (for example, to select the Modigliani \& Modigliani measure, Choice $=$ ' mm '). A single choice is either a string or a scalar cell array with a single code from the table.
Multiple choices can be selected with a cell array of choice codes (for example, to select both Graham-Harvey measures, Choice = \{'gh1', 'gh2' \}). To select all choices, specify Choice = 'all'. If no value is supplied, the default choice is to compute the excess return with Choice $=$ ' xs'. Choice is not case sensitive.

# Description Given NUMSERIES assets with NUMSAMPLES returns in a NUMSAMPLES-by-NUMSERIES matrix Asset, a NUMSAMPLES vector of Benchmark returns, and either a scalar Cash return or a NUMSAMPLES vector of Cash returns, compute risk-adjusted alphas and returns for one or more methods specified by Choice. 

To summarize the outputs of portalpha:

- Alpha is a NUMCHOICES-by-NUMSERIES matrix of risk-adjusted alphas for each series in Asset with each row corresponding to a specified measure in Choice.
- RAReturn is a NUMCHOICES-by-NUMSERIES matrix of risk-adjusted returns for each series in Asset with each row corresponding to a specified measure in Choice.

Note NaN values in the data are ignored and, if NaNs are present, some results could be unpredictable. Although the alphas are comparable across measures, risk-adjusted returns depend on whether the Asset or Benchmark is levered or unlevered to match its risk with the alternative. If Choice = 'all', the order of rows in Alpha and RAReturn follows the order in the table. In addition, Choice = 'all' overrides all other choices.

## Examples

See "Risk-Adjusted Return Example" on page 5-11.

## References

John Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stocks Portfolios and Capital Budgets," Review of Economics and Statistics, Vol. 47, No. 1, February 1965, pp. 13-37.
John R. Graham and Campbell R. Harvey, "Market Timing Ability and Volatility Implied in Investment Newsletters' Asset Allocation Recommendations," Journal of Financial Economics, Vol. 42, 1996, pp. 397-421.

Franco Modigliani and Leah Modigliani, "Risk-Adjusted Performance: How to Measure It and Why," Journal of Portfolio Management, Vol. 23, No. 2, Winter 1997, pp. 45-54.
Jan Mossin, "Equilibrium in a Capital Asset Market," Econometrica, Vol. 34, No. 4, October 1966, pp. 768-783.

William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," Journal of Finance, Vol. 19, No. 3, September 1964, pp. 425-442.

See Also inforatio | sharpe

## Purpose Portfolio constraints

Syntax Conset = portcons(varargin)

Description Using linear inequalities, portcons generates a matrix of constraints for a portfolio of asset investments. The matrix ConSet is defined as ConSet $=\left[\begin{array}{ll}A & b\end{array}\right]$. $A$ is a matrix and $b$ a vector such that $A * P o r t W t s{ }^{\prime}<=$ $b$ sets the value, where PortWts is a 1 -by-number of assets (NASSETS) vector of asset allocations.

ConSet = portcons('ConstType', Data1, ..., DataN) creates a matrix ConSet, based on the constraint type ConstType, and the constraint parameters Data1, ..., DataN.

ConSet $=$ portcons('ConstType1', Data11, ..., Data21, ..., Data2N, ...) creates a matrix ConSet, based on the constraint types ConstTypeN, and the corresponding constraint parameters DataN1, ..., DataNN.

| Constraint Type | Description | Values |
| :--- | :--- | :--- |
| Default | All allocations are <br> $>=0 ;$ no short <br> selling allowed. <br> Combined value <br> of portfolio <br> allocations <br> normalized to 1. | NumAssets (required). Scalar representing <br> number of assets in portfolio. |
| PortValue | Fix total value of <br> portfolio to PVal. | PVal (required). Scalar representing total <br> value of portfolio. <br> NumAssets (required). Scalar representing <br> number of assets in portfolio. See pcpval. |

$\left.\begin{array}{l|l|l}\hline \text { Constraint Type } & \text { Description } & \text { Values } \\ \hline \text { AssetLims } & \begin{array}{l}\text { Minimum and } \\ \text { maximum } \\ \text { allocation per } \\ \text { asset. }\end{array} & \begin{array}{l}\text { AssetMin (required). Scalar or vector of length } \\ \text { NASSETS, specifying minimum allocation per } \\ \text { asset. } \\ \text { AssetMax (required). Scalar or vector of length } \\ \text { NASSETS, specifying maximum allocation per } \\ \text { asset. } \\ \text { NumAssets (optional). See pcalims. }\end{array} \\ \hline \text { GroupLims } & \begin{array}{l}\text { Minimum and } \\ \text { maximum } \\ \text { allocations to asset } \\ \text { group. }\end{array} & \begin{array}{l}\text { Groups (required). NGROUPS-by-NASSETS matrix } \\ \text { specifying which assets belong to each group. } \\ \text { GroupMin (required). Scalar or a vector of } \\ \text { length NGROUPS, specifying minimum combined } \\ \text { allocations in each group. } \\ \text { GroupMax (required). Scalar or a vector of } \\ \text { length NGROUPS, specifying maximum combined } \\ \text { allocations in each group. } \\ \text { See pcglims. }\end{array} \\ \hline \text { GroupComparison } & \begin{array}{l}\text { Group-to-group } \\ \text { comparison } \\ \text { constraints. }\end{array} & \begin{array}{l}\text { GroupA (required). NGROUPS-by-NASSETS matrix } \\ \text { specifying first group in the comparison. } \\ \text { AtoBmin (required). Scalar or vector of }\end{array} \\ \text { length NGROUPS specifying minimum ratios of } \\ \text { allocations in GroupA to allocations in GroupB. }\end{array}\right\}$

| Constraint Type | Description | Values |
| :--- | :--- | :--- |
| Custom | Custom linear <br> inequality <br> constraints <br> A*PortWts ' <= <br> b. | A (required). NCONSTRAINTS-by-NASSETS <br> matrix, specifying weights for each asset in <br> each inequality equation. <br> b (required). Vector of length NCONSTRAINTS <br> specifying the right hand sides of the <br> inequalities. |
| Note For more information using Custom, see <br> "Specifying Additional Constraints" on page <br> $3-17$. |  |  |

## Examples Constrain a portfolio of three assets:

| Asset | IBM | HPQ | XOM |
| :--- | :--- | :--- | :--- |
| Group | A | A | B |
| Min. Wt. | 0 | 0 | 0 |
| Max. Wt. | 0.5 | 0.9 | 0.8 |

```
NumAssets = 3;
PVal = 1; % Scale portfolio value to 1.
AssetMin = 0;
AssetMax = [0.5 0.9 0.8];
GroupA = [llll
GroupB = [lllll}0001]
AtoBmax = 1.5 % Value of assets in Group A at most 1.5 times value
    % in group B.
ConSet = portcons('PortValue', PVal, NumAssets,'AssetLims',...
AssetMin, AssetMax, NumAssets, 'GroupComparison',GroupA, NaN,...
```

AtoBmax, GroupB )
ConSet =

| 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| ---: | ---: | ---: | ---: |
| -1.0000 | -1.0000 | -1.0000 | -1.0000 |
| 1.0000 | 0 | 0 | 0.5000 |
| 0 | 1.0000 | 0 | 0.9000 |
| 0 | 0 | 1.0000 | 0.8000 |
| -1.0000 | 0 | 0 | 0 |
| 0 | -1.0000 | 0 | 0 |
| 0 | 0 | -1.0000 | 0 |
| 1.0000 | 1.0000 | -1.5000 | 0 |

For instance, one possible solution for portfolio weights that satisfy the constraints is $30 \%$ in IBM, $30 \%$ in HPQ, and $40 \%$ in XOM.

## See Also

pcalims | pcgcomp | pcglims | pcpval | portopt

## Superclasses AbstractPortfolio

## Purpose

Description

Portfolio object for mean-variance portfolio optimization and analysis
The portfolio object implements mean-variance portfolio optimization and is derived from the abstract portfolio optimization class AbstractPortfolio. Portfolio objects implement all methods in the AbstractPortfolio class along with methods that are specific to mean-variance portfolio optimization.

The main workflow for portfolio optimization is to create an instance of a portfolio object that completely specifies a portfolio optimization problem and to operate on the portfolio object to obtain and analyze efficient portfolios. A mean-variance optimization problem is completely specified with the following three elements:

- A universe of assets with estimates for the prospective mean and covariance of asset total returns for a period of interest.
- A portfolio set that specifies the set of portfolio choices in terms of a collection of constraints.
- A model for portfolio return and risk, which, for mean-variance optimization, is either the gross or net mean of portfolio returns and the standard deviation of portfolio returns.

After you specify three elements in an unambiguous way, you can solve and analyze portfolio optimization problems. The simplest mean-variance portfolio optimization problem has:

- A mean and covariance of asset total returns
- Nonnegative weights for all portfolios that sum to 1 (the summation constraint is known as a budget constraint)
- Built-in models for portfolio return and risk that use the mean and covariance of asset total returns


## Portfolio

Given mean and covariance of asset returns in the variables AssetMean and AssetCovar, this problem is completely specified by:

```
p = Portfolio('AssetMean', AssetMean, 'AssetCovar', AssetCovar,...
'LowerBound', 0, 'Budget')
```

or equivalently by:

```
p = Portfolio;
p = p.setAssetMoments(AssetMean, AssetCovar);
p = p.setDefaultConstraints;
```

Construction
$p$ = Portfolio constructs an empty portfolio object for mean-variance portfolio optimization and analysis. You can then add elements to the portfolio object using "Add Methods" on page 15-14 and "Set Methods" on page 15-13.
p = Portfolio(Name, Value) constructs a portfolio object for mean-variance portfolio optimization and analysis with additional options specified by one or more Name, Value pair arguments. Name is a property name and Value is its corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, , NameN, ValueN.
p = Portfolio(p, Name, Value) constructs a portfolio object for mean-variance portfolio optimization and analysis using a previously constructed portfolio object $p$ with additional options specified by one or more Name, Value pair arguments.

## Input Arguments

p
(Optional) Previously constructed portfolio object (p).

## Property Name-Value Pair Arguments

Optional comma-separated pairs of Name, Value arguments, where Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, , NameN, ValueN.
AInequalityLinear inequality constraint matrix ([] or [matrix ]).
Default: [ ]
AssetCovarCovariance of asset returns ([] or square matrix.
Default: []
AssetListNames or symbols of assets in universe ([] or [vector cell ofstrings]).
Default: []
AssetMean
Mean of asset returns ([ ] or vector).
Default: []
bInequality
Linear inequality constraint vector ([] or [vector]).
Default: []
BuyCostProportional purchase costs ([] or vector).
Default: []
GroupA

## Portfolio

Group A weights to be bounded by weights in group B ([] or [matrix]).

Default: []

## GroupB

Group B weights ([] or [matrix ]).
Default: []
GroupMatrix
Group membership matrix ([] or [matrix]).
Default: []

## InitPort

Initial portfolio ([] or vector).
Default: [ ]
LowerBudget
Lower-bound budget constraint ([ ] or [scalar]).
Default: []
LowerGroup
Lower-bound group constraint ([] or [vector]).
Default: []

## LowerRatio

Mnimum ratio of allocations between groups A and B ([ ] or [vector]).
Default: [ ]
Name
Name for instance of the portfolio object ([] or [string ]).
Default: []
NumAssets
Number of assets in universe ([ ] or [integer scalar ]).
Default: [ ]
RiskFreeRate
Risk-free rate ([] or scalar).
Default: []
SellCost
Proportional sales costs ([ ] or vector).
Default: []
Turnover
Turnover constraint ([] or [scalar ]).
Default: []
UpperBound
Upper-bound constraint ([] or [vector]).
Default: []
UpperBudget

## Portfolio

Upper-bound budget constraint ([] or [scalar]).
Default: []
UpperGroup
Upper-bound group constraint ([] or [vector]).
Default: []
UpperRatio
Maximum ratio of allocations between groups A and B ([] or [vector]).

Default: []

## Properties

The following properties are from the Portfolio class.
AssetCovar
Covariance of asset returns ([ ] or matrix).
Attributes:
SetAccess public
GetAccess public
AssetMean
Mean of asset returns ([ ] or vector).
Attributes:
SetAccess public
GetAccess public
BuyCost
Proportional purchase costs ([] or vector).
Attributes:
SetAccess public
GetAccess ..... public
RiskFreeRate
Risk-free rate ([] or scalar).
Attributes:
SetAccess ..... public
GetAccess ..... public
SellCost
Proportional sales costs ([ ] or vector).
Attributes:
SetAccess ..... public
GetAccess public
Turnover
Turnover constraint ([] or [scalar]).
Attributes:
SetAccess ..... public
GetAccess public
Inherited PropertiesThe following properties are inherited from the AbstractPortfolioclass.
AEquality

## Portfolio

Linear equality constraint matrix ([] or [matrix]).
Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

AInequality
Linear inequality constraint matrix ([] or [matrix]).
Attributes:
SetAccess public
GetAccess public

## AssetList

Names or symbols of assets in universe ([] or [vector cell of strings]).

Attributes:
SetAccess public
GetAccess public
bEquality
Linear equality constraint vector ([] or [vector]).
Attributes:
SetAccess public
GetAccess public
bInequality
Linear inequality constraint vector ([] or [vector ]).
Attributes:
SetAccess publicGetAccesspublic
GroupAGroup A weights to be bounded by group B ([ ] or [matrix]).
Attributes:
SetAccess ..... public
GetAccess public
GroupB
Group B weights ([] or [matrix]).
Attributes:
SetAccess ..... public
GetAccess public
GroupMatrix
Group membership matrix ([ ] or [matrix]).
Attributes:
SetAccess ..... public
GetAccess public
InitPort
Initial portfolio ([ ] or vector).
Attributes:
SetAccess ..... public
GetAccess public

## Portfolio

## LowerBound

Lower-bound constraint ([] or [vector]).
Attributes:

SetAccess public
GetAccess public

## LowerBudget

Lower-bound budget constraint ([ ] or [scalar]).
Attributes:
SetAccess public
GetAccess public
LowerGroup
Lower-bound group constraint ([] or [vector ]).
Attributes:
SetAccess public
GetAccess public

## LowerRatio

Mnimum ratio of allocations between groups A and B ([] or [vector]).

## Attributes:

SetAccess public
GetAccess public
Name
Name for instance of the portfolio object ([] or [string]).
Attributes:
SetAccess public
GetAccess ..... public
NumAssets
Number of assets in universe ([ ] or [integer scalar ]).
Attributes:
SetAccess ..... public
GetAccess ..... public
UpperBound
Upper-bound constraint ([] or [vector]).
Attributes:
SetAccess ..... public
GetAccess ..... public
UpperBudget
Upper-bound budget constraint ([] or [scalar]).
Attributes:
SetAccess ..... public
GetAccess public
UpperGroup
Upper-bound group constraint ([] or [vector]).
Attributes:

## Portfolio

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

## UpperRatio

Maximum ratio of allocations between groups A and B ([] or [vector]).

## Attributes:

| SetAccess | public |
| :--- | :--- |
| GetAccess | public |

Inherited
Methods

The following methods are inherited from the AbstractPortfolio class.

| addEquality | Add linear equality constraints <br> for portfolio weights to existing <br> constraints |
| :--- | :--- |
| addGroupRatio | Add group ratio constraints for <br> portfolio weights to existing group <br> ratio constraints |
| addGroups | Add group constraints for <br> portfolio weights to existing <br> group constraints |
| addInequality | Add linear inequality constraints <br> for portfolio weights to existing <br> constraints |
| checkFeasibility | Check feasibility of input <br> portfolios against a portfolio <br> object |
| estimateBounds | Estimate global lower and upper <br> bounds for set of portfolios |
|  |  |

estimateFrontier
estimateFrontierByReturn
estimateFrontierByRisk
estimateFrontierLimits
estimatePortReturn
estimatePortRisk
getBounds
getBudget
getEquality
getGroupRatio
getGroups
getInequality
plotFrontier
setAssetList
setBounds
setBudget

Estimate specified number of optimal portfolios over entire efficient frontier

Estimate optimal portfolios with targeted portfolio returns

Estimate optimal portfolios with targeted portfolio risks
Estimate optimal portfolios at endpoints of efficient frontier

Estimate mean of portfolio returns (portfolio return)
Estimate standard deviation of portfolio returns (portfolio risk)

Obtain bounds for portfolio weights from portfolio object
Obtain budget constraint bounds from portfolio object

Obtain equality constraint arrays from portfolio object

Obtain group ratio constraint arrays from portfolio object

Obtain group constraint arrays from portfolio object
Obtain inequality constraint arrays from portfolio object
Plot efficient frontier
Set up list of identifiers for assets
Set up bounds for portfolio weights

Set up budget constraints

## Portfolio

| setDefaultConstraints | Set up portfolio constraints with <br> nonnegative weights that must <br> sum to 1 |
| :--- | :--- |
| setEquality | Set up linear equality constraints <br> for portfolio weights |
| setGroupRatio | Set up group ratio constraints for <br> portfolio weights |
| setGroups | Set up group constraints for <br> portfolio weights |
| setInequality | Set up linear inequality <br> constraints for portfolio weights |
| setInitPort | Set up initial or current portfolio <br> setOptions |
| Set hidden properties in portfolio |  |
| object |  |$\quad$| Choose main solver and specify |
| :--- |
| associated solver options for |
| portfolio optimization |

## Methods

estimateAssetMoments<br>estimatePortMoments<br>getAssetMoments<br>getCosts<br>setAssetMoments

Set up portfolio constraints with nonnegative weights that must sum to 1

Set up linear equality constraints for portfolio weights

Set up group ratio constraints for portfolio weights

Set up group constraints for portfolio weights

Set up linear inequality constraints for portfolio weights Set up initial or current portfolio Set hidden properties in portfolio object

Choose main solver and specify associated solver options for portfolio optimization

Estimate mean and covariance of asset returns from data

Estimate moments of portfolio returns

Obtain mean and covariance of asset returns from portfolio object

Obtain buy and sell transaction costs from portfolio object

Set moments (mean and covariance) of asset returns

## Portfolio

setCosts
setTurnover

Set up proportional transaction costs

Set up maximum portfolio turnover constraint

## Definitions Mean-Variance Portfolio Optimization

For more information on the theory and definition of mean-variance optimization supported by portfolio optimization tools in Financial Toolbox software, see "Portfolio Optimization Theory" on page 4-2.

Copy Value. To learn how value classes affect copy operations, see Copying
Semantics Objects in the MATLAB Programming Fundamentals documentation.

Examples Efficient portfolios can be obtained with:
load CAPMuniverse
p = Portfolio('AssetList', Assets(1:12));
$p$ = p.estimateAssetMoments(Data(:,1:12),'missingdata',true);
p = p.setDefaultConstraints;
p.plotFrontier;
pwgt = p.estimateFrontier(5);
pnames = cell(1,5);
for $\mathrm{i}=1: 5$
pnames\{i\} = sprintf('Port\%d',i);
end

Blotter = dataset([\{pwgt\}, pnames],'obsnames',p.AssetList);
disp(Blotter);

Port1 Port2 Port3 Port4 Port5

## Portfolio

| AAPL | 0.017926 | 0.058247 | 0.097816 | 0.12955 | 0 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| AMZN | 0 | 0 | 0 | 0 | 0 |
| CSCO | 0 | 0 | 0 | 0 | 0 |
| DELL | 0.0041906 | 0 | 0 | 0 | 0 |
| EBAY | 0 | 0 | 0 | 0 | 0 |
| GOOG | 0.16144 | 0.35678 | 0.55228 | 0.75116 | 1 |
| HPQ | 0.052566 | 0.032302 | 0.011186 | 0 | 0 |
| IBM | 0.46422 | 0.36045 | 0.25577 | 0.11928 | 0 |
| INTC | 0 | 0 | 0 | 0 | 0 |
| MSFT | 0.29966 | 0.19222 | 0.082949 | 0 | 0 |
| ORCL | 0 | 0 | 0 | 0 | 0 |
| YHOO | 0 | 0 | 0 | 0 | 0 |


References
Alternatives You can also perform portfolio optimization using a collection of special-purpose functions in Financial Toolbox software. For more information, see "Portfolio Optimization Functions" on page 3-3.
See Also plotFrontier
Tutorials - "Portfolio Optimization Theory" on page 4-2- "Portfolio Object" on page 4-12
How To - "Constructing the Portfolio Object" on page 4-22- Class Attributes- Property Attributes

Purpose
Portfolios on constrained efficient frontier

Syntax<br>[PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ExpCovariance, NumPorts, PortReturn, ConSet, varargin)

## Arguments

| ExpReturn | 1 by number of assets (NASSETS) vector <br> specifying the expected (mean) return of each <br> asset. |
| :--- | :--- |
| ExpCovariance | NASSETS-by-NASSETS matrix specifying the <br> covariance of the asset returns. |
| NumPorts | (Optional) Number of portfolios generated <br> along the efficient frontier. Returns are equally <br> spaced between the maximum possible return <br> and the minimum risk point. If NumPorts is <br> empty (entered as []), computes 10 equally |
| PortReturn | spaced points. <br> (Optional) Expected return of each portfolio. A <br> number of portfolios (NPORTS) by 1 vector. If <br> not entered or empty, NumPorts equally spaced <br> returns between the minimum and maximum <br> possible values are used. |


| Conset | (Optional) Constraint matrix for a portfolio of <br> asset investments, created using portcons. If <br> not specified, a default is created. |
| :---: | :--- |
| varargin | (Optional) varargin supports the following <br> parameter-value pairs: |

- 'algorithm' - Defines which algorithm to use with portopt. Use either a value of 'lcprog' or 'quadprog' to indicate the algorithm to use. The default is 'lcprog'.
- 'maxiter' - Maximum number of iterations before termination of algorithm. The default is 100000 .
- 'tiebreak ' - Method to break ties for pivot selection. This value pair applies only to 'lcprog' algorithm. The default is 'first'. Options are:
- 'first' - Selects pivot with lowest index.
- 'last' - Selects pivot with highest index.
- 'random' - Selects pivot at random.
- 'tolcon' - Tolerance for constraint violations. This value pair applies only to 'lcprog' algorithm. The default is 1.0e-6.
- 'tolpiv' - Pivot value below which a number is considered to be zero. This value pair applies only to 'lcprog' algorithm. The default is $1.0 \mathrm{e}-9$.


## Description [PortRisk, PortReturn, PortWts] = portopt(ExpReturn, ExpCovariance, NumPorts, PortReturn, ConSet, varargin) returns the mean-variance efficient frontier with user-specified

covariance, returns, and asset constraints (ConSet). Given a collection of NASSETS risky assets, computes a portfolio of asset investment weights that minimize the risk for given values of the expected return. The portfolio risk is minimized subject to constraints on the total portfolio value, the individual asset minimum and maximum allocation, the asset group minimum and maximum allocation, or the asset group-to-group comparison.

PortRisk is an NPORTS-by-1 vector of the standard deviation of each portfolio.

PortReturn is an NPORTS-by-1 vector of the expected return of each portfolio.

PortWts is an NPORTS-by-NASSETS matrix of weights allocated to each asset. Each row represents a portfolio. The total of all weights in a portfolio is 1.

If portopt is invoked without output arguments, it returns a plot of the efficient frontier.

## Examples

Plot the risk-return efficient frontier of portfolios allocated among three assets. Connect 20 portfolios along the frontier having evenly spaced returns. By default, choose among portfolios without short-selling and scale the value of the portfolio to 1 .

```
ExpReturn = [0.1 0.2 0.15];
ExpCovariance \(=\begin{array}{lll}0.005 & -0.010 & 0.004\end{array}\)
    \(-0.010 \quad 0.040 \quad-0.002\)
        0.004 -0.002 0.023];
NumPorts = 20;
portopt(ExpReturn, ExpCovariance, NumPorts)
```



Return the two efficient portfolios that have returns of $16 \%$ and $17 \%$. Limit to portfolios that have at least $20 \%$ of the allocation in the first asset, and cap the total value in the first and third assets at $50 \%$ of the portfolio.

```
ExpReturn = [l0.1 0.2 0.15];
ExpCovariance = [l0.005 -0.010 0.004
    -0.010 0.040 -0.002
    0.004 -0.002 0.023];
PortReturn = [0.16
    0.17];
NumAssets = 3;
AssetMin = [0.20 NaN NaN];
Group = [11 0 1];
```

```
GroupMax = 0.50;
ConSet = portcons('Default', NumAssets, 'AssetLims', AssetMin,...
NaN,'GroupLims', Group, NaN, GroupMax);
[PortRisk, PortReturn, PortWts] = portopt(ExpReturn,...
ExpCovariance, [], PortReturn, ConSet)
PortRisk =
    0.0919
    0.1138
    PortReturn =
    0.1600
    0.1700
    PortWts =
\begin{tabular}{lll}
0.3000 & 0.5000 & 0.2000 \\
0.2000 & 0.6000 & 0.2000
\end{tabular}
```

See Also
ewstats | frontcon | frontier | portcons | portstats

## Purpose

Randomized portfolio risks, returns, and weights

```
Syntax
[PortRisk, PortReturn, PortWts] = portrand(Asset, Return,
Points, Method)
portrand(Asset, Return, Points, Method)
```


## Arguments

$\left.\begin{array}{cl}\text { Asset } & \begin{array}{l}\text { Matrix of time series data. Each row is an observation } \\ \text { and each column represents a single security. } \\ \text { Return }\end{array} \\ \text { (Optional) Row vector where each column represents } \\ \text { the rate of return for the corresponding security in } \\ \text { Asset. By default, Return is computed by taking the } \\ \text { average value of each column of Asset. }\end{array}\right\}$

- 'uniform' - Uniformly-distributed portfolio weights (default method). The 'uniform' method generates portfolio weights that are uniformly-distributed on the set of portfolio weights.
- 'geometric' - Concentrated portfolio weights around the geometric center of the set of portfolios. The 'geometric' method generates portfolio weights that are concentrated around the geometric center of the set of portfolio weights.

Note The 'uniform' and 'geometric' methods generate weights that are distributed symmetrically around the geometric center of the set of weights.

## Description

[PortRisk, PortReturn, PortWts] = portrand(Asset, Return, Points, Method) returns the risks, rates of return, and weights of random portfolio configurations.

| PortRisk | Points-by-1 vector of standard deviations. |
| :--- | :--- |
| PortReturn | Points-by-1 vector of expected rates of return. |
| PortWts | Points by number of securities matrix of asset <br> weights. Each row of PortWts is a different portfolio <br> configuration. |

portrand(Asset, Return, Points, Method) plots the points representing each portfolio configuration. It does not return any data to the MATLAB workspace.

Note Portfolios are selected at random from a set of portfolios such that portfolio weights are nonnegative and sum to 1 . The sample mean and covariance of asset returns are used to compute portfolio returns for each random portfolio.

References Bodie, Kane, and Marcus, Investments, Chapter 7.

See Also frontcon | portror | portvar

Purpose Portfolio expected rate of return
Syntax $\quad$ R $=$ portror (Return, Weight)
Arguments

## Description <br> $R=$ portror (Return, Weight) returns a 1-by-M vector for the expected

 rate of return.
## Examples <br> A portfolio is made up of two assets ABC and XYZ having expected rates of return of $10 \%$ and $14 \%$, respectively. If $40 \%$ percent of the portfolio's funds are allocated to asset ABC and the remaining funds are allocated to asset XYZ, the portfolio's expected rate of return is:

```
r = portror([.1 .14],[.4 .6])
r =
0.1240
```


## References <br> Bodie, Kane, and Marcus, Investments, Chapter 7.

See Also frontcon | portrand | portvar

## Purpose Monte Carlo simulation of correlated asset returns

Syntax RetSeries = portsim(ExpReturn, ExpCovariance, NumObs, RetIntervals, NumSim, Method)

## Arguments

| ExpReturn | 1 by number of assets (NASSETS) vector specifying the expected (mean) return of each asset. |
| :---: | :---: |
| ExpCovariance | NASSETS-by-NASSETS matrix of asset return covariances. ExpCovariance must be symmetric and positive semidefinite (no negative eigenvalues). The standard deviations of the returns are ExpSigma = sqrt(diag(ExpCovariance)). |
| NumObs | Positive scalar integer indicating the number of consecutive observations in the return time series. If NumObs is entered as the empty matrix [ ], the length of RetIntervals is used. |
| RetIntervals | (Optional) Positive scalar or number of observations (NUMOBS)-by-1 vector of interval times between observations. If RetIntervals is not specified, all intervals are assumed to have length 1 . |

NumSim (Optional) Positive scalar integer indicating the number of simulated sample paths (realizations) of NUMOBS observations. Default = 1 (single realization of NUMOBS correlated asset returns).

Method (Optional) String indicating the type of Monte Carlo simulation:
'Exact ' (default) generates correlated asset returns in which the sample mean and covariance match the input mean (ExpReturn) and covariance (ExpCovariance) specifications.
'Expected ' generates correlated asset returns in which the sample mean and covariance are statistically equal to the input mean and covariance specifications. (The expected value of the sample mean and covariance are equal to the input mean (ExpReturn) and covariance (ExpCovariance) specifications.)
For either method the sample mean and covariance returned are appropriately scaled by RetIntervals.

## Description

portsim simulates correlated returns of NASSETS assets over NUMOBS consecutive observation intervals. Asset returns are simulated as the proportional increments of constant drift, constant volatility stochastic processes, thereby approximating continuous-time geometric Brownian motion.

RetSeries is a NUMOBS-by-NASSETS-by-NUMSIM three-dimensional array of correlated, normally distributed, proportional asset returns. Asset returns over an interval of length $d t$ are given by

$$
\frac{d S}{S}=\mu d t+\sigma d z=\mu d t+\sigma \varepsilon \sqrt{d t}
$$

where $S$ is the asset price, $\mu$ is the expected rate of return, $\sigma$ is the volatility of the asset price, and $\varepsilon$ represents a random drawing from a standardized normal distribution.

## Notes

- When Method is 'Exact', the sample mean and covariance of all realizations (scaled by RetIntervals) match the input mean and covariance. When the returns are subsequently converted to asset prices, all terminal prices for a given asset are in close agreement. Although all realizations are drawn independently, they produce similar terminal asset prices. Set Method to 'Expected' to avoid this behavior.
- The returns from the portfolios in PortWts are given by PortReturn = PortWts * RetSeries(:,:,1)', where PortWts is a matrix in which each row contains the asset allocations of a portfolio. Each row of PortReturn corresponds to one of the portfolios identified in PortWts, and each column corresponds to one of the observations taken from the first realization (the first plane) in RetSeries. See portopt and portstats for portfolio specification and optimization.


## Examples

## Example 1. Distinction Between Simulation Methods

This example highlights the distinction between the Exact and Expected methods of simulation.

Consider a portfolio of five assets with the following expected returns, standard deviations, and correlation matrix based on daily asset returns (where ExpReturn and Sigmas are divided by 100 to convert percentages to returns):

| ExpReturn | $=[0.0246$ | 0.0189 | 0.0273 | 0.0141 | 0.0311]/100; |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sigmas | $=[0.9509$ | 1.4259 | 1.5227 | 1.1062 | 1.0877]/100; |
| Correlations | $=[1.0000$ | 0.4403 | 0.4735 | 0.4334 | 0.6855 |
|  | 0.4403 | 1.0000 | 0.7597 | 0.7809 | 0.4343 |
|  | 0.4735 | 0.7597 | 1.0000 | 0.6978 | 0.4926 |
|  | 0.4334 | 0.7809 | 0.6978 | 1.0000 | 0.4289 |
|  | 0.6855 | 0.4343 | 0.4926 | 0.4289 | 1.0000]; |

Convert the correlations and standard deviations to a covariance matrix.

```
ExpCovariance = corr2cov(Sigmas, Correlations);
ExpCovariance =
```

1.0e-003 *

| 0.0904 | 0.0597 | 0.0686 | 0.0456 | 0.0709 |
| :--- | :--- | :--- | :--- | :--- |
| 0.0597 | 0.2033 | 0.1649 | 0.1232 | 0.0674 |
| 0.0686 | 0.1649 | 0.2319 | 0.1175 | 0.0816 |
| 0.0456 | 0.1232 | 0.1175 | 0.1224 | 0.0516 |
| 0.0709 | 0.0674 | 0.0816 | 0.0516 | 0.1183 |

Assume that there are 252 trading days in a calendar year, and simulate two sample paths (realizations) of daily returns over a two-year period. Since ExpReturn and ExpCovariance are expressed daily, set RetIntervals = 1 .

```
StartPrice = 100;
NumObs = 504; % two calendar years of daily returns
NumSim = 2;
RetIntervals = 1; % one trading day
NumAssets = 5;
```

To illustrate the distinction between methods, simulate two paths by each method, starting with the same random number state.

```
randn('state',0);
```

```
RetExact = portsim(ExpReturn, ExpCovariance, NumObs, ...
RetIntervals, NumSim, 'Exact');
randn('state',0);
RetExpected = portsim(ExpReturn, ExpCovariance, NumObs, ...
RetIntervals, NumSim, 'Expected');
```

If you compare the mean and covariance of RetExact with the inputs (ExpReturn and ExpCovariance), you will observe that they are almost identical.

At this point, RetExact and RetExpected are both 504-by-5-by-2 arrays. Now assume an equally weighted portfolio formed from the five assets and create arrays of portfolio returns in which each column represents the portfolio return of the corresponding sample path of the simulated returns of the five assets. The portfolio arrays PortRetExact and PortRetExpected are 504-by-2 matrices.

```
Weights = ones(NumAssets, 1)/NumAssets;
PortRetExact = zeros(NumObs, NumSim);
PortRetExpected = zeros(NumObs, NumSim);
for i = 1:NumSim
    PortRetExact(:,i) = RetExact(:,:,i) * Weights;
    PortRetExpected(:,i) = RetExpected(:,:,i) * Weights;
end
```

Finally, convert the simulated portfolio returns to prices and plot the data. In particular, note that since the Exact method matches expected return and covariance, the terminal portfolio prices are virtually identical for each sample path. This is not true for the Expected simulation method.

Although this example examines portfolios, the same methods apply to individual assets as well. Thus, Exact simulation is most appropriate when unique paths are required to reach the same terminal prices.

```
PortExact = ret2tick(PortRetExact, ...
repmat(StartPrice,1,NumSim));
```

```
PortExpected = ret2tick(PortRetExpected, ...
repmat(StartPrice,1,NumSim));
subplot(2,1,1), plot(PortExact, '-r')
ylabel('Portfolio Prices')
title('Exact Method')
subplot(2,1,2), plot(PortExpected, '-b')
ylabel('Portfolio Prices')
title('Expected Method')
```



Example 2. Interaction Between ExpReturn, ExpCovariance and RetIntervals
Recall that portsim simulates correlated asset returns over an interval of length $d t$, given by the equation

$$
\frac{d S}{S}=\mu d t+\sigma d z=\mu d t+\sigma \varepsilon \sqrt{d t}
$$

where $S$ is the asset price, $\mu$ is the expected rate of return, $\sigma$ is the volatility of the asset price, and $\varepsilon$ represents a random drawing from a standardized normal distribution.

The time increment $d t$ is determined by the optional input RetIntervals, either as an explicit input argument or as a unit time increment by default. Regardless, the periodicity of ExpReturn, ExpCovariance and RetIntervals must be consistent. For example, if ExpReturn and ExpCovariance are annualized, then RetIntervals must be in years. This point is often misunderstood.

To illustrate the interplay among ExpReturn, ExpCovariance, and RetIntervals, consider a portfolio of five assets with the following expected returns, standard deviations, and correlation matrix based on daily asset returns.

| ExpReturn | $=[0.0246$ | 0.0189 | 0.0273 | 0.0141 | 0.0311]/100; |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sigmas | $=[0.9509$ | 1.4259 | 1.5227 | 1.1062 | 1.0877]/100; |
| Correlations | $=[1.0000$ | 0.4403 | 0.4735 | 0.4334 | 0.6855 |
|  | 0.4403 | 1.0000 | 0.7597 | 0.7809 | 0.4343 |
|  | 0.4735 | 0.7597 | 1.0000 | 0.6978 | 0.4926 |
|  | 0.4334 | 0.7809 | 0.6978 | 1.0000 | 0.4289 |
|  | 0.6855 | 0.4343 | 0.4926 | 0.4289 | 1.0000]; |

Convert the correlations and standard deviations to a covariance matrix of daily returns.

```
ExpCovariance = corr2cov(Sigmas, Correlations);
```

Assume 252 trading days per calendar year, and simulate a single sample path of daily returns over a four-year period. Since the ExpReturn and ExpCovariance inputs are expressed daily, set RetIntervals = 1.

```
StartPrice = 100;
NumObs = 1008; % four calendar years of daily returns
RetIntervals = 1; % one trading day
```

```
NumAssets = length(ExpReturn);
randn('state',0);
RetSeries1 = portsim(ExpReturn, ExpCovariance, NumObs, ...
RetIntervals, 1, 'Expected');
```

Now annualize the daily data, thereby changing the periodicity of the data, by multiplying ExpReturn and ExpCovariance by 252 and dividing RetIntervals by 252 (RetIntervals = 1/252 of a year).

Resetting the random number generator to its initial state, you can reproduce the results.

```
randn('state',0);
RetSeries2 = portsim(ExpReturn*252, ExpCovariance*252, ...
NumObs, RetIntervals/252, 1, 'Expected');
```

Assume an equally weighted portfolio and compute portfolio returns associated with each simulated return series.

```
Weights = ones(NumAssets, 1)/NumAssets;
PortRet1 = RetSeries2 * Weights;
PortRet2 = RetSeries2 * Weights;
```

Comparison of the data reveals that PortRet1 and PortRet2 are identical.

Example 3. Univariate Geometric Brownian Motion
This example simulates a univariate geometric Brownian motion process. It is based on an example found in Hull, Options, Futures, and Other Derivatives, 5th Edition (see example 12.2 on page 236). In addition to verifying Hull's example, it also graphically illustrates the lognormal property of terminal stock prices by a rather large Monte Carlo simulation.

First, assume you own a stock with an initial price of $\$ 20$, an annualized expected return of $20 \%$ and volatility of $40 \%$. Simulate the daily price
process for this stock over the course of one full calendar year (252 trading days).

```
StartPrice = 20;
ExpReturn = 0.2;
ExpCovariance = 0.4^2;
NumObs = 252;
NumSim = 10000;
RetIntervals = 1/252;
```

Note that RetIntervals is expressed in years, consistent with the fact that ExpReturn and ExpCovariance are annualized. Also, note that ExpCovariance is entered as a variance rather than the more familiar standard deviation (volatility).

Now set the random number generator state, and simulate 10,000 trials (realizations) of stock returns over a full calendar year of 252 trading days.

```
randn('state',10);
RetSeries = squeeze(portsim(ExpReturn, ExpCovariance, NumObs, ...
RetIntervals, NumSim, 'Expected'));
```

The squeeze function reformats the output array of simulated returns from a 252 -by- 1 -by- 10000 array to more convenient 252 -by- 10000 array. (Recall that portsim is fundamentally a multivariate simulation engine).

In accordance with Hull's equations 12.4 and 12.5 on page 236

$$
\begin{aligned}
E\left(S_{T}\right) & =S_{0} e^{\mu T} \\
\operatorname{var}\left(S_{T}\right) & =S_{0}{ }^{2} e^{2 \mu T}\left(e^{\sigma^{2} T}-1\right)
\end{aligned}
$$

convert the simulated return series to a price series and compute the sample mean and the variance of the terminal stock prices.

```
StockPrices = ret2tick(RetSeries, repmat(StartPrice, 1, NumSim));
```

```
SampMean = mean(StockPrices(end,:))
SampMean =
    24.4587
SampVar = var(StockPrices(end,:))
SampVar =
    104.2016
```

Compare these values with the values you obtain by using Hull's equations.

```
ExpValue = StartPrice*exp(ExpReturn)
ExpValue =
    24.4281
ExpVar = ...
StartPrice*StartPrice*exp(2*ExpReturn)*(exp((ExpCovariance)) - 1)
ExpVar =
    103.5391
```

These results are very close to the results shown in Hull's example 12.2.
Next, display the sample density function of the terminal stock price after one calendar year. From the sample density function, the lognormal distribution of terminal stock prices is apparent.

```
[count, BinCenter] = hist(StockPrices(end,:), 30);
figure
bar(BinCenter, count/sum(count), 1, 'r')
xlabel('Terminal Stock Price')
```



## References

See Also

Hull, John, C., Options, Futures, and Other Derivatives, Upper Saddle River, New Jersey: Prentice-Hall. 5th ed., 2003, ISBN 0-13-009056-5.
ewstats | portopt | portstats | randn | ret2tick

Purpose Portfolio expected return and risk

```
Syntax
[PortRisk, PortReturn] = portstats(ExpReturn, ExpCovariance,
PortWts)
```


## Arguments

| ExpReturn | 1-by-number of assets (NASSETS) vector <br> specifying the expected (mean) return of each <br> asset. |
| :--- | :--- |
| ExpCovariance | NASSETS-by-NASSETS matrix specifying the <br> covariance of the asset returns. |
| PortWts | (Optional) Number of portfolios (NPORTS) <br> by NASSETS matrix of weights allocated to <br> each asset. Each row represents a different <br> weighting combination. Default $=1 /$ NASSETS <br> (equally weighted). |

## Description

[PortRisk, PortReturn] = portstats(ExpReturn, ExpCovariance, PortWts) computes the expected rate of return and risk for a portfolio of assets.

PortRisk is an NPORTS-by-1 vector of the standard deviation of each portfolio.

PortReturn is an NPORTS-by-1 vector of the expected return of each portfolio.

## Examples

```
ExpReturn = [0.1 0.2 0.15];
ExpCovariance = [l0.0100 -0.0061 0.0042
    -0.0061 0.0400 -0.0252
    0.0042 -0.0252 0.0225 ];
```

```
PortWts=[0.4 0.2 0.4; 0.2 0.4 0.2];
[PortRisk, PortReturn] = portstats(ExpReturn, ExpCovariance,...
PortWts)
PortRisk =
    0.0560
    0.0550
PortReturn =
    0.1400
    0.1300
```

See Also
frontcon

Purpose Variance for portfolio of assets
Syntax $\quad V=\operatorname{portvar}($ Asset, Weight)
Arguments

| Description | $V=$ portvar (Asset, Weight) returns the portfolio variance as an <br> R-by-1vector (assuming Weight is a matrix of size $R$-by-N) with each row <br> representing a variance calculation for each row of Weight. |
| :--- | :--- |
| $V=$ portvar (Asset) assigns each security an equal weight when <br> calculating the portfolio variance. |  |
| References | Bodie, Kane, and Marcus, Investments, Chapter 7. |
| See Also | frontcon \| portrand | portror |

Purpose Portfolio value at risk (VaR)<br>Syntax ValueAtRisk = portvrisk(PortReturn, PortRisk, RiskThreshold, PortValue)

## Arguments

| PortReturn | Number of portfolios (NPORTS)-by-1 vector or <br> scalar of the expected return of each portfolio <br> over the period. |
| :--- | :--- |
| PortRisk | NPORTS-by-1 vector or scalar of the standard <br> deviation of each portfolio over the period. |
| RiskThreshold | (Optional) NPORTS-by-1 vector or scalar <br> specifying the loss probability. Default = 0.05 <br> (5\%). |
| PortValue | (Optional) NPORTS-by-1 vector or scalar <br> specifying the total value of asset portfolio. <br> Default $=1$. |

## Description

ValueAtRisk = portvrisk(PortReturn, PortRisk, RiskThreshold, PortValue) returns the maximum potential loss in the value of a portfolio over one period of time, given the loss probability level RiskThreshold.

ValueAtRisk is an NPORTS-by-1 vector of the estimated maximum loss in the portfolio, predicted with a confidence probability of 1 RiskThreshold. portvrisk calculates ValueAtRisk using a normal distribution.

If PortValue is not given, ValueAtRisk is presented on a per-unit basis. $A$ value of 0 indicates no losses.

Examples This example computes ValueAtRisk on a per-unit basis.

```
PortReturn = 0.29/100;
PortRisk = 3.08/100;
RiskThreshold = [0.01;0.05;0.10];
PortValue = 1;
ValueAtRisk = portvrisk(PortReturn,PortRisk,...
RiskThreshold,PortValue)
ValueAtRisk =
```

    0.0688
    0.0478
    0.0366
    This example computes ValueAtRisk with actual values.

```
PortReturn = [0.29/100;0.30/100];
PortRisk = [3.08/100;3.15/100];
RiskThreshold = 0.10;
PortValue = [1000000000;500000000];
ValueAtRisk = portvrisk(PortReturn,PortRisk,...
RiskThreshold,PortValue)
ValueAtRisk =
    1.0e+007 *
    3.6572
    1.8684
```


## See Also frontcon | portopt

## Purpose Positive volume index

```
Syntax pvi = posvolidx(closep, tvolume, initpvi)
pvi = posvolidx([closep tvolume], initpvi)
pvits = posvolidx(tsobj)
pvits = posvolidx(tsobj, initpvi, ParameterName, ParameterValue, ...)
```


## Arguments

| closep | Closing price (vector). |
| :--- | :--- |
| tvolume | Volume traded (vector). |
| initpvi | (Optional) Initial value for positive volume index. |
|  | Default $=100$. |
| tsobj | Financial time series object. |

## Description

pvi $=$ posvolidx(closep, tvolume, initpvi) calculates the positive volume index from a set of stock closing prices (closep) and volume traded (tvolume) data. pvi is a vector representing the positive volume index. If initpvi is specified, posvolidx uses that value instead of the default (100).
pvi = posvolidx([closep tvolume], initpvi) accepts a two-column matrix, the first column representing the closing prices (closep) and the second representing the volume traded (tvolume). If initpvi is specified, posvolidx uses that value instead of the default (100).
pvits $=$ posvolidx(tsobj) calculates the positive volume index from the financial time series object tsobj. The object must contain, at least, the series Close and Volume. The pvits output is a financial time series object with dates similar to tsobj and a data series named PVI. The initial value for the positive volume index is arbitrarily set to 100 .
pvits = posvolidx(tsobj, initpvi, ParameterName, ParameterValue,...) accepts parameter name/parameter value pairs
as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- CloseName: closing prices series name
- VolumeName: volume traded series name

Parameter values are the strings that represent the valid parameter names.

## Examples

Compute the positive volume index for Disney stock and plot the results:

```
load disney.mat
dis_PosVol = posvolidx(dis)
plot(dis_PosVol)
title('Positive Volume Index for Disney')
```



## References

Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 236-238.

## See Also

Purpose Financial time series power

```
Syntax newfts = tsobj .^ array
newfts = array .^tsobj
newfts = tsobj_1 .^ tsobj_2
```


## Arguments

tsobj Financial time series object.
array Scalar value or array with the number of rows equal to the number of dates in tsobj and the number of columns equal to the number of data series in tsobj.
tsobj_1, tsobj_2 Pair of financial time series objects.

## Description

## See Also

minus | plus | rdivide | times

## Purpose Price bonds in portfolio by set of zero curves

Syntax BondPrices = prbyzero(Bonds, Settle, ZeroRates, ZeroDates)

## Arguments



Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

EndMonthRule (Optional) End-of-month rule. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
Settle Serial date number of the settlement date.
ZeroRates NUMDATES-by-NUMCURVES matrix of observed zero rates, as decimal fractions. Each column represents a rate curve. Each row represents an observation date.

ZeroDates
NUMDATES-by- 1 column of dates for observed zeros

## Description

BondPrices = prbyzero(Bonds, Settle, ZeroRates, ZeroDates) computes the bond prices in a portfolio using a set of zero curves.

BondPrices is a NUMBONDS-by-NUMCURVES matrix of clean bond prices. Each column is derived from the corresponding zero curve in ZeroRates.

In addition, you can use the Fixed-Income Toolbox ${ }^{\text {TM }}$ method getZeroRates for an IRDataCurve object with a Dates property to create a vector of dates and data acceptable for prbyzero. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object".

## Examples

This example uses zbtprice to compute a zero curve given a portfolio of coupon bonds and their prices. It then reverses the process, using the zero curve as input to prbyzero to compute the prices.

```
Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
    datenum('7/1/2000') 0.06 100 2 0 0;
    datenum('7/1/2000') 0.09375 100 6 1 0;
    datenum('6/30/2001') 0.05125 100 1 3 1;
    datenum('4/15/2002') 0.07125 100 4 1 0;
    datenum('1/15/2000') 0.065 100 2 0 0;
    datenum('9/1/1999') 0.08 100 3 3 0;
    datenum('4/30/2001') 0.05875 100 2 0 0;
    datenum('11/15/1999') 0.07125 100 2 0 0;
    datenum('6/30/2000') 0.07 100 2 3 1;
    datenum('7/1/2001') 0.0525 100 2 3 0;
    datenum('4/30/2002') 0.07 100 2 0 0];
Prices = [ 99.375;
        99.875;
        105.75;
            96.875;
        103.625;
        101.125;
        103.125;
            99.375;
        101.0 ;
        101.25 ;
        96.375;
        102.75 ];
Settle = datenum('12/18/1997');
```

Set semiannual compounding for the zero curve, on an actual/365 basis.
OutputCompounding = 2;
Execute zbtprice
[ZeroRates, ZeroDates] = zbtprice(Bonds, Prices, Settle,... OutputCompounding)
which returns the zero curve at the maturity dates.

$$
\begin{aligned}
& \text { ZeroRates }= \\
& 0.0616 \\
& 0.0609 \\
& 0.0658 \\
& 0.0590 \\
& 0.0648 \\
& 0.0655 \\
& 0.0606 \\
& 0.0601 \\
& 0.0642 \\
& 0.0621 \\
& 0.0627 \\
& \text { ZeroDates }= \\
& \\
& 729907 \\
& 730364 \\
& 730439 \\
& 730500 \\
& 730667 \\
& 730668 \\
& 730971 \\
& 731032 \\
& 731033 \\
& 731321 \\
& 731336
\end{aligned}
$$

Now execute prbyzero

```
BondPrices = prbyzero(Bonds, Settle, ZeroRates, ZeroDates)
```

which returns

```
BondPrices =
```

99.38
98.80
106.83
96.88
103.62
101.13
103.12
99.36
101.00
101.25
96.37
102.74

In this example zbtprice and prbyzero do not exactly reverse each other. Many of the bonds have the end-of-month rule off (EndMonthRule $=0)$. The rule subtly affects the time factor computation. If you set the rule on (EndMonthRule $=1$ ) everywhere in the Bonds matrix, then prbyzero returns the original prices, except when the two incompatible prices fall on the same maturity date.

## See Also

tr2bonds | zbtprice

## Purpose Price rate of change

```
Syntax proc = prcroc(closep, nTimes)
procts = prcroc(tsobj, nTimes)
procts = prcroc(tsobj, nTimes, ParameterName, ParameterValue)
```


## Arguments

| closep | Closing price |
| :--- | :--- |
| nTimes | (Optional) Time difference. Default $=12$. |
| tsobj | Financial time series object |

## Description

proc $=$ prcroc(closep, $n$ Times) calculates the price rate of change proc from the closing price closep. If $n$ Times time is specified, the price rate of change is calculated between the current closing price and the closing price nTimes ago.
procts = prcroc(tsobj, nTimes) calculates the price rate of change procts from the financial time series object tsobj. tsobj must contain a data series named Close. The output procts is a financial time series object with similar dates as tsobj and a data series named PriceROC. If $n$ Times is specified, the price rate of change is calculated between the current closing price and the closing price nTimes ago.
procts $=$ prcroc(tsobj, nTimes, ParameterName, ParameterValue) specifies the name for the required data series when it is different from the default name. The valid parameter name is

- CloseName: closing price series name

The parameter value is a string that represents the valid parameter name.

Note, to compute a quantity over $n$ periods, you must specify $n+1$ for $n T i m e s$. If you specify $n T i m e s=0$, the function returns your original time series.

## Examples

Compute the price rate of change for Disney stock and plot the results:

```
load disney.mat
dis_PriceRoc = prcroc(dis)
plot(dis_PriceRoc)
title('Price Rate of Change for Disney')
```



## References

Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 243-245.

## See Also <br> volroc

## Purpose Price of discounted security

```
Syntax Price = prdisc(Settle, Maturity, Face, Discount, Basis)
```


## Arguments

| Settle | Enter as serial date number or date string. <br> Settle must be earlier than Maturity. |
| :--- | :--- |
| Maturity | Enter as serial date number or date string. |
| Face | Redemption (par, face) value. |
| Discount | Bank discount rate of the security. Enter as decimal | fraction.

Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- 3 = actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- $7=$ actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)


## prdisc

- 13 = BUS/252

For more information, seebasis on page Glossary-1.

```
Description Price = prdisc(Settle, Maturity, Face, Discount, Basis)
returns the price of a security whose yield is quoted as a bank discount
rate (for example, U. S. Treasury bills).
```


## Examples Using this data

```
    Settle = '10/14/2000';
    Maturity = '03/17/2001';
    Face = 100;
    Discount = 0.087;
    Basis = 2;
    Price = prdisc(Settle, Maturity, Face, Discount, Basis)
returns
    Price =
    96.2783
```

References Mayle, Standard Securities Calculation Methods, Volumes I-II, 3rd edition. Formula 2.
See Also acrudisc | bndprice | discrate | prmat | ylddisc

## Purpose Price and volume chart <br> Syntax priceandvol(X) <br> Arguments

Description

## Examples

M-by- 6 matrix where the columns are date, open, high, low, close, and volume.
priceandvol(X) plots the asset data displaying the open, high, low, and closing prices on one axis and the volume on a second axis.

If asset $X$ is an $M$-by- 6 matrix for date, open, high, low, close, and volume:

| $X=[\ldots$ |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 733299.00 | 41.93 | 42.15 | 41.83 | 41.99 | $15045445.00 ; \ldots$ |
| 733300.00 | 42.09 | 42.24 | 41.76 | 42.14 | $15346658.00 ; \ldots$ |
| 733303.00 | 42.00 | 42.20 | 41.78 | 41.93 | $9034397.00 ; \ldots$ |
| 733304.00 | 41.82 | 42.16 | 41.70 | 41.98 | $14486275.00 ; \ldots$ |
| 733305.00 | 41.94 | 42.19 | 41.70 | 41.75 | $16389872.00 ; \ldots$ |
| 733306.00 | 42.00 | 42.57 | 41.50 | 41.61 | $20475208.00 ; \ldots$ |
| 733307.00 | 41.93 | 42.35 | 41.74 | 42.29 | $14833200.00 ; \ldots$ |
| 733310.00 | 42.01 | 42.70 | 42.01 | 42.19 | $18945176.00 ; \ldots$ |
| 733311.00 | 42.18 | 42.72 | 41.73 | 41.82 | $25188101.00 ; \ldots$ |
| 733312.00 | 42.57 | 42.57 | 41.33 | 41.93 | $22689878.00 ; \ldots$ |
| 733313.00 | 41.86 | 42.35 | 41.71 | 41.81 | $21084723.00 ; \ldots$ |
| 733314.00 | 41.70 | 41.90 | 41.04 | 41.37 | $27963619.00 ; \ldots$ |
| 733317.00 | 40.98 | 41.49 | 40.82 | 41.17 | $20385033.00 ; \ldots$ |
| 733318.00 | 41.50 | 42.15 | 41.21 | 42.02 | $27783775.00]$ |

then the price volume chart is
priceandvol(X)
which plots the asset data displaying the open, high, low, and closing prices on one axis and the volume on a second axis.


See Also
bolling | candle | highlow | kagi | linebreak | movavg | pointfig | renko | volarea

## Purpose

Price with interest at maturity

Syntax<br>[Price, AccruInterest] = prmat(Settle, Maturity, Issue, Face, CouponRate, Yield, Basis)

## Arguments

Settle Enter as serial date number or date string. Settle must be earlier than Maturity.
Maturity Enter as serial date number or date string.
Issue Enter as serial date number or date string.
Face Redemption (par, face) value.
CouponRate Enter as decimal fraction.
Yield Annual yield. Enter as decimal fraction.
Basis
(Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- $2=$ actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

## Description

[Price, AccruInterest] = prmat(Settle, Maturity, Issue, Face, CouponRate, Yield, Basis) returns the price and accrued interest of a security that pays interest at maturity. This function also applies to zero-coupon bonds or pure discount securities by setting CouponRate $=0$.

Examples Using this data

```
Settle = '02/07/2002';
Maturity = '04/13/2002';
Issue = '10/11/2001';
Face = 100;
CouponRate = 0.0608;
Yield = 0.0608;
Basis = 1;
[Price, AccruInterest] = prmat(Settle, Maturity, Issue, Face,...
CouponRate, Yield, Basis)
returns
Price =
    99.9784
AccruInterest =
1.9591
```

References Mayle, Standard Securities Calculation Methods, Volumes I-II, 3rd edition. Formula 4.
See Also acrubond | acrudisc | bndprice | prdisc | yldmat

Purpose Price of Treasury bill
Syntax Price = prtbill(Settle, Maturity, Face, Discount)

Arguments
Description

Price = prtbill(Settle, Maturity, Face, Discount) returns the price for a Treasury bill.
Examples The settlement date of a Treasury bill is February 10, 2002, the maturity date is August 6, 2002, the discount rate is $3.77 \%$, and the par value is $\$ 1000$. Using this data

```
Price = prtbill('2/10/2002', '8/6/2002', 1000, 0.0377)
```

returns
Price =
981.4642
References Bodie, Kane, and Marcus, Investments, pages 41-43.
See Also
beytbill | yldtbill
Purpose Present value with fixed periodic payments
Syntax PresentVal = pvfix(Rate, NumPeriods, Payment, ExtraPayment, Due)
Arguments

| rate | Periodic interest rate, as a decimal fraction. |
| :--- | :--- |
| NumPeriods | Number of periods. <br> Payment |
| ExtraPayment | Periodic payment. <br> (Optional) Payment received other than <br> Payment in the last period. Default = 0. |
| Due | (Optional) When payments are due or made: 0 <br> = end of period (default), or $1=$ beginning of <br> period. |

Description PresentVal = pvfix(Rate, NumPeriods, Payment, ExtraPayment, Due) returns the present value of a series of equal payments.
Examples $\$ 200$ is paid monthly into a savings account earning $6 \%$. The payments are made at the end of the month for five years. To find the present value of these payments
PresentVal = pvfix(0.06/12, 5*12, 200, 0, 0)
returns
PresentVal =
10345.11
See Also fvfix | fvvar | payper | pvvar

## Purpose Price and Volume Trend (PVT)

```
Syntax pvt = pvtrend(closep, tvolume)
pvt = pvtrend([closep tvolume])
pvtts = pvtrend(tsobj)
pvtts = pvtrend(tsobj, ParameterName, ParameterValue, ...)
```


## Arguments

| closep | Closing price. |
| :--- | :--- |
| tvolume | Volume traded. |
| tsobj | Financial time series object. |
| ParameterName | Valid parameter names are: |
| ParameterValue | - CloseName: closing prices series name |
|  | - VolumeName: volume traded series name <br> Parameter values are the strings that <br> represent the valid parameter names. |

## Description

pvt = pvtrend(closep, tvolume) calculates the Price and Volume Trend (PVT) from the stock closing price (closep) data and the volume traded (tvolume) data.
pvt = pvtrend([closep tvolume]) accepts a two-column matrix in which the first column contains the closing prices (closep) and the second contains the volume traded (tvolume).
pvtts = pvtrend(tsobj) calculates the PVT from the stock data contained in the financial time series object tsobj. The object tsobj must contain the closing price series Close and the volume traded series Volume. The output pvtts is a financial time series object with dates similar to tsobj and a data series named PVT.
pvtts = pvtrend(tsobj, ParameterName, ParameterValue, ...) accepts parameter name/ parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Parameter values are the strings that represent the valid parameter names.

## Examples

References

Compute the PVT for Disney stock and plot the results:

```
load disney.mat
dis_PVTrend = pvtrend(dis)
plot(dis_PVTrend)
title('Price and Volume Trend for Disney')
```



Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 239-240.

Purpose Present value of varying cash flow

Syntax PresentVal = pvvar(CashFlow, Rate, IrrCFDates)

## Arguments

| CashFlow | A vector of varying cash flows. Include the initial <br> investment as the initial cash flow value (a negative <br> number). If CashFlow is a matrix, each column is <br> treated as a separate cash-flow stream. |
| :--- | :--- |
| Rate | Periodic interest rate. Enter as a decimal fraction. If <br> CashFlow is a matrix, a scalar Rate is allowed when <br> the same rate applies to all cash-flow streams in <br> CashFlow. When multiple cash-flow streams require <br> different discount rates, Rate must be a vector whose <br> length equals the number of columns in CashFlow. |
|  | (Optional) A vector of serial date numbers or date <br> strings on which the cash flows occur. Specify |
| IrrCFDates when there are irregular (nonperiodic) |  |
| cash flows. The default assumes that CashFlow |  |
| contains regular (periodic) cash flows. If CashFlow is |  |
| a matrix, and all cash-flow streams share the same |  |
| dates, Ir rcFDates can be a vector whose length |  |
| matches the number of rows in CashFlow. When |  |
| different cash-flow streams have different payment |  |
| dates, specify IrrCFDates as a matrix the same size |  |
| as CashFlow. |  |

PresentVal $=$ pvvar (CashFlow, Rate, IrrCFDates) returns the net present value of a varying cash flow. Present value is calculated at the time the first cash flow occurs.

Examples
This cash flow represents the yearly income from an initial investment of $\$ 10,000$. The annual interest rate is $8 \%$.

| Year 1 | $\$ 2000$ |
| :--- | :--- |
| Year 2 | $\$ 1500$ |
| Year 3 | $\$ 3000$ |
| Year 4 | $\$ 3800$ |
| Year 5 | $\$ 5000$ |

To calculate the net present value of this regular cash flow
PresentVal $=$ pvvar $([-10000 \quad 2000 \quad 150030003800$ 5000], 0.08)
returns

```
PresentVal =
```

1715.39

An investment of $\$ 10,000$ returns this irregular cash flow. The original investment and its date are included. The periodic interest rate is $9 \%$.

## Cash Flow

| $(\$ 10000)$ | January 12, 1987 |
| :--- | :--- |
| $\$ 2500$ | February 14, 1988 |
| $\$ 2000$ | March 3, 1988 |
| $\$ 3000$ | June 14, 1988 |
| $\$ 4000$ | December 1, 1988 |

To calculate the net present value of this irregular cash flow

```
CashFlow = [-10000, 2500, 2000, 3000, 4000];
```

```
    IrrCFDates = ['01/12/1987'
        '02/14/1988'
        '03/03/1988'
        '06/14/1988'
        '12/01/1988'];
    PresentVal = pvvar(CashFlow, 0.09, IrrCFDates)
returns
PresentVal =
```

142.16

The net present value of the same investment under different discount rates of $7 \%, 9 \%$, and $11 \%$ is obtained in a single call:

```
PresentVal = pvvar(repmat(CashFlow,1,3), [.07 .09 .11], IrrCFDates)
```

```
pv =
    419.0136 142.1648-122.1275
```

See Also fvfix | fvvar | irr | payuni | pvfix

Purpose<br>Zero curve given par yield curve<br>Syntax<br>[ZeroRates, CurveDates] = pyld2zero(ParRates, CurveDates, Settle, Compounding, Basis, InputCompounding)

## Arguments

| ParRates | Column vector of annualized implied par <br> yield rates, as decimal fractions. (Par yields <br> = coupon rates.) In aggregate, the yield rates <br> in ParRates constitute an implied par yield <br> curve for the investment horizon represented <br> by CurveDates. |
| :--- | :--- |
| CurveDates | Column vector of maturity dates (as serial date <br> numbers) that correspond to the par rates. |
| Settle | Serial date number that is the common <br> settlement date for the par rates. <br> (Optional) Scalar value representing the |
| Compounding | periodicity in which the output zero rates are <br> compounded when annualized. Allowed values <br> are: <br> 1 |
| 2 Annual compounding |  |
| 3 | Semiannual compounding (default) |

Basis (Optional) Day-count basis used to annualize the zero rates.

- 0 = actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
InputCompounding (Optional) Scalar value representing the periodicity in which the input par rates were compounded when annualized. The default is the value for Compounding.

Description [ZeroRates, CurveDates] = pyld2zero(ParRates, CurveDates, Settle, Compounding, Basis, InputCompounding) returns a zero curve given a par yield curve and its maturity dates.

Note pyld2zero uses zbtprice for computations.

| ZeroRates | Column vector of decimal fractions. In aggregate, the <br> rates in ZeroRates constitute a zero curve for the <br> investment horizon represented by CurveDates. |
| :---: | :--- |
| CurveDates | Column vector of maturity dates (as serial date <br> numbers) corresponding to the zero rates. This vector <br> is the same as the input vector CurveDates. |

## Examples Given

- A par yield curve over a set of maturity dates
- A settlement date
- Annual compounding for the input par rates and monthly compounding for the output zero curve
compute a zero yield curve.

```
ParRates = [0.0479
            0.0522
            0.0540
            0.0540
            0.0536
            0.0532
            0.0532
            0.0539
            0.0558
            0.0543];
                                    CurveDates = [datenum('06-Nov-2000')
```

```
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
InputCompounding = 1;
Compounding = 12;
[ZeroRates, CurveDates] = pyld2zero(ParRates, CurveDates,...
Settle, Compounding, [], InputCompounding)
ZeroRates =
    0.0484
    0.0529
    0.0549
    0.0550
    0.0547
    0.0544
    0.0545
    0.0551
    0.0572
    0.0557
CurveDates =
    730796
    7 3 0 8 3 1
    7 3 0 8 6 6
    730887
    7 3 0 9 1 4
```

730943
730971
731027
731098
731167

For readability, ParRates and ZeroRates are shown only to the basis point. However, MATLAB computes them at full precision. If you enter ParRates as shown, ZeroRates may differ due to rounding.

## See Also <br> zero2pyld

How To . "Term Structure of Interest Rates" on page 2-36

## rdivide

Purpose Financial time series division

```
Synfax newfts = tsobj_1 ./ tsobj_2
newfts = tsobj ./ array
newfts = array ./ tsobj
```


## Arguments

$$
\begin{array}{ll}
\text { tsobj_1, tsobj_2 } & \begin{array}{l}
\text { Pair of financial time series objects. } \\
\text { array }
\end{array} \\
& \begin{array}{l}
\text { Scalar value or array with the number of rows } \\
\text { equal to the number of dates in tsobj and the } \\
\text { number of columns equal to the number of data } \\
\text { series in tsobj. }
\end{array}
\end{array}
$$

## Description

The rdivide method divides, element by element, the components of one financial time series object by the components of the other. You can also divide the whole object by an array or divide a financial time series object into an array.

If an object is to be divided by another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when an object is divided by another object, follows the order of the first object.
newfts = tsobj_1 ./ tsobj_2 divides financial time series objects element by element.
newfts = tsobj ./ array divides a financial time series object element by element by an array.
newfts = array ./ tsobj divides an array element by element by a financial time series object.

For financial time series objects, the rdivide operation is identical to the mrdivide operation.

## rdivide

See Also minus | mrdivide | plus | times
Purpose Renko chart

```
Syntax renko(X)
renko(X, threshold)
```

Arguments

## Description

renko( X ) plots asset price with respect to dates.
renko(X, threshold) plots the asset data, X , adding a new box only when the price has changed but at least the value specified by threshold.

Examples If asset $X$ is an $M$-by-2 matrix of date numbers and asset price:

| $X=[\ldots$ |  |
| :--- | :--- |
|  |  |
| 733299.00 | $41.99 ; \ldots$ |
| 733300.00 | $42.14 ; \ldots$ |
| 733303.00 | $41.93 ; \ldots$ |
| 733304.00 | $41.98 ; \ldots$ |
| 733305.00 | $41.75 ; \ldots$ |
| 733306.00 | $41.61 ; \ldots$ |
| 733307.00 | $42.29 ; \ldots$ |
| 733310.00 | $42.19 ; \ldots$ |
| 733311.00 | $41.82 ; \ldots$ |
| 733312.00 | $41.93 ; \ldots$ |
| 733313.00 | $41.81 ; \ldots$ |
| 733314.00 | $41.37 ; \ldots$ |
| 733317.00 | $41.17 ; \ldots$ |

then the Renko chart is
renko (X)
which plots the asset prices with respect to dates.

bolling | candle | highlow | kagi | linebreak | movavg | pointfig priceandvol | volarea

Purpose Downsample data
Syntax newfts = resamplets(oldfts, samplestep)

Description newfts = resamplets(oldfts, samplestep) downsamples the data contained in the financial time series object oldfts every samplestep periods. For example, to have the new financial time series object contain every other data element from oldfts, set samplestep to 2.
newfts is a financial time series object containing the same data series (names) as the input oldfts.

See Also<br>filter

| Purpose | Convert return series to price series |
| :--- | :--- |
| Syntax | [TickSeries, TickTimes] = ret2tick(RetSeries, StartPrice, |
|  | RetIntervals, StartTime, Method) |

## Arguments

| RetSeries | Number of observations (NUMOBS) by number of assets (NASSETS) time series array of asset returns associated with the prices in TickSeries. The $i$ th return is quoted for the period TickTimes(i) to TickTimes(i+1) and is not normalized by the time increment between successive price observations. |
| :---: | :---: |
| StartPrice | (Optional) 1-by-NASSETS vector of initial asset prices or a single scalar initial price applied to all assets. Prices start at 1 if StartPrice is not specified. |
| RetIntervals | (Optional) Scalar or NUMOBS-by-1 vector of interval times between observations. If this argument is not specified, all intervals are assumed to have length 1. |
| StartTime | (Optional) Starting time for first observation, applied to the price series of all assets. The default is zero. |
| Method | (Optional) Character string indicating the method to convert asset returns to prices. Must be 'Simple' (default) or 'Continuous'. If Method is 'Simple', ret2tick uses simple periodic returns. If Method is 'Continuous ', the function uses continuously compounded returns. Case is ignored for Method. |

## Description

## Examples

[TickSeries, TickTimes] = ret2tick(RetSeries, StartPrice, RetIntervals, StartTime, Method) generates price values from the starting prices of NASSETS investments and NUMOBS incremental return observations.

TickSeries is a NUMOBS+1-by-NASSETS times series array of equity prices. The first row contains the oldest observations and the last row the most recent. Observations across a given row occur at the same time for all columns. Each column is a price series of an individual asset. If Method is unspecified or 'Simple', the prices are

```
TickSeries(i+1) = TickSeries(i)*[1 + RetSeries(i)]
```

If Method is 'Continuous ', the prices are

```
TickSeries(i+1) = TickSeries(i)*exp[RetSeries(i)]
```

TickTimes is a NUMOBS+1 column vector of monotonically increasing observation times associated with the prices in TickSeries. The initial time is zero unless specified in StartTime, and sequential observation times occur at unit increments unless specified in RetIntervals.

Compute the price increase of two stocks over a year's time based on three incremental return observations.

```
RetSeries = [0.10 0.12
    0.05 0.04
    -0.05 0.05];
RetIntervals = [182
            91
            92];
StartTime = datenum('18-Dec-2000');
[TickSeries,TickTimes] = ret2tick(RetSeries,[],RetIntervals,...
StartTime)
```

TickSeries $=$

| 1.0000 | 1.0000 |
| :--- | :--- |
| 1.1000 | 1.1200 |
| 1.1550 | 1.1648 |
| 1.0973 | 1.2230 |

TickTimes =

730838
731020
731111
731203
datestr(TickTimes)
ans =

18-Dec-2000
18-Jun-2001
17-Sep-2001
18-Dec-2001
See Also portsim \| tick2ret

## Purpose Convert return series to price series for time series object

```
Syntax priceFts = ret2tick(returnFts)
priceFts = ret2tick(returnFts, 'PARAM1', VALUE1,
'PARAM2', VALUE2', ...)
```


## Arguments

returnFts Financial time series object of returns.
'PARAM1 ' (Optional) StartPrice is a Numeric value and is a scalar or 1-by-N vector of initial prices for each asset. If StartPrice is unspecified or empty, the initial price of all assets is 1.
'PARAM2 ' (Optional) StartTime is Date value for a scalar date number or a single date string specifying the starting time for the first observation. This date is applied to the price series of all assets.

Note The first period price value of the resulting price series will not be reported if StartTime is not specified. The resulting price series will be scaled based on the StartPrice, even if StartTime is not supplied.
'PARAM3 ${ }^{\text {' (Optional) Method is a character string }}$ indicating the method to convert asset returns to prices. The value must be defined as 'Simple' (default) or 'Continuous'. If Method is 'Simple', ret2tick uses simple periodic returns. If Method is 'Continuous', the function uses continuously compounded returns. Case is ignored for Method.

## Description

priceFts = ret2tick(returnFts, 'PARAM1', VALUE1, 'PARAM2', VALUE2 ' , ...) generates a financial time series object of prices.

If Method is unspecified or 'Simple', the prices are

```
PriceSeries(i+1) = PriceSeries(i)*[1 + ReturnSeries(i)]
```

If Method is 'Continuous', the prices are

```
PriceSeries(i+1) = PriceSeries(i)*exp[ReturnSeries(i)]
```

Examples Compute the price series from the following return series:

```
RetSeries = [0.10 0.12
    0.05 0.04
    -0.05 0.05]
```

Use the following dates:

```
Dates = {'18-Jun-2001'; '17-Sep-2001'; '18-Dec-2001'}
```

where

```
ret = fints(Dates, RetSeries)
ret =
desc: (none)
freq: Unknown (0)
```

| 'dates: (3) |  | 'series1: (3)' | 'series2: (3)' |
| :--- | :--- | :--- | :--- |
| '18-Jun-2001' | $[$ | $0.1000]$ | $[$ |

PriceFtS is computed as:

```
PriceFts = ret2tick(ret, 'StartPrice', 100, 'StartTime', '18-Dec-2000')
```

| PriceFts = |  |  |
| :---: | :---: | :---: |
| desc: (none) |  |  |
| freq: Unknown |  |  |
| 'dates: (4)' | 'series1: (4)' | 'series2: (4)' |
| '18-Dec-2000' | 100] | 100] |
| '18-Jun-2001' | 110.0000] | $112.0000]$ |
| '17-Sep-2001' | 115.5000] | $116.4800]$ |
| '18-Dec-2001' | $109.7250]$ | 122.3040] |

See Also portsim | tick2ret

## Purpose Remove data series

Syntax fts = rmfield(tsobj, fieldname)

Arguments
tsobj Financial time series object.
fieldname String array containing the data series name to remove a single series from the object. Cell array of data series names to remove multiple data series from the object at the same time.
$\begin{array}{ll}\text { Description } & \begin{array}{l}\mathrm{fts}=\text { rmfield(tsobj, fieldname) } \\ \text { fieldname and its contents from the financial time data series object tsobj. }\end{array}\end{array}$
See Also
chfield | extfield | fieldnames | getfield | isfield

Purpose Relative Strength Index (RSI)

```
Syntax rsi = rsindex(closep, nperiods)
rsits = rsindex(tsobj, nperiods)
rsits = rsindex(tsobj, nperiods, ParameterName, ParameterValue)
```


## Arguments

| closep | Vector of closing prices. |
| :--- | :--- |
| nperiods | (Optional) Number of periods. Default $=14$. |
| tsobj | Financial time series object. |

## Description

rsi = rsindex(closep, nperiods) calculates the Relative Strength Index (RSI) from the closing price vector closep.
rsits = rsindex(tsobj, nperiods) calculates the RSI from the closing price series in the financial time series object tsobj. The object tsobj must contain at least the series Close, representing the closing prices. The output rsits is a financial time series object whose dates are the same as tsobj and whose data series name is RSI.
rsits = rsindex(tsobj, nperiods, ParameterName, ParameterValue) accepts a parameter name/parameter value pair as input. This pair specifies the name for the required data series if it is different from the expected default name. The valid parameter name is

CloseName: closing prices series name
The parameter value is the string that represents the valid parameter name.

1 The relative strength factor is calculated by dividing the average of the gains by the average of the losses within a specified time period: RS = (average gains)/(average losses).

2 The first value of RSI, RISI (1), is set as NaN to preserve the dimensions of CLOSEP.

## Examples

## References

See Also negvolidx | posvolidx
Purpose Seconds of date or time
Syntax Seconds = second(Date)
Description Seconds $=$ second(Date) returns the seconds given a serial date number or a date string.
Examples Seconds = second(738647.558427893)
or
Seconds = second('06-May-2022, 13:24:08.17')returns
Seconds =
8.1700
See Also datevec | hour | minute

## Purpose Portfolio configurations from 3-D efficient frontier

Syntax PortConfigs = selectreturn(AllMean, All Covariance, Target)

## Arguments

\(\left.$$
\begin{array}{ll}\text { AllMean } & \begin{array}{l}\text { Number of curves (NCURVES) by 1 cell array } \\
\text { where each element is a 1-by-NASSETS (number } \\
\text { of assets) vector of the expected asset returns } \\
\text { used to generate each curve on the surface. }\end{array} \\
\text { AllCovariance } & \begin{array}{l}\text { NCURVES-by-1 cell array where each element is } \\
\text { an NASSETS-by-NASSETS vector of the covariance }\end{array}
$$ <br>
matrix used to generate each curve on the <br>

surface.\end{array}\right\}\)| Target return value for each curve in the |
| :--- |
| frontier. |

## Description PortConfigs = selectreturn(AllMean, All Covariance, Target) returns the portfolio configurations for a target return given the average return and covariance for a rolling efficient frontier. <br> PortConfigs is a NASSETS-by-NCURVES matrix of asset allocation weights needed to obtain the target rate of return.

See Also frontier

## Portfolio.setAssetList

Superclasses AbstractPortfolio
Purpose Set up list of identifiers for assets
Syntax obj = setAssetList(obj, varargin)
Description
obj = setAssetList(obj, varargin) to set up a list of identifiers for the assets.

Tips

- Use dot notation to set up list of identifiers for assets:
obj = obj.setAssetList(varargin);
- To clear an AssetList, call this method with[] or\{[ ]\}.

Input
obj
A portfolio object [Portfolio].

## varargin

Asset identifiers. Either a comma-separated list of strings or a vector cell array of strings. Each string is an asset identifier.
If an asset list is entered as an input, this method overwrites an existing asset list in the object if one exists.
If no asset list is entered as an input, three actions can occur:

- If NumAssets is nonempty and AssetList is empty, AssetList becomes a numbered list of assets with default names according to the hidden property in defaultforAssetList (currently 'Asset').
- If NumAssets is nonempty and AssetList is nonempty, nothing happens.
- If NumAssets is empty and AssetList is empty, the default NumAssets $=1$ is set and a default asset list is created (currently 'Asset1').

For more information on setting up a list of asset identifiers, see "Common Operations on the Portfolio Object" on page 4-29.

## Output <br> Arguments <br> obj

Updated portfolio object [Portfolio].

Note The underlying object has a number of public hidden properties to format the asset list:

- defaultforAssetList - Default name for assets, currently 'Asset'. Change this name to create default asset names such as 'ETF', 'Bond', and so on.
- sortAssetList - Reserved for future implementation.
- uppercaseAssetList - If true, make all asset identifiers uppercase strings. Otherwise does nothing. Default is false.


## Attributes

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

## Examples

To create a default list of asset names with three assets:

```
p = Portfolio('NumAssets',3);
p = p.setAssetList;
disp(p.AssetList);
    'Asset1' 'Asset2' 'Asset3'
```


## Portfolio.setAssetList

To create a list of asset names for three equities AGG, EEM, and VEU:

```
p = Portfolio;
p = p.setAssetList('AGG', 'EEM', 'VEU');
disp(p.AssetList);
    'AGG' 'EEM' 'VEU'
```


## See Also Portfolio |

Tutorials . "Common Operations on the Portfolio Object" on page 4-29
How To

- "Setting Up a List of Asset Identifiers" on page 4-30


## Portfolio.setAssetMoments

Purpose Set moments (mean and covariance) of asset returns
Syntax
Description
Tips
InputArguments
obj = setAssetMoments(obj, AssetMean)
obj = setAssetMoments(obj, AssetMean, AssetCovar, NumAssets)
obj = setAssetMoments(obj, AssetMean) to set the mean of asset returns.
obj = setAssetMoments(obj, AssetMean, AssetCovar, NumAssets) to set moments (mean and covariance) of the asset returns with additional options for AssetCovar and NumAssets.

- Use dot notation to set moments (mean and covariance) of the asset returns:

```
obj = obj.setAssetMoments(obj, AssetMean, AssetCovar, NumAssets);
```

- To clear AssetMean and AssetCovar, use this method to set these respective inputs to [].


## obj

A portfolio object [Portfolio].

## AssetMean

Mean of asset returns [vector].

Note If AssetMean is a scalar and the number of assets is known, scalar expansion occurs. If the number of assets cannot be determined, this method assumes that NumAssets $=1$.

## AssetCovar

(Optional) Covariance of asset returns [matrix].

## Portfolio.setAssetMoments

## Note AssetCovar must be a symmetric positive-semidefinite matrix.

If AssetCovar is a scalar and the number of assets is known, a diagonal matrix is formed with the scalar value along the diagonals. If it is not possible to determine the number of assets, this method assumes that NumAssets $=1$.

If AssetCovar is a vector, a diagonal matrix is formed with the vector along the diagonal.

## NumAssets

(Optional) Number of assets [integer].

Note If NumAssets is not already set in the object, NumAssets can be entered to resolve array expansions with AssetMean or AssetCovar.

## Output Arguments <br> obj <br> Updated portfolio object [Portfolio].

## Attributes

## Examples

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Set the asset moment properties, given the mean and covariance of asset returns in the variables m and C :

```
m = [ 0.05; 0.1; 0.12; 0.18 ];
C = [ 0.0064 0.00408 0.00192 0;
    0.00408 0.0289 0.0204 0.0119;
    0.00192 0.0204 0.0576 0.0336;
    0 0.0119 0.0336 0.1225 ];
m = m/12;
C = C/12;
p = Portfolio;
p = p.setAssetMoments(m, C);
[assetmean, assetcovar] = p.getAssetMoments
assetmean =
    0.0042
    0.0083
    0.0100
    0.0150
assetcovar =
\begin{tabular}{rrrr}
0.0005 & 0.0003 & 0.0002 & 0 \\
0.0003 & 0.0024 & 0.0017 & 0.0010 \\
0.0002 & 0.0017 & 0.0048 & 0.0028 \\
0 & 0.0010 & 0.0028 & 0.0102
\end{tabular}
```

[^10]
## Portfolio.setBounds

Superclasses AbstractPortfolio
Purpose Set up bounds for portfolio weights

Syntax<br>Description

obj = setBounds(obj, LowerBound)
$\mathrm{obj}=$ setBounds(obj, LowerBound, UpperBound, NumAssets)

Tips
Use dot notation to set up the bounds for portfolio weights:

```
obj = obj.setBounds(LowerBound, UpperBound, NumAssets);
```


## Input <br> obj

Arguments
obj = setBounds(obj, LowerBound) to set up the lower bound for portfolio weights.
obj = setBounds(obj, LowerBound, UpperBound, NumAssets) to set up bounds for portfolio weights with additional options for UpperBound, and NumAssets.

Given bound constraints LowerBound and UpperBound, every weight in a portfolio Port must satisfy:

```
LowerBound <= Port <= UpperBound
```

A portfolio object [Portfolio].
LowerBound
Lower-bound weight for each asset [vector].

## UpperBound

(Optional) Upper-bound weight each asset [vector].

## NumAssets

(Optional) Number of assets in portfolio [scalar]. NumAssets cannot be used to change the dimension of a portfolio object.

## Portfolio.setBounds

Note If either LowerBound or UpperBound are input as empties with [ ], the corresponding attributes in the portfolio object are cleared and set to [].

If LowerBound or UpperBound are specified as scalars and NumAssets exists or can be imputed, then they undergo scalar expansion. The default value for NumAssets is 1 .

If both LowerBound and UpperBound exist and they are not ordered correctly, this method switches bounds if necessary.

## Output obj <br> Arguments

Updated portfolio object [Portfolio].

## Attributes

## Examples

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Suppose you have a balanced fund with stocks that can range between $50 \%$ and $75 \%$ of your portfolio and bonds that can range between $25 \%$ and $50 \%$ of your portfolio. The bound constraints for a balanced fund can be set with:

```
lb = [ 0.5; 0.25 ];
ub = [ 0.75; 0.5 ];
p = Portfolio;
p = p.setBounds(lb, ub);
disp(p.NumAssets);
```


## Portfolio.setBounds

disp(p.LowerBound);
disp(p.UpperBound);
2
0.5000
0.2500
0.7500
0.5000
See Also Portfolio | getBounds
Tutorials - "Bound Constraints" on page 4-7
How To - "Working with Bound Constraints" on page 4-55

| Superclasses | AbstractPortfolio |
| :---: | :---: |
| Purpose | Set up budget constraints |
| Syntax | obj = setBudget(obj, LowerBudget) <br> obj = setBudget(obj, LowerBudget, UpperBudget) |
| Description | obj = setBudget(obj, LowerBudget) to set up the lower budget constraint. <br> obj = setBudget(obj, LowerBudget, UpperBudget) to set up budget constraints with an additional option for UpperBudget. |
| Tips | Use dot notation to set up the budget constraints: obj = obj.setBudget(LowerBudget, UpperBudget); |
| Input Arguments | obj $\quad$ A portfolio object [Portfolio]. |
|  | LowerBudget <br> Lower-bound for budget constraint [scalar]. |
|  | UpperBudget (Optional) Upper-bound for budget constraint [scalar]. |

## Portfolio.setBudget

Note Given bounds for a budget constraints in either LowerBudget or UpperBudget, budget constraints requires any portfolio in Port to satisfy:

```
LowerBudget <= sum(Port) <= UpperBudget
```

One or both constraints may be specified. The usual budget constraint for a fully-invested portfolio is to have LowerBudget = UpperBudget $=1$. However, if the portfolio has allocations in cash, the budget constraints can be used to specify the cash constraints. For example, if the portfolio can hold between $0 \%$ and $10 \%$ in cash, the budget constraint would be set up with

```
obj = obj.setBudget(0.9, 1)
```


## Output obj <br> Updated portfolio object [Portfolio].

## Attributes

## Examples

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Assume you have a fund that permits up to $10 \%$ leverage, which means that your portfolio can be between $100 \%$ and $110 \%$ invested in risky assets. Given a portfolio object p , set the budget constraint:

```
p = Portfolio;
p = p.setBudget(1, 1.1);
```

disp(p.LowerBudget);
disp(p.UpperBudget);
1
1.1000

## See Also Portfolio | getBudget

Tułorials . "Budget Constraints" on page 4-8
How To . "Working with Budget Constraints" on page 4-57

## Portfolio.setCosts

Superclasses AbstractPortfolio
Purpose Set up proportional transaction costs

Syntax<br>Description

obj = setCosts(obj, BuyCost)
obj = setCosts(obj, BuyCost, SellCost, InitPort, NumAssets)
obj = setCosts(obj, BuyCost) to set up proportional transaction costs to purchase assets.
obj = setCosts(obj, BuyCost, SellCost, InitPort, NumAssets) to set up proportional transaction costs with additional options specified for SellCost, InitPort, and NumAssets.

Given proportional transaction costs and an initial portfolio in the variables BuyCost, SellCost, and InitPort, the transaction costs for any portfolio Port reduce expected portfolio return by:

```
BuyCost' * max{ 0, Port - InitPort } + SellCost' * max{ 0, InitPort - Port }
```

Tips

Input Arguments

- Use dot notation to set up proportional transaction costs:

```
obj = obj.setCosts(BuyCost, SellCost, InitPort, NumAssets);
```

- If BuyCost or SellCost are input as empties with [], the corresponding attributes in the portfolio object are cleared and set to [ ]. If InitPort is set to empty with [ ], it will only be cleared and set to [] if BuyCost, SellCost, and Turnover are also empty. Otherwise, it is an error.
obj
A portfolio object [Portfolio].


## BuyCost

Proportional transaction cost to purchase each asset [vector].

## SellCost

Proportional transaction cost to sell each asset [vector].

## InitPort

Initial or current portfolio weights [vector].

Note If no InitPort is specified, that value is assumed to be 0 .

## NumAssets

Number of assets in portfolio [scalar]. NumAssets cannot be used to change the dimension of a portfolio object.

Note If BuyCost, SellCost, or InitPort are specified as scalars and NumAssets exists or can be imputed, then these values undergo scalar expansion. The default value for NumAssets is 1 .

Transaction costs in BuyCost and SellCost are positive valued if they introduce a cost to trade. In some cases, they can be negative valued, which implies trade credits

## Output <br> Arguments <br> obj <br> Updated portfolio object [Portfolio].

## Attributes

| Access | public |
| :--- | :--- |
| Static | false |
| Hidden | false |

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

## Portfolio.setCosts

```
Examples Assume you have the same costs and initial portfolio as in the previous example. Given a portfolio object \(p\) with an initial portfolio already set, use setCosts to set up transaction costs:
```

```
bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
```

bc = [ 0.00125; 0.00125; 0.00125; 0.00125; 0.00125 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
sc = [ 0.00125; 0.007; 0.00125; 0.00125; 0.0024 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
x0 = [ 0.4; 0.2; 0.2; 0.1; 0.1 ];
p = Portfolio('InitPort', x0);
p = Portfolio('InitPort', x0);
p = p.setCosts(bc, sc);
p = p.setCosts(bc, sc);
disp(p.NumAssets);
disp(p.NumAssets);
disp(p.BuyCost);
disp(p.BuyCost);
disp(p.SellCost);
disp(p.SellCost);
disp(p.InitPort);
disp(p.InitPort);
5
0.0013
0.0013
0.0013
0.0013
0.0013
0.0013
0.0070
0.0013
0.0013
0.0024
0.4000
0.2000
0.2000
0.1000
0.1000
See Also
setInitPort | getCosts | Portfolio |

```

How To . "Working with Transaction Costs" on page 4-49

\section*{Portfolio.setDefaultConstraints}

Superclasses AbstractPortfolio

Purpose

Syntax

Description

Tips

Input
Arguments

Set up portfolio constraints with nonnegative weights that must sum to 1
obj = setDefaultConstraints(obj)
obj = setDefaultConstraints(obj, NumAssets)
obj \(=\) setDefaultConstraints(obj) to set up the portfolio constraints with nonnegative weights that must sum to 1 .
obj \(=\) setDefaultConstraints(obj, NumAssets) to set up the portfolio constraints with nonnegative weights that must sum to 1 with an additional option for NumAssets.
A "default" portfolio set has LowerBound \(=0\) and LowerBudget \(=\) UpperBudget \(=1\) such that a portfolio Port must satisfy sum (Port) = 1 with Port >= 0 .
- Use dot notation to set up the default portfolio set:
obj = obj.setDefaultConstraints(NumAssets);
- This method does not modify any existing constraints in a portfolio object other than the bound and budget constraints. If an UpperBound constraint exists, it is cleared and set to [].
obj
A portfolio object [Portfolio].
NumAssets
(Optional) Number of assets in portfolio [scalar]. NumAssets cannot be used to change the dimension of a portfolio object.

Default: 1

\section*{Portfolio.setDefaultConstraints}

\section*{Output \\ Arguments}
obj
Updated portfolio object [Portfolio].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Examples \\ Assume you have 20 assets, you can define the "default" portfolio set} as follows:
```

p = Portfolio('NumAssets', 20);
p = p.setDefaultConstraints;
disp(p);
disp(p);
Portfolio
Properties:
BuyCost: []
SellCost: []
RiskFreeRate: []
AssetMean: []
AssetCovar: []
Turnover: []
Name: []
NumAssets: 20
AssetList: []
InitPort: []
AInequality: []
bInequality: []
AEquality: []

```

\section*{Portfolio.setDefaultConstraints}
bEquality: []
LowerBound: [20x1 double]
UpperBound: ..... []
LowerBudget: ..... 1
UpperBudget: ..... 1
GroupMatrix: ..... []
LowerGroup: ..... []
UpperGroup: ..... []
GroupA: ..... []
GroupB: ..... []
LowerRatio: ..... []
UpperRatio: ..... [ ]
Methods, Superclasses
See AlsosetBounds | getBounds | setBudget | Portfolio
How To - "Setting Default Constraints for Portfolio Weights" on page 4-52

\section*{Portfolio.setEquality}

\section*{Superclasses AbstractPortfolio}

Purpose
Syntax
Description

Tips

\section*{Input Arguments}

Set up linear equality constraints for portfolio weights
obj = setEquality(obj, AEquality, bEquality)
obj = setEquality(obj, AEquality, bEquality) to set up linear equality constraints for portfolio weights.

Given linear equality constraint matrix AEquality and vector bEquality, every weight in a portfolio Port must satisfy:
```

AEquality * Port = bEquality

```
- Use dot notation to set up linear equality constraints for portfolio weights:
```

obj = obj.setEquality(AEquality, bEquality);

```
- Linear equality constraints can be removed from a portfolio object by entering [] for each property you want to remove.

\section*{obj}

A portfolio object [Portfolio].
AEquality
Matrix to form linear equality constraints [matrix].
bEquality
Vector to form linear equality constraints [vector].

Note An error results if AEquality is empty and bEquality is nonempty or if AEquality is nonempty and bEquality is empty.

\section*{Portfolio.setEquality}
Output ..... objArguments
Attributes
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}
Examples object \(p\), set the linear equality constraints with:
```

A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = p.setEquality(A, b);
disp(p.NumAssets);
disp(p.AEquality);
disp(p.bEquality);

```
5
\(\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}\)
0.5000

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.Suppose you have a portfolio of five assets, and you want to ensure thatthe first three assets are exactly \(50 \%\) of your portfolio. Given a portfolio
See Also addEquality | getEquality | Portfolio
Tutorials - "Linear Equality Constraints" on page 4-7
How To - "Working with Linear Equality Constraints" on page 4-66

\section*{Portfolio.setGroupRatio}

\section*{Superclasses AbstractPortfolio}

Purpose
Set up group ratio constraints for portfolio weights
obj = setGroupRatio(obj, GroupA)
obj = setGroupRatio(obj, GroupA, GroupB, LowerRatio, UpperRatio)
obj = setGroupRatio(obj, GroupA) to set up the group ratio constraints for portfolio weights with lower bound on the ratio between groups.
obj = setGroupRatio(obj, GroupA, GroupB, LowerRatio, UpperRatio) to set up group ratio constraints for portfolio weights with an additional option specified for UpperRatio.

Given base and comparison group matrices GroupA and GroupB and LowerRatio or UpperRatio bounds, group ratio constraints require any portfolio in Port to satisfy:
```

(GroupB * Port) .* LowerRatio <= GroupA * Port <= (GroupB * Port) .* UpperRatio

```

\section*{Caution}

This collection of constraints usually require that portfolio weights be nonnegative and that the products GroupA * Port and GroupB * Port are always nonnegative. Although negative portfolio weights and non-Boolean group ratio matrices are supported, use with caution.

Tips
- Use dot notation to set up group ratio constraints for portfolio weight:
```

obj = obj.setGroupRatio(GroupA, GroupB, LowerRatio, UpperRatio);

```
- To remove group ratio constraints, enter empty arrays for the corresponding arrays. To add to existing group ratio constraints, use addGroupRatio.

\section*{Portfolio.setGroupRatio}

\author{
Input obj Arguments \\ A portfolio object [Portfolio]. GroupA \\ Matrix that forms base groups for comparison [matrix]. GroupB \\ Matrix that forms comparison groups [matrix].
}

Note The group matrices GroupA and GroupB are usually indicators of membership in groups, which means that their elements are usually either 0 or 1 . Because of this interpretation, GroupA and GroupB matrices can be either logical or numerical arrays.

\section*{LowerGroup}

Lower-bound for ratio of GroupB groups to GroupA groups [vector].

Note If input is scalar, LowerGroup undergoes scalar expansion to be conformable with the group matrices.

\section*{UpperRatio}
(Optional) Upper-bound for ratio of GroupB groups to GroupA groups [vector].

Note If input is scalar, UpperRatio undergoes scalar expansion to be conformable with the group matrices.

\section*{Portfolio.setGroupRatio}

\section*{Output Arguments}
obj
Updated portfolio object [Portfolio].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Examples}

Suppose you want to make sure that the ratio of financial to nonfinancial companies in your portfolio never goes above 50\%. Assume you have 12 assets with 6 financial companies (assets 1-6) and 6 nonfinanical companies (assets 7-12). Group ratio constraints can be set with:
```

GA = [ true true true false false false ]; % financial companies
GB = [ false false false true true true ]; % non-financial companies
p = Portfolio;
p = p.setGroupRatio(GA, GB, [], 0.5);
disp(p.NumAssets);
disp(p.GroupA);
disp(p.GroupB);
disp(p.UpperRatio);
6
1
0
0.5000

```
```

See Also
addGroupRatio | getGroupRatio | Portfolio |

```

\section*{Portfolio.setGroupRatio}

\author{
Tutorials \\ - "Group Ratio Constraints" on page 4-10 \\ How To . "Working with Group Ratio Constraints" on page 4-62
}

\section*{Portfolio.setGroups}
\begin{tabular}{|c|c|}
\hline Superclasses & AbstractPortfolio \\
\hline Purpose & Set up group constraints for portfolio weights \\
\hline Syntax & \begin{tabular}{l}
obj \(=\) setGroups (obj, GroupMatrix, LowerGroup) \\
obj = setGroups(obj, GroupMatrix, LowerGroup, UpperGroup)
\end{tabular} \\
\hline Description & \begin{tabular}{l}
obj = setGroups(obj, GroupMatrix, LowerGroup) to set up group constraints for portfolio weights subject to a lower bound on groups. \\
obj = setGroups(obj, GroupMatrix, LowerGroup, UpperGroup) to set up group constraints for portfolio weights with an additional options specified for UpperGroup.
\end{tabular} \\
\hline & \begin{tabular}{l}
Given GroupMatrix and either LowerGroup or UpperGroup, a portfolio Port must satisfy: \\
LowerGroup <= GroupMatrix * Port <= UpperGroup
\end{tabular} \\
\hline Tips & \begin{tabular}{l}
- Use dot notation to set up group constraints for portfolio weights:
obj = obj.setGroups(GroupMatrix, LowerGroup, UpperGroup); \\
- To remove group constraints, enter empty arrays for the corresponding arrays. To add to existing group constraints, use addGroups.
\end{tabular} \\
\hline Input Arguments & obj A portfolio object [Portfolio]. \\
\hline & GroupMatrix \\
\hline & Group constraint matrix [matrix]. \\
\hline
\end{tabular}

\section*{Portfolio.setGroups}

Note The group matrix GroupMatrix is usually an indicator of membership in groups, which means that its elements are usually either 0 or 1 . Because of this interpretation, GroupMatrix can be either a logical or numerical matrix.

\section*{LowerGroup}

Lower-bound for group constraints [vector].

Note If input is scalar, LowerGroup undergoes scalar expansion to be conformable with GroupMatrix.

UpperGroup
Upper-bound for group constraints [vector].

Note If input is scalar, UpperGroup undergoes scalar expansion to be conformable with GroupMatrix.

\section*{Output Arguments}
obj
Updated portfolio object [Portfolio].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Portfolio.setGroups}
Examples Suppose you have a portfolio of five assets and you want to ensure thatthe first three assets constitute no more than \(30 \%\) of your portfolio.Given a portfolio object \(p\), set the group constraints with:
```

G = [ true true true false false ];
p = Portfolio;
p = p.setGroups(G, [], 0.3);
disp(p.NumAssets);
disp(p.GroupMatrix);
disp(p.UpperGroup);

```
5
\(\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}\)
0.3000
See Also addGroups | Portfolio | getGroups
Tutorials . "Group Constraints" on page 4-9
How To • "Working with Group Constraints" on page 4-59

\section*{Portfolio.setInequality}

Superclasses AbstractPortfolio
Purpose Set up linear inequality constraints for portfolio weights

Tips

\section*{Input \\ Arguments}

Syntax obj = setInequality (obj, AInequality, bInequality)
Description \(\quad o b j=\) setInequality (obj, AInequality, bInequality) to set up linear inequality constraints for portfolio weights.
Given a linear inequality constraint matrix AInequality and vector bInequality, every weight in a portfolio Port must satisfy:
```

```
AInequality * Port <= bInequality
```

```
```

```
AInequality * Port <= bInequality
```

```
- Use dot notation to set up linear inequality constraints for portfolio weights:
```

obj = obj.setInequality(AInequality, bInequality);

```
- To remove inequality constraints enter empty arguments. To add to existing inequality constraints, use addInequality.
obj
A portfolio object [Portfolio].
AEquality
Matrix to form linear inequality constraints [matrix].
bEquality
Vector to form linear inequality constraints [vector].

Note An error results if AInequality is empty and bInequality is nonempty or if AInequality is nonempty and bInequality is empty.

\section*{Portfolio.setInequality}

\section*{Output Arguments \\ Attributes}
obj
Updated portfolio object [Portfolio].
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Examples}

Suppose you have a portfolio of five assets and you want to ensure that the first three assets are no more than \(50 \%\) of your portfolio. Given a portfolio object \(p\), set the linear inequality constraints with:
```

A = [ 1 1 1 1 0 0 ];
b = 0.5;
p = Portfolio;
p = p.setInequality(A, b);
disp(p.NumAssets);
disp(p.AInequality);
disp(p.bInequality);

```
5
\(\begin{array}{lllll}1 & 1 & 1 & 0 & 0\end{array}\)
0.5000
See Also addInequality | Portfolio | getInequality
Tutorials - "Linear Inequality Constraints" on page 4-6
How To - "Working with Linear Inequality Constraints" on page 4-68

\section*{Portfolio.setlnitPort}

Superclasses AbstractPortfolio
Purpose Set up initial or current portfolio
Syntax obj = setInitPort(obj, InitPort)
obj = setInitPort(obj, InitPort, NumAssets)
Description

Tips

Input
Arguments
obj = setInitPort(obj, InitPort) to set up the initial or current portfolio.
obj = setInitPort(obj, InitPort, NumAssets) to set up the initial or current portfolio with an additional options specified for NumAssets.
- Use dot notation to set up initial or current portfolio:
obj = obj.setInitPort(InitPort, NumAssets);
- To remove an initial portfolio, call this method with an empty argument [] for InitPort.
obj
A portfolio object [Portfolio].
InitPort
Initial or current portfolio weights [vector].

Note If InitPort is specified as a scalar and NumAssets exists, then InitPort undergoes scalar expansion.

\section*{NumAssets}
(Optional) Number of assets in portfolio [scalar].

\section*{Portfolio.setInitPort}

Note If it is not possible to obtain a value for NumAssets, it is assumed that NumAssets is 1 .

\section*{Output obj \\ Arguments \\ Updated portfolio object [Portfolio]. \\ Attributes \\ \begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\section*{Examples Given an initial portfolio in x 0 , use setInitPort to set the InitPort} property:
```

p = Portfolio('NumAssets', 4, 'InitPort', 1/4);
x0 = [ 0.3; 0.2; 0.2; 0.0 ];
p = p.setInitPort(x0);
disp(p.InitPort);
disp(p.InitPort);
0.3000
0.2000
0.2000
0

```

To create an equally-weighted portfolio of four assets, use setInitPort:
```

p = p.setInitPort(1/4, 4);
disp(p.InitPort);
0.2500

```

\section*{Portfolio.setlnitPort}
0.2500
0.2500
0.2500
See Also setTurnover | setCosts | Portfolio
Tutorials . "Turnover Constraints" on page 4-10
How To . "Working with Turnover Constraints" on page 4-70

\section*{Superclasses AbstractPortfolio}

\section*{Purpose \\ Set hidden properties in portfolio object}

Syntax obj = setOptions(obj)
obj = setOptions(obj, varargin)
Description \(\quad \mathrm{obj}=\) setOptions(obj) to set the hidden properties in a portfolio object.
obj = setOptions(obj, varargin) to set the hidden properties in a portfolio object with additional options specified by one or more Name, Value pair arguments.

\section*{Warning}

This method is currently nonfunctional and issues the following warning message:

Warning: The method setOptions, which will enable modification of hidden properties, is not supported yet

The only way to modify hidden properties in a portfolio object is by direct assignment. An exception is the method setSolver, which permits modification of hidden properties associated with the solvers used by the portfolio object. In addition, the only way to get or display hidden properties is by direct access.

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

\author{
See Also setSolver | ।
}

\section*{Portfolio.setSolver}

Superclasses AbstractPortfolio
Purpose Choose main solver and specify associated solver options for portfolio optimization

\author{
Syntax \\ Description
}
obj = setSolver(obj, solverType)
obj = setSolver(obj, solverType, varargin)
obj = setSolver(obj, solverType) to choose solver for portfolio optimization.
obj = setSolver(obj, solverType, varargin) to choose solver and specify associated solver options with additional options specified by one or more Name, Value pair arguments or an optimset struct.
After you specify a solver, the varargin argument accepts either name-value pairs to set options or, for the case of solvers from Optimization Toolbox software, a structure created by optimset.

Tips

Input
Arguments
Use dot notation to choose the solver and specify associated solver options:
```

obj = obj.setSolver(solverType, varargin);

```
obj
A portfolio object [Portfolio].
solverType
Solver to use for portfolio optimization [string]. The default solver for the portfolio object is 'lcprog' with the control variables 'maxiter', 'tiebreak', 'tolpiv'.

Default: 'lcprog'

\section*{Name-Value Pair Arguments or optimset Struct}

Optional comma-separated pairs of Name, Value arguments, where Name is the argument name and Value is the corresponding value. Name must
appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, , NameN, ValueN.
varargin
Options to control the solver specified in solverType as [name/value pairs or an optimset struct].

Note The default solver for the portfolio object is 'lcprog'. The portfolio object can also use 'quadprog', which has several different options that can be set with optimset. Unlike Optimization Toolbox software which uses the trust-region-reflective algorithm as the default algorithm for quadprog, the portfolio optimization tools use the interior-point-convex algorithm. For more information about quadprog and quadratic programming algorithms and options, see "Quadratic Programming Algorithms".

\section*{Output \\ Arguments}
obj
Updated portfolio object [Portfolio].

\section*{Attributes}

\section*{Examples}

If you use quadprog as the solverType, the default is the interior-point-convex version of quadprog:
p = p.setSolver('quadprog');
display(p.solverType);

\section*{Portfolio.setSolver}
quadprog
You can switch back to lcprog with:
p = p.setSolver('lcprog');
display(p.solverType);
lcprog
See Also setOptions | optimset | quadprog |
Tutorials . "Choosing and Controlling the Solver" on page 4-86

\section*{Portfolio.setTurnover}
Purpose Set up maximum portfolio turnover constraint

Set up maximum portfolio turnover constraint

\section*{Tips}

\section*{Input Arguments}
Syntax
obj = setTurnover(obj, Turnover)
obj = setTurnover(obj, Turnover, InitPort, NumAssets)
obj = setTurnover(obj, Turnover, InitPort, NumAssets)
obj = setTurnover(obj, Turnover) to set up the maximum portfolio turnover constraint.
obj = setTurnover(obj, Turnover, InitPort, NumAssets) to set up the maximum portfolio turnover constraint with additional options specified by InitPort and NumAssets.
Given an upper bound for portfolio turnover in Turnover and an initial portfolio in InitPort, the turnover constraint requires any portfolio in Port to satisfy:
```

1' * | Port - InitPort | <= Turnover

```
Use dot notation to set up the maximum portfolio turnover constraint:
```

obj = obj.setTurnover(Turnover, InitPort, NumAssets);

```
obj
A portfolio object [Portfolio].

\section*{Turnover}
Portfolio turnover constraint [scalar].

Note Turnover must be nonnegative and finite.

\section*{InitPort}
(Optional) Initial or current portfolio weights [vector].

\section*{Portfolio.setTurnover}

> Note InitPort must be a finite vector with NumAssets \(>0\) elements.
> If no InitPort is specified, that value is assumed to be 0 .
> If InitPort is specified as a scalar and NumAssets exists, then InitPort undergoes scalar expansion.

\section*{NumAssets}
(Optional) Number of assets in portfolio [scalar].

Note If it is not possible to obtain a value for NumAssets, it is assumed that NumAssets is 1.

\section*{Output \\ obj} Arguments

Updated portfolio object [Portfolio].

\section*{Attributes}
\begin{tabular}{ll} 
Access & public \\
Static & false \\
Hidden & false
\end{tabular}

To learn about attributes of methods, see Method Attributes in the MATLAB Object-Oriented Programming documentation.

Examples Given a portfolio object \(p\), to ensure that average turnover is no more than \(30 \%\) with an initial portfolio of 10 assets in a variable \(\times 0\), use setTurnover to set the turnover constraint:
```

x0 = [ 0.12; 0.09; 0.08; 0.07; 0.1; 0.1; 0.15; 0.11; 0.08; 0.1 ];
p = Portfolio('InitPort', x0);

```
```

p = p.setTurnover(0.3);
disp(p.NumAssets);
disp(p.Turnover);
disp(p.InitPort);
1 0
0.3000
0.1200
0.0900
0.0800
0.0700
0.1000
0.1000
0.1500
0.1100
0.0800
0.1000

```
\begin{tabular}{ll} 
See Also & setInitPort | Portfolio \\
Tutorials & . "Turnover Constraints" on page 4-10 \\
How To & . "Working with Turnover Constraints" on page 4-70
\end{tabular}

\section*{Purpose Set content of specific field}
```

Syntax
newfts = setfield(tsobj, field, V)
newfts = setfield(tsobj, field, {dates}, V)

```

\section*{Description}

\section*{Examples}
setfield treats the contents of fields in a time series object ( tsobj ) as fields in a structure.
newfts = setfield(tsobj, field, V) sets the contents of the specified field to the value \(V\). This is equivalent to the syntax S.field \(=\mathrm{V}\).
newfts = setfield(tsobj, field, \{dates\}, V) sets the contents of the specified field for the specified dates. dates can be individual cells of date strings or a cell of a date string range using the :: operator, for example,
'03/01/99::03/31/99'. Dates can contain time-of-day information.

Example 1. Set the closing value for all days to 3890.
```

load dji30short
format bank
myfts1 = setfield(myfts1, 'Close', 3890);

```

Example 2. Set values for specific times on specific days.
First create a financial time series containing time-of-day data.
```

dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
'02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
times]);
myfts = fints(dates_times,[(1:4)'; nan; 6],{'Data1'},1,...
'My FINTS')
myfts =

```
```

desc: My FINTS
freq: Daily (1)

```
\begin{tabular}{|c|c|c|c|c|}
\hline 'dates: (6)' & 'times: & (6) ' & 'Data1: & (6) \({ }^{\prime}\) \\
\hline '01-Jan-2001' & '11:00' & & [ & \(1]\) \\
\hline " ' & '12:00' & & [ & \(2]\) \\
\hline '02-Jan-2001' & '11:00' & & [ & 3] \\
\hline ' " ' & '12:00' & & [ & 4] \\
\hline '03-Jan-2001' & '11:00' & & [ & NaN ] \\
\hline ' " ' & '12:00' & & [ & \(6]\) \\
\hline
\end{tabular}

Now use setfield to replace the data in myfts with new data starting at 12:00 on January 1, 2001 and ending at 11:00 on January 3, 2001.
```

S = setfield(myfts,'Data1',...
{'01-Jan-2001 12:00::03-Jan-2001 11:00'},(102:105)')

```
S =
    desc: My FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|c|c|}
\hline dates: (6)' & 'times: & (6) ' & & 1: (6)' \\
\hline '01-Jan-2001' & '11:00' & & [ & 1.00] \\
\hline " ' & '12:00' & & [ & 102.00] \\
\hline '02-Jan-2001' & '11:00' & & [ & 103.00] \\
\hline " ' & '12:00' & & [ & 104.00] \\
\hline '03-Jan-2001' & '11:00' & & [ & 105.001 \\
\hline " ' & '12:00' & & [ & 6.00] \\
\hline
\end{tabular}

\section*{See Also}
```

chfield| fieldnames | getfield | isfield | rmfield

```

Purpose Compute Sharpe ratio for one or more assets

\author{
Syntax \\ sharpe(Asset) \\ sharpe(Asset, Cash) \\ Ratio = sharpe(Asset, Cash)
}

\section*{Arguments}
\begin{tabular}{ll} 
Asset & \begin{tabular}{l} 
NUMSAMPLES-by-NUMSERIES matrix with \\
NUMSAMPLES observations of asset returns for \\
Cash \\
\\
NUMSERIES asset return series.
\end{tabular} \\
& \begin{tabular}{l} 
(Optional) Either a scalar return for a riskless \\
asset or a vector of asset returns to be a proxy \\
for a riskless asset. In either case, the return \\
periodicity must be the same as the periodicity \\
of Asset. For example, if Asset is monthly \\
data, then Cash must be monthly returns. If \\
no value is supplied, the default value for Cash \\
returns is 0.
\end{tabular}
\end{tabular}

Description
Given NUMSERIES assets with NUMSAMPLES returns for each asset in a NUMSAMPLES-by-NUMSERIES matrix Asset and given either a scalar Cash asset return or a vector of Cash asset returns, the Sharpe ratio is computed for each asset.

The output is Ratio, a 1-by-NUMSERIES row vector of Sharpe ratios for each series in Asset. Any series in Asset with standard deviation of returns equal to 0 will have a NaN value for its Sharpe ratio.

\title{
Note If Cash is a vector, Asset and Cash need not have the same number of returns but must have the same periodicity of returns. The classic Sharpe ratio assumes that Cash is riskless. In reality, a short-term cash rate is not necessarily riskless. NaN values in the data are ignored.
}

\author{
Examples See "Sharpe Ratio Example" on page 5-6. \\ References William F. Sharpe, "Mutual Fund Performance," Journal of Business, Vol. 39, No. 1, Part 2, January 1966, pp. 119-138.
}

\author{
See Also \\ inforatio | portalpha
}

Purpose Number of dates and data series
```

Syntax szfts = size(tsobj, dim)
[numRows, numCols] = size(tsobj)

```

\section*{Arguments}

\section*{Description}

See Also length \| size extract. (columns).
szfts = size(tsobj) returns the number of dates (rows) and the number of data series (columns) in the financial time series object tsobj. The result is returned in the vector szfts, whose first element is the number of dates and second is the number of data series.
szfts = size(tsobj, dim) specifies the dimension you want to
numRows returns a scalar representing the number of dates (rows).
numCols returns a scalar representing the number of data series
Financial time series object.
dim (Optional) A scalar that specifies the following dimension:
dim \(=1\) returns number of dates (rows).
dim = 2 returns number of data series (columns).

\section*{Purpose Smooth data}
```

Syntax output = smoothts(input)
output = smoothts(input, 'b', wsize)
output = smoothts(input, 'g', wsize, stdev)
output = smoothts(input, 'e', n)

```

\section*{Arguments}
\begin{tabular}{|c|c|}
\hline input & Financial time series object or a row-oriented matrix In a row-oriented matrix, each row represents an individual set of observations. \\
\hline \[
\begin{aligned}
& \text { 'b', 'g', or } \\
& \text { 'e' }
\end{aligned}
\] & Smoothing method (essentially the type of filter used). Can be Exponential (e), Gaussian (g), or Box (b). Default = b. \\
\hline wsize & Window size (scalar). Default \(=5\). \\
\hline stdev & Scalar that represents the standard deviation of the Gaussian window. Default \(=0.65\). \\
\hline n & \begin{tabular}{l}
For Exponential method, specifies window size or exponential factor, depending upon value. \\
- \(n>1\) (window size) or period length \\
- \(\mathrm{n}<1\) and > 0 (exponential factor: alpha) \\
- \(n=1\) (either window size or alpha)
\end{tabular} \\
\hline & If \(n\) is not supplied, the defaults are wsize \(=5\) and alpha \(=0.3333\). \\
\hline
\end{tabular}

Description
smoothts smooths the input data using the specified method.
output = smoothts(input) smooths the input data using the default Box method with window size, wsize, of 5 .
output = smoothts(input, 'b', wsize) smooths the input data using the Box (simple, linear) method. wsize specifies the width of the box to be used.
output = smoothts(input, 'g', wsize, stdev) smooths the input data using the Gaussian window method.
output \(=\) smoothts(input, 'e', n) smooths the input data using the Exponential method. n can represent the window size (period length) or alpha. If \(n>1, n\) represents the window size. If \(0<n<1, n\) represents alpha, where
\[
\alpha=\frac{2}{w \operatorname{size}+1} .
\]

If input is a financial time series object, output is a financial time series object identical to input except for contents. If input is a row-oriented matrix, output is a row-oriented matrix of the same length.

\section*{See Also \\ tsmovavg}

\section*{Purpose Sort financial time series}
```

Syntax sfts = sortfts(tsobj)
sfts = sortfts(tsobj, flag)
sfts = sortfts(tsobj, seriesnames, flag)
[sfts, sidx] = sortfts(...)

```

\section*{Arguments}
\begin{tabular}{ll} 
tsobj & Financial time series object. \\
flag & (Optional) Sort order: \\
& flag \(=1\); increasing order (default) \\
& flag \(=-1\); decreasing order \\
seriesnames & \begin{tabular}{l} 
(Optional) String containing a data series name or \\
cell array containing a list of data series names.
\end{tabular}
\end{tabular}
sfts = sortfts(tsobj) sorts the financial time series object tsobj in increasing order based only upon the 'dates' vector if tsobj does not contain time-of-day information. If the object includes time-of-day information, the sort is based upon a combination of the 'dates' and 'times ' vectors. The 'times' vector cannot be sorted individually.
sfts \(=\) sortfts(tsobj, flag) sets the order of the sort. flag = 1 : increasing date and time order. flag \(=-1\) : decreasing date and time order.
sfts = sortfts(tsobj, seriesnames, flag) sorts the financial time series object tsobj based upon the data series name(s) seriesnames. The seriesnames argument can be a single string containing a data series name or a cell array containing a list of data series names. If the optional flag is set to -1 , the sort is in decreasing order.
[sfts, sidx] = sortfts(...) additionally returns the index of the original object tsobj sorted based on 'dates ' or specified data series name(s).

See Also issorted | sort | sortrows

\section*{Purpose Slow stochastics}
```

Syntax
[spctk, spctd] = spctkd(fastpctk, fastpctd)
[spctk, spctd] = spctkd([fastpctk fastpctd])
[spctk, spctd] = spctkd(fastpctk, fastpctd, dperiods, dmamethod)
[spctk, spctd] = spctkd([fastpctk fastpctd], dperiods, dmamethod)
skdts = spctkd(tsobj)
skdts = spctkd(tsobj, dperiods, dmamethod)
skdts = spctkd(tsobj, dperiods, dmamethod, ParameterName,
ParameterValue, ...)

```

\section*{Arguments}
\begin{tabular}{ll} 
fastpctk & Fast stochastic F\%K (vector). \\
fastpctd & Fast stochastic F\%D (vector). \\
dperiods & (Optional) \%D periods. Default = 3. \\
dmamethod & (Optional) \%D moving average method. Default = \\
& 'e' (exponential). \\
tsobj & Financial time series object.
\end{tabular}

\section*{Description}
[spctk, spctd] = spctkd(fastpctk, fastpctd) calculates the slow stochastics \(\mathrm{S} \% \mathrm{~K}\) and \(\mathrm{S} \% \mathrm{D}\). spctk and spctd are column vectors representing the respective slow stochastics. The inputs must be single column-oriented vectors containing the fast stochastics F\%K and F\%D.
[spctk, spctd] = spctkd([fastpctk fastpctd]) accepts a two-column matrix as input. The first column contains the fast stochastic F\%K values, and the second contains the fast stochastic F\%D values.
[spctk, spctd] = spctkd(fastpctk, fastpctd, dperiods, dmamethod) calculates the slow stochastics, S\%K and S\%D, using the value of dperiods to set the number of periods and dmamethod
to indicate the moving average method. The inputs fastpctk and fastpctk must contain the fast stochastics, F\%K and F\%D, in column orientation. spctk and spctd are column vectors representing the respective slow stochastics.

Valid moving average methods for \%D are exponential ('e'), triangular ('t'), and modified ('m'). See tsmovavg for explanations of these methods.
[spctk, spctd] = spctkd([fastpctk fastpctd], dperiods, dmamethod) accepts a two-column matrix rather than two separate vectors. The first column contains the F\%K values, and the second contains the \(\mathrm{F} \% \mathrm{D}\) values.
skdts \(=\) spctkd(tsobj) calculates the slow stochastics, \(\mathrm{S} \% \mathrm{~K}\) and S\%D. tsobj must contain the fast stochastics, F\%K and F\%D, in data series named PercentK and PercentD. The skdts output is a financial time series object with the same dates as tsobj. Within tsobj the two series SlowPctK and SlowPctD represent the respective slow stochastics.
skdts = spctkd(tsobj, dperiods, dmamethod) lets you specify the length and the method of the moving average used to calculate \(\mathrm{S} \% \mathrm{D}\) values.
skdts = spctkd(tsobj, dperiods, dmamethod, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are
- KName: F\%K series name
- DName: F\%D series name

Parameter values are the strings that represent the valid parameter names.

\section*{Examples Compute the slow stochastics for Disney stock and plot the results:}
```

load disney.mat

```
```

dis_FastStoch = fpctkd(dis);
dis_SlowStoch = spctkd(dis_FastStoch);
plot(dis_SlowStoch)
title('Slow Stochastics for Disney')

```


\section*{References}

See Also

Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 268-271.
fpctkd | stochosc | tsmovavg
Purpose Standard deviation
```

Syntax tsstd = std(tsobj)
tsstd = std(tsobj, flag)

```

Arguments
\begin{tabular}{ll} 
tsobj & Financial time series object. \\
flag & (Optional) Normalization factor: \\
& flag \(=1\) normalizes by \(n\) (number of observations). \\
& flag \(=0\) normalizes by n-1.
\end{tabular}

Description tsstd \(=\operatorname{std}(\mathrm{tsobj})\) computes the standard deviation of each data series in the financial time series object tsobj and returns the results in tsstd. The tsstd output is a structure with field name(s) identical to the data series name(s).
tsstd = std(tsobj, flag) normalizes the data as indicated by flag.
See Also hist | mean

\section*{Purpose}

Stochastic oscillator
Syntax
```

stosc = stochosc(highp, lowp, closep)
stosc = stochosc([highp lowp closep])
stosc = stochosc(highp, lowp, closep, kperiods, dperiods, dmamethod)
stosc = stochosc([highp lowp closep], kperiods, dperiods, dmamethod)
stoscts = stochosc(tsobj, kperiods, dperiods, dmamethod)
stoscts = stochosc(tsobj, kperiods, dperiods, dmamethod,
ParameterName, ParameterValue, ...)

```

\section*{Arguments}
\begin{tabular}{ll} 
highp & High price (vector). \\
lowp & Low price (vector). \\
closep & Closing price (vector). \\
kperiods & (Optional) \%K periods. Default \(=10\). \\
dperiods & (Optional) \%D periods. Default \(=3\). \\
damethod & (Optional) \%D moving average method. Default \(=\) \\
& 'e' (exponential). \\
tsobj & Financial time series object.
\end{tabular}

\section*{Description}
stosc \(=\) stochosc(highp, lowp, closep) calculates the fast stochastics F\%K and F\%D from the stock price data highp (high prices), lowp (low prices), and closep (closing prices). stosc is a two-column matrix whose first column is the \(\mathrm{F} \% \mathrm{~K}\) values and second is the \(\mathrm{F} \% \mathrm{D}\) values.
stosc \(=\) stochosc([highp lowp closep]) accepts a three-column matrix of high (highp), low (lowp), and closing prices (closep), in that order.
stosc \(=\) stochosc(highp, lowp, closep, kperiods, dperiods, dmamethod) calculates the fast stochastics F\%K and F\%D from the stock price data highp (high prices), lowp (low prices), and closep (closing prices). kperiods sets the \(\% \mathrm{~K}\) period. dperiods sets the \(\% \mathrm{D}\) period. damethod specifies the \(\% \mathrm{D}\) moving average method. Valid moving average methods for \(\% \mathrm{D}\) are exponential ('e') and triangular ('t'). See tsmovavg for explanations of these methods.
stosc= stochosc([highp lowp closep], kperiods, dperiods, dmamethod) accepts a three-column matrix of high (highp), low (lowp), and closing prices (closep), in that order.
stoscts \(=\) stochosc(tsobj, kperiods, dperiods, dmamethod) calculates the fast stochastics \(\mathrm{F} \% \mathrm{~K}\) and \(\mathrm{F} \% \mathrm{D}\) from the stock price data in the financial time series object tsobj. tsobj must minimally contain the series High (high prices), Low (low prices), and Close (closing prices). stoscts is a financial time series object with similar dates to tsobj and two data series named SOK and SOD.
stoscts \(=\) stochosc(tsobj, kperiods, dperiods, dmamethod, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are
- HighName: high prices series name
- LowName: low prices series name
- CloseName: closing prices series name

Parameter values are the strings that represent the valid parameter names.

\section*{Examples}

Compute the stochastic oscillator for Disney stock and plot the results:
```

load disney.mat
dis_StochOsc = stochosc(dis)
plot(dis_StochOsc)

```


Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 268-271.

\section*{See Also}

\section*{Purpose Content assignment}
subasgn assigns content to a component within a financial time series object. subasgn supports integer indexing or date string indexing into the time series object with values assigned to the designated components. Serial date numbers cannot be used as indices. To use date string indexing, enclose the date string(s) in a pair of single quotation marks ' '.

You can use integer indexing on the object as in any other MATLAB matrix. It will return the appropriate entry(ies) from the object.

You must specify the component to which you want to assign values. An assigned value must be either a scalar or a column vector.

\section*{Examples}

Given a time series myfts with a default data series name of series1,
```

myfts.series1('07/01/98::07/03/98') = [1 2 3]';

```
assigns the values 1,2 , and 3 corresponding to the first three days of July, 1998.
```

myfts('07/01/98::07/05/98')
ans =

```
```

desc: Data Assignment

```
desc: Data Assignment
freq: Daily (1)
'dates: (5)' 'series1: (5)'
'01-Jul-1998' [ 1]
'02-Jul-1998' [ 2]
'03-Jul-1998' [ 3]
'04-Jul-1998' [ 4561.2]
'05-Jul-1998' [ 5612.3]
```

When the financial time series object contains a time-of-day specification, you can assign data to a specific time on a specific day. For
example, create a financial time series object called timeday containing both dates and times:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
'02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
times]);
timeday = fints(dates_times,(1:6)',{'Data1'},1,'My first FINTS')
timeday =
    desc: My first FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|}
\hline dates: (6)' & 'times: (6)' & 'Data1: (6)' \\
\hline '01-Jan-2001' & '11:00' & \(1]\) \\
\hline " ' & '12:00' & \(2]\) \\
\hline '02-Jan-2001' & '11:00' & 3] \\
\hline ' " ' & '12:00' & \(4]\) \\
\hline '03-Jan-2001' & '11:00' & 5] \\
\hline ' " ' & '12:00' & \(6]\) \\
\hline
\end{tabular}
```

Use integer indexing to assign the value 999 to the first item in the object.

```
timeday(1) = 999
timeday =
    desc: My first FINTS
freq: Daily (1)
\begin{tabular}{|c|c|c|c|c|}
\hline ates: (6)' & 'times: & (6) \({ }^{\prime}\) & 'Data1: & (6) \({ }^{\prime}\) \\
\hline '01-Jan-2001' & '11:00' & & [ & 999] \\
\hline ' & '12:00' & & [ & 2] \\
\hline '02-Jan-2001' & '11:00' & & & \(3]\) \\
\hline
\end{tabular}
```

| ' " ' | '12:00' | [ | 4] |
| :---: | :---: | :---: | :---: |
| '03-Jan-2001' | '11:00' | [ | 5] |
| ' " ' | '12:00' | [ | $6]$ |

For value assignment using date strings, enclose the string in single quotation marks. If a date has multiple times, designating only the date and assigning a value results in every element of that date taking on the assigned value. For example, to assign the value 0.5 to all times-of-day on January 1, 2001, enter

```
timedata('01-Jan-2001') = 0.5
```

The result is

```
timedata =
    desc: My first FINTS
    freq: Daily (1)
\begin{tabular}{|c|c|c|c|}
\hline s: (6)' & times: & (6) ' & 'Data1: (6)' \\
\hline '01-Jan-2001' & '11:00' & & \(0.5000]\) \\
\hline " ' & '12:00' & & \(0.5000]\) \\
\hline '02-Jan-2001' & '11:00' & & \(3]\) \\
\hline " ' & '12:00' & & \(4]\) \\
\hline '03-Jan-2001' & '11:00' & & 5] \\
\hline " ' & '12:00' & & \(6]\) \\
\hline
\end{tabular}
```

To access the individual components of the financial time series object, use the structure syntax. For example, to assign a range of data to all the data items in the series Data1, you can use

```
timedata.Data1 = (0: .1 : .5)'
timedata =
    desc: My first FINTS
    freq: Daily (1)
```

| 'dates: (6)' | 'times: | (6) ${ }^{\prime}$ | 'Data1: (6)' |
| :---: | :---: | :---: | :---: |
| '01-Jan-2001' | '11:00' |  | $0]$ |
| ' " ' | '12:00' |  | $0.1000]$ |
| '02-Jan-2001' | '11:00' |  | $0.2000]$ |
| ' " ' | '12:00' |  | $0.3000]$ |
| '03-Jan-2001' | '11:00' |  | $0.4000]$ |
| ' " ' | '12:00' |  | $0.5000]$ |

## See Also <br> datestr | subsref

## Purpose Subscripted reference

Description
subsref implements indexing for a financial time series object. Integer indexing or date (and time) string indexing is allowed. Serial date numbers cannot be used as indices.

To use date string indexing, enclose the date string(s) in a pair of single quotation marks ' '.

You can use integer indexing on the object as in any other MATLAB matrix. It returns the appropriate entry(ies) from the object.
Additionally, subsref lets you access the individual components of the object using the structure syntax.

## Examples Create a time series named myfts:

```
myfts = fints((datenum('07/01/98'):datenum('07/01/98')+4)', ...
[1234.56; 2345.61; 3456.12; 4561.23; 5612.34], [], 'Daily',...
'Data Reference');
```

Extract the data for the single day July 1, 1998:

```
myfts('07/01/98')
ans =
    desc: Data Reference
    freq: Daily (1)
    'dates: (1)' 'series1: (1)'
    '01-Jul-1998' [ 1234.6]
```

Now, extract the data for the range of dates July 1, 1998, through July 5, 1998:

```
myfts('07/01/98::07/03/98')
ans =
    desc: Data Reference
```

| freq: Daily (1) |  |  |
| :--- | :--- | :--- |
| 'dates: (3)' |  |  |
| 'series1: | (3) |  |
| '01-Jul-1998' | [ | $1234.6]$ |
| '02-Jul-1998' | [ | $2345.6]$ |
| '03-Jul-1998' | [ | $3456.1]$ |

You can use the MATLAB structure syntax to access the individual components of a financial time series object. To get the description field of myfts, enter

```
myfts.desc
```

at the command line, which returns
ans =
Data Reference
Similarly
myfts.series1
returns
ans =
desc: Data Reference
freq: Daily (1)
'dates: (5)' 'series1: (5)'
'01-Jul-1998' [ 1234.6]
'02-Jul-1998' [ 2345.6]
'03-Jul-1998' [ 3456.1]
'04-Jul-1998' [ 4561.2]
'05-Jul-1998' [ 5612.3]
The syntax for integer indexing is the same as for any other MATLAB matrix. Create a new financial time series object containing both dates and times:

```
dates = ['01-Jan-2001';'01-Jan-2001'; '02-Jan-2001'; ...
```

```
    '02-Jan-2001'; '03-Jan-2001';'03-Jan-2001'];
times = ['11:00';'12:00';'11:00';'12:00';'11:00';'12:00'];
dates_times = cellstr([dates, repmat(' ',size(dates,1),1),...
    times]);
anewfts = fints(dates_times,(1:6)',{'Data1'},1,'Another FinTs');
```

Use integer indexing to extract the second and third data items from the object.

```
anewfts(2:3)
ans =
    desc: Another FinTs
    freq: Daily (1)
\begin{tabular}{lllll} 
'dates: (2)' & 'times: (2)' & 'Data1: (2) ' \\
'01-Jan-2001' & '12:00' & & [ & \(2]\) \\
'02-Jan-2001' & \(' 11: 00 '\) & {\([\)} & \(3]\)
\end{tabular}
```

For date or string enclose the indexing string in a pair of single quotation marks.

If there is one date with multiple times, indexing with only the date returns all the times for that specific date:

```
anewfts('01-Jan-2001')
ans =
    desc: Another FinTs
    freq: Daily (1)
\begin{tabular}{|c|c|c|}
\hline 'dates: (2)' & 'times: (2)' & 'Data1: (2)' \\
\hline '01-Jan-2001' & '11:00' & [ \\
\hline , " . & '12:00' & [ 2 ] \\
\hline
\end{tabular}
```

To specify one specific date and time, index with that date and time:

```
anewfts('01-Jan-2001 12:00')
ans =
    desc: Another FinTs
    freq: Daily (1)
\begin{tabular}{llll} 
'dates: (1)' & 'times: (1)' & 'Data1: (1)' \\
'01-Jan-2001' & '12:00' & {\([\)} & \(2]\)
\end{tabular}
```

To specify a range of dates and times, use the double colon (: :) operator:


To request all the dates, times, and data, use the :: operator without specifying any specific date or time:

```
anewfts('::')
```

See Also
datestr | fts2mat | subsasgn

Purpose Portfolio weight accuracy
Syntax return $=$ targetreturn(Universe, Window, Offset, Weights)

Arguments
Universe Number of observations (NUMOBS) by number of assets plus one (NASSETS + 1) array containing total return data for a group of securities. Each row represents an observation. Column 1 contains MATLAB serial date numbers. The remaining columns contain the total return data for each security.
Window Number of data periods used to calculate frontier.
Offset Increment in number of periods at which each frontier is generated.
Weights $\quad$ Number of assets (NASSETS) by number of curves (NCURVES) matrix of asset allocation weights needed to obtain the target rate of return.
return $=$ targetreturn(Universe, Window, Offset, Weights) computes target return values for each window of data and given portfolio weights. These values should match the input target return used with selectreturn.

See Also frontier | portopt | selectreturn

| Purpose | After-tax rate of return |
| :--- | :--- |
| Syntax | Return $=$ taxedrr (PreTaxReturn, TaxRate $)$ |

Arguments

| PreTaxReturn | Nominal rate of return. Enter as a decimal <br> fraction. |
| :--- | :--- |
| TaxRate | Tax rate. Enter as a decimal fraction. |

Description $\quad \begin{aligned} & \text { Return }=\text { taxedrr (PreTaxReturn, TaxRate) calculates the after-tax } \\ & \text { rate of return. }\end{aligned}$

## Examples

An investment has a $12 \%$ nominal rate of return and is taxed at a $30 \%$ rate. The after-tax rate of return is

```
Return = taxedrr(0.12, 0.30)
Return =
    0.0840
```

or $8.4 \%$
See Also effrr | irr | mirr | nomrr | xirr

## Purpose

Syntax [TBondMatrix, Settle] = tbl2bond(TBillMatrix)

## Arguments

$\left.\left.\begin{array}{ll}\text { TBillMatrix } & \begin{array}{l}\text { Treasury bill parameters. An n-by-5 matrix } \\ \text { where each row describes a Treasury bill. } \mathrm{n}\end{array} \\ \text { is the number of Treasury bills. Columns are } \\ \text { [Maturity DaysMaturity Bid Asked AskYield] } \\ \text { where: }\end{array}\right\} \begin{array}{l}\text { Maturity date, as a serial date number. Use datenum } \\ \text { to convert date strings to serial date numbers. }\end{array}\right\}$

## Description

[TBondMatrix, Settle] = tbl2bond(TBillMatrix) restates U.S. Treasury bill market parameters in U.S. Treasury bond form as zero-coupon bonds. This function makes Treasury bills directly comparable to Treasury bonds and notes.

| TBondMatrix | Treasury bond parameters. An N-by-5 matrix <br> where each row describes an equivalent <br> Treasury (zero-coupon) bond. Columns are <br> [CouponRate Maturity Bid Asked AskYield] <br> where |
| :--- | :--- |
| CouponRate | Coupon rate, which is always 0. |
| Maturity | Maturity date, as a serial date number. This date is <br> the same as the Treasury bill Maturity date. |
| Bid | Bid price based on $\$ 100$ face value. |
| Asked | Asked price based on $\$ 100$ face value. |
| AskYield | Asked yield to maturity: the effective return from <br> holding the bond to maturity, annualized on a <br> compound-interest basis. |

Examples Given published Treasury bill market parameters for December 22, 1997

```
TBill = [datenum('jan 02 1998') 10 0.0526 0.0522 0.0530
    datenum('feb 05 1998') 44 0.0537}00.0533 0.0544
    datenum('mar 05 1998') 72 0.0529 0.0527 0.0540];
```

Execute the function.

```
TBond = tbl2bond(TBill)
TBond =
```

    \(1.0 \mathrm{e}+005\) *
    | 0 | 7.2976 | 0.0010 | 0.0010 | 0.0000 |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 7.2979 | 0.0010 | 0.0010 | 0.0000 |
| 0 | 7.2982 | 0.0010 | 0.0010 | 0.0000 |

See Also tr2bonds
How To . "Term Structure of Interest Rates" on page 2-36

## Purpose Find third Wednesday of month

Syntax<br>[BeginDates, EndDates] = thirdwednesday(Month, Year)

## Arguments

| Month | Month of delivery for Eurodollar futures. |
| :--- | :--- |
| Year | Four-digit year of delivery for Eurodollar futures, <br> in sequence corresponding to a month in the Month <br> input argument. |

Inputs can be scalars or $n$-by- 1 vectors.

## Description

[BeginDates, EndDates] = thirdwednesday(Month, Year)
computes the beginning and end period date for a LIBOR contract (third Wednesdays of delivery months).

BeginDates is the beginning of three-month period contract as specified by Month and Year.

EndDates is the end of three-month period contract as specified by Month and Year.

## Notes

1. All dates are returned as serial date numbers. Convert to strings using datestr.
2. The function returns duplicates if you supply identical months and years.
3. The function supports dates from January 2000 to December 2099.

Examples Find the third Wednesday dates for swaps commencing in the month of October in the years 2002, 2003, and 2004.

```
Months = [10; 10; 10];
Year = [2002; 2003; 2004];
[BeginDates, EndDates] = thirdwednesday(Months, Year);
datestr(BeginDates)
ans =
16-0ct-2002
15-0ct-2003
20-0ct-2004
datestr(EndDates)
ans =
16-Jan-2003
15-Jan-2004
20-Jan-2005
```

Purpose Thirty-second quotation to decimal
Syntax OutNumber = thirtytwo2dec(InNumber, InFraction)
Arguments
Description OutNumber = thirtytwo2dec(InNumber, InFraction) changes the price quotation for a bond or bond future from a fraction with a denominator of 32 to a decimal.
OutNumber represents the sum of InNumber and InFraction expressed as a decimal.
Examples Two bonds are quoted as 101-25 and 102-31. Convert these prices to decimal.

```
InNumber = [101; 102];
InFraction = [25; 31]
OutNumber = thirtytwo2dec(InNumber, InFraction)
OutNumber =
```

    101.7813
    102.9688
    See Also dec2thirtytwo

Purpose Convert price series to return series<br>Syntax [RetSeries, RetIntervals] = tick2ret(TickSeries, TickTimes, Method)

## Arguments

## Description

[RetSeries, RetIntervals] = tick2ret(TickSeries, TickTimes, Method) computes the asset returns realized between NUMOBS observations of prices of NASSETS assets.

RetSeries is a (NUMOBS-1)-by-NASSETS time series array of asset returns associated with the prices in TickSeries. The $i$ th return is quoted for the period TickTimes(i) to TickTimes(i+1) and is not normalized by
the time increment between successive price observations. If Method is unspecified or 'Simple', the returns are:

```
RetSeries(i) = TickSeries(i+1)/TickSeries(i) - 1
```

If Method is 'Continuous ', the returns are:

```
RetSeries(i) = log[TickSeries(i+1)/TickSeries(i)]
```

RetIntervals is a (NUMOBS-1)-by-1 column vector of interval times between observations. If TickTimes is empty or unspecified, all intervals are assumed to have length 1.

## Examples

Compute the periodic returns of two stocks observed in the first, second, third, and fourth quarters.

```
TickSeries = [100 80
    11090
    11588
    110 91];
TickTimes = [0
    6
    9
    12];
[RetSeries, RetIntervals] = tick2ret(TickSeries, TickTimes)
RetSeries =
    0.1000 0.1250
    0.0455 -0.0222
    -0.0435 0.0341
RetIntervals =
```


## tick2ret

## See Also <br> ewstats | ret2tick

## Purpose

Convert price series to return series for time series object
Syntax

```
returnFts = tick2ret(priceFts)
returnFts = tick2ret(priceFts, 'PARAM1', VALUE1,
'PARAM2', VALUE2', ...)
```


## Arguments

'PARAM1 ${ }^{\text {I }} \quad$ (Optional) Method is a character string indicating the method to convert asset returns to prices. The value must be defined as 'Simple' (default) or 'Continuous'. If Method is 'Simple', tick2ret uses simple periodic returns. If Method is 'Continuous', the function uses continuously compounded returns. Case is ignored for Method.

## Description

returnFts = tick2ret(priceFts, 'PARAM1', VALUE1, 'PARAM2', VALUE2', ...) generates a financial time series object of returns.

Note The $i$ 'th return is quoted for the period PriceSeries $(i)$ to PriceSeries( $i+1$ ) and is not normalized by the time increment between successive price observations.

If Method is unspecified or 'Simple', the prices are

```
ReturnSeries(i) = PriceSeries(i+1)/PriceSeries(i)-1
```

If Method is 'Continuous ', the prices are

```
ReturnSeries(i) = log[PriceSeries(i+1)/PriceSeries(i)]
```


## Examples Compute the return series from the following price series:

```
PriceSeries = [100.0000 100.0000
110.0000 112.0000
115.5000 116.4800
109.7250 122.3040]
```

Use the following dates:

```
Dates = {'18-Dec-2000'
'18-Jun-2001'
'17-Sep-2001'
'18-Dec-2001'}
```

where

```
p = fints(Dates, PriceSeries)
```

returnFtS is computed as:

```
returnFts = tick2ret(p)
returnFts =
desc: (none)
freq: Unknown (0)
\begin{tabular}{lllll} 
'dates: (3)' & 'series1: (3)' & 'series2: (3)' \\
'18-Jun-2001' & [ & \(0.1000]\) & [ & \(0.1200]\) \\
'17-Sep-2001' & [ & \(0.0500]\) & [ & \(0.0400]\) \\
'18-Dec-2001' & [ & \(-0.0500]\) & [ & \(0.0500]\)
\end{tabular}
```


## See Also

portsim | ret2tick

Purpose Dates from time and frequency<br>Syntax Dates = time2date(Settle, TFactors, Compounding, Basis, EndMonthRule)

## Arguments

| Settle | Settlement date. A vector of serial date numbers or date strings. |
| :---: | :---: |
| TFactors | A vector of time factors corresponding to the compounding value. TFactors must be equal to or greater than zero. |
| Compounding | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default $=2$. This argument determines the formula for the discount factors: |
|  | Compounding $=1,2,3,4,6,12$ <br> Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the compounding frequency, $Z$ is the zero rate, and T is the time in periodic units; for example, $\mathrm{T}=$ $F$ is one year. |
|  | Compounding $=365$ |
|  | Disc $=(1+Z / F)^{\wedge}(-T)$, where $F$ is the number of days in the basis year and $T$ is a number of days elapsed computed by basis. |
|  | Compounding $=-1$ |
|  | Disc $=\exp \left(-T^{*} Z\right)$, where $T$ is time in years. |

Basis (Optional) Day-count basis of the instrument. A vector of integers.

- $0=$ actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual/365
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- $12=$ actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
EndMonthRule (Optional) End-of-month rule. A vector. This rule applies only when Maturity is an end-of-month date for a month having 30 or fewer days. $0=$ ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. $1=$ set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.

```
Description Dates = time2date(Settle, TFactors, Compounding, Basis,
EndMonthRule) computes dates corresponding to the times occurring
beyond the settlement date.
The time2date function is the inverse of date2time.
Examples Show that date2time and time2date are the inverse of each other. First compute the time factors using date2time.
```

```
Settle = '1-Sep-2002';
```

Settle = '1-Sep-2002';
Dates = datenum(['31-Aug-2005'; '28-Feb-2006'; '15-Jun-2006';
Dates = datenum(['31-Aug-2005'; '28-Feb-2006'; '15-Jun-2006';
'31-Dec-2006']);
'31-Dec-2006']);
Compounding = 2;
Compounding = 2;
Basis = 0;
Basis = 0;
EndMonthRule = 1;
EndMonthRule = 1;
TFactors = date2time(Settle, Dates, Compounding, Basis,...
TFactors = date2time(Settle, Dates, Compounding, Basis,...
EndMonthRule)
EndMonthRule)
TFactors =
TFactors =
5.9945
5.9945
6.9945
6.9945
7.5738
7.5738
8.6576

```
    8.6576
```

Now use the calculated TFactors in time2date and compare the calculated dates with the original set.

```
Dates_calc = time2date(Settle, TFactors, Compounding, Basis,...
    EndMonthRule)
Dates_calc =
    732555
    732736
```


## time2date

732843
733042
datestr(Dates_calc)
ans $=$
31-Aug-2005
28-Feb-2006
15-Jun-2006
31-Dec-2006

See Also cftimes | date2time

## Purpose Financial time series multiplication

```
Syntax
newfts = tsobj_1 .* tsobj_2
newfts = tsobj .* array
```

newfts = array .* tsobj

## Arguments

## Description

See Also
tsobj_1, tsobj_2 Pair of financial time series objects.
array A scalar value or array with the number of rows equal to the number of dates in tsobj and the number of columns equal to the number of data series in tsobj.

The times method multiplies element by element the components of one financial time series object by the components of the other. You can also multiply the entire object by an array.
If an object is to be multiplied by another object, both objects must have the same dates and data series names, although the order need not be the same. The order of the data series, when an object is multiplied by another object, follows the order of the first object.
newfts = tsobj_1 .* tsobj_2 multiplies financial time series objects element by element.
newfts = tsobj .* array multiplies a financial time series object element by element by an array.
newfts = array .* tsobj newfts = array / tsobj multiplies an array element by element by a financial time series object.

For financial time series objects, the times operation is identical to the mtimes operation.

```
minus | mtimes | plus | rdivide
```


## Purpose Convert to annual

```
Syntax
newfts = toannual(oldfts)
newfts = toannual(oldfts, 'ParameterName', ParameterValue, ...)
```


## Arguments

$$
\text { oldfts } \quad \text { Financial time series object. }
$$

newfts = toannual(oldfts) converts a financial time series of any frequency to one of an annual frequency. The default end-of-year is the last business day of the December.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as '00:00' for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for toannual will trigger the use of the defaults.
newfts $=$ toannual(oldfts, 'ParameterName', ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.

| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :--- | :--- |
| CalcMethod | CumSum | Returns the cumulative sum of the values <br> within each year. Data for missing dates <br> are given the value 0. |
|  | Exact | Returns the exact value at the end-of-year <br> date. No data manipulation occurs. |


| Parameter <br> Name | Parameter <br> Value | Dearest <br> Desciption |
| :--- | :--- | :--- |
| SimpAvg | (Default) Returns the values located at <br> the end-of-year dates. If there is missing <br> data, Nearest returns the nearest data <br> point preceding the end-of-year date. <br> Returns an averaged annual value that <br> only takes into account dates with data <br> (nonNaN) within each year. |  |
| v21x | This mode is compatible with previous <br> versions of this function (Version 2.1.x <br> and earlier). It returns an averaged <br> end-of-year value using a previous <br> toannual algorithm. This algorithm <br> takes into account all dates and data. For <br> dates that do not contain any data, the <br> data is assumed to be 0. |  |

Note If you set CalcMethod to v21x, settings for all of the following parameter name/parameter value pairs are not supported.

| BusDays 0 | Returns a financial time series that <br> ranges from (or between) the first date to <br> the last date in oldfts (includes NYSE <br> nonbusiness days and holidays). |
| :---: | :--- |

$\left.\begin{array}{lll}\hline \begin{array}{l}\text { Parameter } \\ \text { Name }\end{array} & \begin{array}{l}\text { Parameter } \\ \text { Value }\end{array} & \begin{array}{l}\text { Description }\end{array} \\ \text { (Default) Generates a monthly financial } \\ \text { time series that ranges from the first } \\ \text { date to the last date in oldfts (excludes } \\ \text { NYSE nonbusiness days and holidays } \\ \text { and weekends based on Altholidays and } \\ \text { Weekend). If an end-of-month date falls } \\ \text { on a nonbusiness day or NYSE holiday, } \\ \text { returns the last business day of the } \\ \text { month. }\end{array}\right\}$

| Parameter Name | Parameter <br> Value | Description |
| :---: | :---: | :---: |
| ED | 0 | Annual period ends on the last day or last business day of the month. |
|  | 1-31 | Specifies a particular annual day. Months that do not contain the specified day return the last day (or last business day) of the month (for example, ED $=31$ does not exist for February.) |
| EM | 1-12 | (Default) The annual period ends on the last day (or last business day) of the specified month All subsequent annual dates are calculated from this month. Default annual month is December (12). |
| EndPtTol | [Begin, End] | Denotes the minimum number of days that constitute an odd annual period at the endpoints of the time series (before the first time series date and after the last end-of-year date). <br> Begin and End must be -1 or any positive integer greater than or equal to 0 . <br> A single value input for 'EndPtTol' is the same as specifying that single value for Begin and End. <br> - 1 Exclude odd annual period dates and data from calculations. <br> 0 (Default) Include odd annual period dates and data in calculations. <br> n Number of days (any positive integer) that constitute an odd annual period. If there are insufficient days for a complete year, the endpoint data is ignored. |

## Parameter Parameter Name Value Description

The following diagram is a general depiction of the factors involved in the determination of endpoints for this function.


| TimeSpec First | Returns only the observation that occurs <br> at the first (earliest) time for a specific <br> date. |
| :--- | :---: |
| AltHolidays | (Default) Returns only the observation <br> that occurs at the last (latest) time for a <br> specific date. |
| Weekend | Vector of dates specifying an alternate set <br> of market closure dates. |
|  | Excludes all holidays. |
| Vector of length 7 containing 0's and 1's. <br> The value 1 indicates a weekend day. The |  |
| first element of this vector corresponds |  |
| to Sunday. For example, when Saturday |  |
| and Sunday are weekend days (default) |  |
| then Weekend = $\left.\begin{array}{ll}10000001\end{array}\right]$. |  |

```
Purpose Convert to daily
Syntax newfts = todaily(oldfts)
newfts = todaily(oldfts, 'ParameterName', ParameterValue, ...)
```


## Arguments

$$
\text { oldfts } \quad \text { Financial time series object }
$$

Description newfts = todaily (oldfts) converts a financial time series of any frequency to a daily frequency.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as '00:00' for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for todaily will trigger the use of the defaults.
newfts = todaily(oldfts, 'ParameterName', ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.

| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :--- | :--- |
| CalcMethod Exact | Returns the value located at specific <br> dates/times. No data manipulation <br> occurs. |  |
| v21x | This mode is compatible with previous <br> versions of this function (Version 2.1.x <br> and earlier). It returns a five-day <br> business week that starts on Monday and <br> ends on Friday. |  |

Note If you set CalcMethod to v21x, settings for all of the following parameter name/parameter value pairs are not supported.

| BusDays 0 | Generates a financial time series that <br> ranges from (or between) the first date to <br> the last date in oldfts (includes NYSE <br> nonbusiness days and holidays). |
| :---: | :---: |
| (Default) Generates a daily financial |  |
| time series that ranges from the first |  |
| date to the last date in oldfts (excludes |  |
| NYSE nonbusiness days and holidays |  |
| and weekends based on AltHolidays and |  |
| Weekend). |  |

NYSE market closures, holidays, and weekends are observed if AltHolidays and Weekend are not supplied or empty ([]).

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| DateFilter | Absolute | (Default) Displays all daily dates between the start and end dates of oldfts. Some dates may be disregarded if BusDays $=1$. |
|  |  | Note The default is to create a time series with every date at the specified periodicity, which is with DateFilter = Absolute. If you use DateFilter $=$ Relative, the endpoint effects do not apply since only your data defines which dates will appear in the output time series object. |
| TimeSpec | Relative | Displays only dates that exist in oldfts. Some dates may be disregarded if BusDays $=1$. |
|  | First | Returns only the observation that occurs at the first (earliest) time for a specific date. |
|  | Last | (Default) Returns only the observation that occurs at the last (latest) time for a specific date. |
| AltHolidays |  | Vector of dates specifying an alternate set of market closure dates. |
|  | -1 | Excludes all holidays. |
| Weekend |  | Vector of length 7 containing 0's and 1's. The value 1 indicates a weekend day. The first element of this vector corresponds to Sunday. For example, when Saturday and Sunday are weekend days (default) then Weekend $=\left[\begin{array}{lllllll}1 & 0 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |

## todaily

## See Also convertto | toannual | tomonthly | toquarterly | tosemi | toweekly

Purpose Current date
Syntax Datenum = today
Description Datenum = today returns the current date as a serial date number.
Examples Datenum = today
returns
Datenum =730695
on July 28, 2000.
See Also datenum | datestr | now

## todecimal

Purpose Fractional to decimal conversion

```
Syntax usddec = todecimal(quote, fracpart)
```

Description usddec = todecimal(quote, fracpart) returns the decimal equivalent, usddec, of a security whose price is normally quoted as a whole number and a fraction (quote). fracpart indicates the fractional base (denominator) with which the security is normally quoted (default $=32$ ).

Examples In the Wall Street Journal, bond prices are quoted in fractional form based on a denominator of 32 . For example, if you see the quoted price is 100:05 it means $1005 / 32$. To find the equivalent decimal value, enter

```
usddec = todecimal(100.05)
usddec =
    100.1563
usddec = todecimal(97.04, 16)
usddec =
    97.2500
```

Note The convention of using . (period) as a substitute for : (colon) in the input is adopted from Excel software.

## See Also toquoted

## Purpose Convert to monthly

```
Syntax newfts = tomonthly(oldfts)
newfts = tomonthly(oldfts, 'ParameterName', ParameterValue, ...)
```


## Arguments

$$
\text { oldfts } \quad \text { Financial time series object. }
$$

Description newfts = tomonthly(oldfts) converts a financial time series of any frequency to a monthly frequency. The default end-of-month day is the last business day of the month.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as 00:00 for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for tomonthly will trigger the use of the defaults.
newfts = tomonthly(oldfts, 'ParameterName', ParameterValue, ...) accepts parameter name/parameter value pairs as input, as specified in the following table.
$\left.\begin{array}{lll}\hline \begin{array}{l}\text { Parameter } \\ \text { Name }\end{array} & \begin{array}{l}\text { Parameter } \\ \text { Value }\end{array} & \begin{array}{l}\text { Description }\end{array} \\ \text { CalcMethod } & \text { CumSum } & \begin{array}{l}\text { Returns the cumulative sum of the values } \\ \text { within each month. Data for missing } \\ \text { dates are given the value 0. }\end{array} \\ \text { Exact } & \begin{array}{l}\text { Returns the exact value at the } \\ \text { end-of-month date. No data manipulation } \\ \text { occurs. }\end{array} \\ \text { (Default) Returns the values located at } \\ \text { the end-of-month date. If there is missing } \\ \text { data, 'Nearest' returns the nearest data } \\ \text { point preceding the end-of-month date. } \\ \text { Returns an averaged monthly value that } \\ \text { only takes into account dates with data } \\ \text { (nonNaN) within each month. }\end{array}\right\}$

Note If you set CalcMethod to v21x, settings for all of the following parameter name/parameter value pairs are not supported.

BusDays $0 \quad$| Generates a monthly financial time |
| :--- |
| series that ranges from the first date to |
| the last date in oldfts (includes NYSE |
| nonbusiness days and holidays). |

| Parameter <br> Name | Parameter <br> Value | Description |
| :---: | :---: | :---: |
| DateFilter | 1 | (Default) Generates a monthly financial time series that ranges from the first date to the last date in oldfts (excludes NYSE nonbusiness days and holidays and weekends based on AltHolidays and Weekend). If an end-of-month date falls on a nonbusiness day or NYSE holiday, returns the last business day of the month. |
|  |  | NYSE market closures, holidays, and weekends are observed if AltHolidays and Weekend are not supplied or empty ([). |
|  | Absolute | (Default) Returns all monthly dates between the start and end dates of oldfts. Some dates may be disregarded if BusDays $=1$. |
|  |  | Note The default is to create a time series with every date at the specified periodicity, which is with DateFilter = Absolute. If you use DateFilter = Relative, the endpoint effects do not apply since only your data defines which dates will appear in the output time series object. |
|  | Relative | Returns only monthly dates that exist in oldfts. Some dates may be disregarded if BusDays $=1$. |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| ED | 0 | (Default) The end-of-month date is the last day (or last business day) of the month. |
|  | 1-31 | Returns values on the specified end-of-month day. Months that do not contain the specified end-of-month day return the last day of the month instead (for example, ED = 31 does not exist for February). <br> If end-of-month falls on a NYSE non-business day or holiday, the previous business day is returned if BusDays $=1$. |
| EndPtTol | [Begin, End] | Denotes the minimum number of days that constitute an odd month at the end points of the time series (before the first whole period and after the last whole period). <br> Begin and End must be -1 or any positive integer greater than or equal to 0 . <br> A single value input for EndPtTol is the same as specifying that single value for Begin and End. <br> -1 Do not include odd month dates and data in calculations. <br> 0 (Default) Include all odd month dates and data in calculations. <br> n Number of days that constitute an odd month. If the minimum number of days is not met, the odd month dates and data are ignored. |

## Parameter Parameter Name Value Description

The following diagram is a general depiction of the factors involved in the determination of end points for this function.


| TimeSpec | First | Returns only the observation that occurs at the first (earliest) time for a specific date. |
| :---: | :---: | :---: |
|  | Last | (Default) Returns only the observation that occurs at the last (latest) time for a specific date. |
| AltHolidays |  | Vector of dates specifying an alternate set of market closure dates. |
|  | -1 | Excludes all holidays. |
| Weekend |  | Vector of length 7 containing 0's and 1's. The value 1 indicates a weekend day. The first element of this vector corresponds to Sunday. For example, when Saturday and Sunday are weekend days (default) then Weekend $=\left[\begin{array}{lllllll}1 & 0 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |

## toquarterly

Purpose Convert to quarterly

```
Syntax newfts = toquarterly(oldfts)
newfts = toquarterly(oldfts, 'ParameterName', ParameterValue, ...)
```


## Arguments

$$
\text { oldfts } \quad \text { Financial time series object }
$$

## Description

newfts = toquarterly (oldfts) converts a financial time series of any frequency to a quarterly frequency. The default quarterly days are the last business day of March, June, September, and December.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as 00:00 for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for toquarterly will trigger the use of the defaults.

```
newfts = toquarterly(oldfts, 'ParameterName',
ParameterValue, ...) accepts parameter name/parameter value
pairs as input, as specified in the following table.
```

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| CalcMethod | CumSum | Returns the cumulative sum of the values between each quarter. Data for missing dates are given the value 0 . |
|  | Exact | Returns the exact value at the end-of-quarter date. No data manipulation occurs. |
|  | Nearest | (Default) Returns the values located at the end-of-quarter date. If there is missing data, Nearest returns the nearest data point preceding the end-of-quarter date. |
|  | SimpAvg | Returns an averaged quarterly value that only takes into account dates with data (nonNaN) within each quarter. |
|  | v21x | This mode is compatible with previous versions of this function (Version 2.1.x and earlier). It returns an averaged end-of-quarter value using a previous toquarterly algorithm. This algorithm takes into account all dates and data. For dates that do not contain any data, the data is assumed to be 0 . |
|  | Note If you set CalcMethod to v21x, settings for all of the following parameter name/parameter value pairs are not supported. |  |

## toquarterly

| Parameter <br> Name | Parameter <br> Value | Description |
| :--- | :--- | :--- |
| BusDays | 0 | Generates a financial time series that <br> ranges from (or between) the first date to <br> the last date in oldfts (includes NYSE |
| nonbusiness days and holidays). |  |  |


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| ED | Relative | Returns only quarterly dates that exist in oldfts. Some dates may be disregarded if BusDays $=1$. |
|  | 0 | (Default) The end-of-quarter date is the last day (or last business day) of the quarter. |
|  | 1-31 | Specifies a particular end-of-quarter day. Months that do not contain the specified end-of-quarter day return the last day of the quarter instead (for example, ED $=31$ does not exist for February). |
| EM | 1-12 | Last month of the first quarter. All subsequent quarterly dates are based on this month. The default end-of-first-quarter month is March (3). |
| EndPtTol | [Begin, End] | Denotes the minimum number of days that constitute a odd quarter at the endpoints of the time series (before the first whole period and after the last whole period). <br> Begin and End must be -1 or any positive integer greater than or equal to 0 . <br> A single value input for EndPtTol is the same as specifying that single value for Begin and End. <br> -1 Do not include odd quarter dates and data in calculations. <br> 0 (Default) Include all odd quarter dates and data in calculations. <br> n Number of days (any positive integer) that constitute an odd quarter. If there are insufficient days for a complete |

## Parameter Parameter Name Value Description

quarter, the odd quarter dates and data are ignored.

The following diagram is a general depiction of the factors involved in the determination of endpoints for this function.


| TimeSpec First | Returns only the observation that occurs <br> at the first (earliest) time for a specific <br> date. |
| :--- | :---: |
| AltHolidays | (Default) Returns only the observation <br> that occurs at the last (latest) time for a <br> specific date. |
| Weekend | Vector of dates specifying an alternate set <br> of market closure dates. |
|  | Excludes all holidays. |
| Vector of length 7 containing 0's and 1's. <br> The value 1 indicates a weekend day. The <br> first element of this vector corresponds <br> to Sunday. For example, when Saturday <br> and Sunday are weekend days (default) <br> then Weekend $=[10000001]$. |  |

See Also convertto | toannual | todaily | tomonthly | tosemi | toweekly

## Purpose Decimal to fractional conversion

```
Syntax quote = toquoted(usddec, fracpart)
```

Description quote $=$ toquoted(usddec, fracpart) returns the fractional equivalent, quote, of the decimal figure, usddec, based on the fractional base (denominator), fracpart. The fractional bases are the ones used for quoting equity prices in the United States (denominator 2, 4, 8, 16, or 32 ). If fracpart is not entered, the denominator 32 is assumed.

Examples A United States equity price in decimal form is 101.625. To convert this to fractional form in eighths of a dollar:

```
quote = toquoted(101.625, 8)
quote =
    101.05
```

The answer is interpreted as $1015 / 8$.

Note The convention of using . (period) as a substitute for : (colon) in the output is adopted from Excel software.
See Also ..... todecimal

```
Purpose Convert to semiannual
Syntax newfts = tosemi(oldfts)
newfts = tosemi(oldfts, 'ParameterName', ParameterValue, ...)
```


## Arguments

$$
\text { oldfts } \quad \text { Financial time series object. }
$$

Description newfts = tosemi(oldfts) converts a financial time series of any frequency to a semiannual frequency. The default semiannual days are the last business day of June and December.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as 00:00 for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for tosemi will trigger the use of the defaults.

```
newfts = tosemi(oldfts, 'ParameterName', ParameterValue,
...) accepts parameter name/parameter value pairs as input, as
specified in the following table.
```

\(\left.$$
\begin{array}{lll}\hline \begin{array}{l}\text { Parameter } \\
\text { Name }\end{array} & \begin{array}{l}\text { Parameter } \\
\text { Value }\end{array} & \begin{array}{l}\text { Description }\end{array} \\
\text { CalcMethod } & \text { CumSum } & \begin{array}{l}\text { Returns the cumulative sum of the values } \\
\text { within each semiannual period. Data for } \\
\text { missing dates are given the value 0. }\end{array} \\
\text { Exact } & \begin{array}{l}\text { Returns the exact value at the } \\
\text { end-of-period date. No data manipulation } \\
\text { occurs. }\end{array} \\
\text { Nearest } & \begin{array}{l}\text { (Default) Returns the values located at } \\
\text { the end-of-period date. If there is missing } \\
\text { data, Nearest returns the nearest data } \\
\text { point preceding the end-of-period date. } \\
\text { Returns an averaged semiannual value }\end{array}
$$ <br>
simpAvg only takes into account dates with <br>
data (nonNaN) within each semiannual <br>

period.\end{array}\right]\)| This mode is compatible with previous |
| :--- |
| versions of this function (Version 2.1.x |
| and earlier). It returns an averaged |
| end-of-period value using a previous |
| tosemi algorithm. This algorithm takes |
| into account all dates and data. For dates |
| that do not contain any data, the data is |
| assumed to be 0. |

Note If you set CalcMethod to v21x, settings for all of the following parameter name/parameter value pairs are not supported.
$\left.\left.\begin{array}{lll}\hline \begin{array}{ll}\text { Parameter } \\ \text { Name }\end{array} & \begin{array}{l}\text { Parameter } \\ \text { Value }\end{array} & \begin{array}{l}\text { Description }\end{array} \\ \text { BusDays } & 0 & \begin{array}{l}\text { Generates a financial time series that } \\ \text { ranges from (or between) the first date to } \\ \text { the last date in oldfts (includes NYSE } \\ \text { nonbusiness days and holidays). }\end{array} \\ \text { (Default) Generates a financial time } \\ \text { series that ranges from the first date } \\ \text { to the last date in oldfts (excludes } \\ \text { NYSE nonbusiness days and holidays } \\ \text { and weekends based on AltHolidays and } \\ \text { Weekend). If an end-of-quarter date falls } \\ \text { on a nonbusiness day or NYSE holiday, } \\ \text { returns the last business day of the } \\ \text { quarter. }\end{array}\right] \begin{array}{l}\text { NYSE market closures, holidays, and } \\ \text { weekends are observed if AltHolidays } \\ \text { and Weekend are not supplied or empty }\end{array}\right\}$

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
|  | Relative | Returns only semiannual dates that exist in oldfts. Some dates may be disregarded if BusDays $=1$. |
| ED | 0 | (Default) The end-of-period date is the last day (or last business day) of the semiannual period. |
|  | 1-31 | Specifies a particular end-of-period day. Months that do not contain the specified end-of-period day return the last day of the semiannual period instead (for example, ED $=31$ does not exist for February). |
| EM | 1-12 | End month of the first semiannual period. All subsequent period dates are based on this month. The default end of period months are June (6) and December (12). |
| EndPtTol | [Begin, End] | Denotes the minimum number of days that constitute an odd semiannual period at the endpoints of the time series (before the first whole period and after the last whole period). <br> Begin and End must be -1 or any positive integer greater than or equal to 0 . <br> A single value input for EndPtTol is the same as specifying that single value for Begin and End. <br> - 1 Do not include odd period dates and data in calculations. <br> 0 (Default) Include all odd period dates and data in calculations. <br> n Number of days (any positive integer) that constitute an odd period. If there |

## Parameter Parameter Name Value Description

are insufficient days for a complete semiannual period, the odd period dates and data are ignored.
The following diagram is a general depiction of the factors involved in the determination of endpoints for this function.


| TimeSpec First | Returns only the observation that occurs <br> at the first (earliest) time for a specific <br> date. |
| :--- | :---: | :--- |
| AltHolidays | (Default) Returns only the observation <br> that occurs at the last (latest) time for a <br> specific date. |
| Veekend | Vector of dates specifying an alternate set <br> of market closure dates. |
|  | Excludes all holidays. |
| Vector of length 7 containing 0's and 1's. <br> The value 1 indicates a weekend day. The <br> first element of this vector corresponds <br> to Sunday. For example, when Saturday <br> and Sunday are weekend days (default) <br> then Weekend $=[10000001]$. |  |

## See Also <br> convertto | toannual | todaily | tomonthly | toquarterly | toweekly

## Purpose Total return price time series

```
Syntax Return = totalreturnprice(Price, Action, Dividend)
```


## Arguments

## Description

Return = totalreturnprice(Price, Action, Dividend) generates a total return price time series given price data, action or split data, and dividend data.

Return is NUMOBS-by-2 array of price data, where NUMOBS reflects the number of observations of price data. Column 1 contains MATLAB serial date numbers. Column 2 contains total return price values.

See Also periodicreturns

## Purpose Convert to weekly

```
Syntax newfts = toweekly(oldfts)
newfts = toweekly(oldfts, 'ParameterName', ParameterValue, ...)
```


## Arguments

$$
\text { oldfts } \quad \text { Financial time series object. }
$$

## Description

newfts $=$ toweekly(oldfts) converts a financial time series of any frequency to a weekly frequency. The default weekly days are Fridays or the last business day of the week.

Note If oldfts contains time-of-day information, newfts displays the time-of-day as 00:00 for those days that did not previously exist in oldfts.

Empty ([ ]) passed as inputs for parameter pair values for toweekly will trigger the use of the defaults.

```
newfts = toweekly(oldfts, 'ParameterName', ParameterValue,
...) accepts parameter name/parameter value pairs as input, as
specified in the following table.
```

| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| CalcMethod | CumSum | Returns the cumulative sum of the values within each week. Data for missing dates are given the value 0 . |
|  | Exact | Returns the exact value at the end-of-week dates. No data manipulation occurs. |
|  | Nearest | (Default) Returns the values located at the end-of-week dates. If there is missing data, Nearest returns the nearest data point preceding the end-of-week date. |
|  | SimpAvg | Returns an averaged weekly value that only takes into account dates with data (nonNaN) within each week. |
|  | v21x | This mode is compatible with previous versions of this function (Version 2.1.x and earlier). It returns an averaged end-of-weekly value using a previous toquarterly algorithm. This algorithm takes into account all dates and data. For dates that do not contain any data, the data is assumed to be 0 . |
|  | Note If you set CalcMethod to v21x, settings for all of the following parameter name/parameter value pairs are not supported. |  |
| BusDays | 0 | Generates a financial time series that ranges from (or between) the first date to the last date in oldfts (includes NYSE nonbusiness days and holidays). |

$\left.\begin{array}{lll}\hline \begin{array}{l}\text { Parameter } \\ \text { Name }\end{array} & \begin{array}{l}\text { Parameter } \\ \text { Value }\end{array} & \begin{array}{l}\text { (Default) Generates a financial time } \\ \text { Description }\end{array} \\ \text { series that ranges from the first date } \\ \text { to the last date in oldfts (excludes } \\ \text { NYSE nonbusiness days and holidays } \\ \text { and weekends based on AltHolidays } \\ \text { and Weekend). If an end-of-quarter date } \\ \text { falls on a nonbusiness day or NYSE } \\ \text { holiday, returns the last business day of } \\ \text { the quarter. }\end{array}\right\}$

| Parameter <br> Name | Parameter <br> Value | Description <br> EndPtTol <br> [Begin, <br> End] |
| :--- | :--- | :--- |
|  |  | Denotes the minimum number of days <br> that constitute a odd week at the <br> endpoints of the time series (before the <br> first whole period and after the last whole <br> period). <br> Begin and End must be -1 or any positive <br> integer greater than or equal to 0. |
| A single value input for EndPtTol is the |  |  |
| same as specifying that single value for |  |  |
| Begin and End. |  |  |

The following diagram is a general depiction of the factors involved in the determination of endpoints for this function.


| Parameter Name | Parameter Value | Description |
| :---: | :---: | :---: |
| EOW | 0-6 | Specifies the end-of-week day: <br> - 0 Friday (default) <br> - 1 Saturday <br> - 2 Sunday <br> - 3 Monday <br> - 4 Tuesday <br> - 5 Wednesday <br> - 6 Thursday |
| TimeSpec | First | Returns only the observation that occurs at the first (earliest) time for a specific date. |
|  | Last | (Default) Returns only the observation that occurs at the last (latest) time for a specific date. |
| AltHolidays |  | Vector of dates specifying an alternate set of market closure dates. |
|  | -1 | Excludes all holidays. |
| Weekend |  | Vector of length 7 containing 0's and 1's. The value 1 indicates a weekend day. The first element of this vector corresponds to Sunday. For example, when Saturday and Sunday are weekend days (default) then Weekend $=\left[\begin{array}{lllllll}1 & 0 & 0 & 0 & 0 & 0 & 1\end{array}\right]$. |

## Purpose

Term-structure parameters given Treasury bond parameters

## Syntax

## Arguments

| TreasuryMatrix | Treasury bond parameters. An n-by-5 <br> matrix, where each row describes <br> a Treasury bond. Columns are <br> [CouponRate Maturity Bid Asked AskYield] <br> where: |
| :--- | :--- |
| CouponRate | Coupon rate, as a decimal fraction. <br> Maturity |
| Maturity date, as a serial date number. Use <br> datenum to convert date strings to serial date <br> numbers. |  |
| Bid | Bid price based on $\$ 100$ face value. |
| Asked | Asked price based on $\$ 100$ face value. |
| AskYield | Asked yield to maturity, as a decimal fraction. <br> (Optional) Date string or serial date number of |
| Settle | the settlement date for the analysis. |

[Bonds, Prices, Yields] = tr2bonds(TreasuryMatrix, Settle) returns term-structure parameters (bond information, prices, and yields) sorted by ascending maturity date, given Treasury bond parameters. The formats of the output matrix and vectors meet requirements for input to the zbtprice and zbtyield zero-curve bootstrapping functions.

## tr2bonds

\(\left.$$
\begin{array}{ll}\text { Bonds } & \begin{array}{l}\text { Coupon bond information. An n-by-6 matrix } \\
\text { where each row describes a bond. Columns are } \\
\text { [Maturity CouponRate Face Period Basis } \\
\text { EndMonthRule] where: }\end{array} \\
\text { Maturity } & \begin{array}{l}\text { Maturity date of the bond, as a serial date } \\
\text { number. Use datestr to convert serial date } \\
\text { numbers to date strings. }\end{array} \\
\text { CouponRate } & \begin{array}{l}\text { Coupon rate of the bond, as a decimal fraction. }\end{array} \\
\text { Face } & \begin{array}{l}\text { Redemption or face value of the bond, always } \\
\text { 100. }\end{array} \\
\text { Period } & \begin{array}{l}\text { Coupons per year of the bond, always } 2 .\end{array} \\
\text { Basis } & \begin{array}{l}\text { Day-count basis of the bond, possible values } \\
\text { include: }\end{array} \\
& \begin{array}{l}\text { - } 0=\text { actual/actual (default) }\end{array}
$$ <br>
\& - 1=30 / 360 (SIA) <br>
- 2=actual/360 <br>

- 3=actual/365\end{array}\right]\)| For more information, seebasis on page |
| :--- |
| Glossary-1. |

Prices | Yrices. Column vector containing the price of |
| :--- |
| each bond in bonds, respectively. The number |
| of rows $(\mathrm{n})$ matches the number of rows in |
| bonds. |

Examples Given published Treasury bond market parameters for December 22, 1997

```
Matrix =[0.0650 datenum('15-apr-1999') 101.03125 101.09375 0.0564
    0.05125 datenum('17-dec-1998') 99.4375 99.5 0.0563
    0.0625 datenum('30-jul-1998') 100.3125 100.375 0.0560
    0.06125 datenum('26-mar-1998') 100.09375 100.15625 0.0546];
```

Execute the function.

| Bonds = |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 729840 | 0.06125 | 100 | 2 | 0 | 1 |
| 729966 | 0.0625 | 100 | 2 | 0 | 1 |
| 730106 | 0.05125 | 100 | 2 | 0 | 1 |
| 730225 | 0.065 | 100 | 2 | 0 | 1 |

100.1563
100.3750

## tr2bonds

$$
\text { Yields = } \begin{array}{r}
99.5000 \\
101.0938 \\
\\
\\
\\
\\
\\
\\
\\
0.0546 \\
\\
0.056 \\
0.0564
\end{array}
$$

(Example output has been formatted for readability.)

## See Also <br> tbl2bond | zbtprice | zbtyield

How To

- "Term Structure of Interest Rates" on page 2-36


## Purpose

Estimation of transition probabilities from credit ratings data

## Syntax

## Description

## Input Arguments

[transMat, sampleTotals, idTotals] = transprob(data)
[transMat, sampleTotals, idTotals] = transprob(data, Name, Value)
[transMat, sampleTotals, idTotals] = transprob(data) constructs a transition matrix from historical data of credit ratings.
[transMat, sampleTotals, idTotals] = transprob(data, Name, Value) constructs a transition matrix from historical data of credit ratings with additional options specified by one or more Name, Value pair arguments.
data
Historical input data for credit ratings. Cell array of size
nRecords-by- 3 containing the credit ratings. Each row contains an ID (column 1), a date (column 2), and a credit rating (column 3 ). The assigned credit rating corresponds to the associated ID on the associated date. All information corresponding to the same ID must be stored in contiguous rows. Sorting this information by date is not required but is recommended. IDs, dates, and ratings are usually stored in string format, but they can also be entered in numeric format. Here is an example with all of the information in string format:

| '00010283' | '10-Nov-1984' | 'CCC' |
| :--- | :--- | :--- |
| '00010283' | '12-May-1986' | 'B' |
| '00010283' | '29-Jun-1988' | 'CCC' |
| '00010283' | '12-Dec-1991' | 'D' |
| '00013326' | '09-Feb-1985' | 'A' |
| $00013326^{\prime}$ | $' 24-$ Feb-1994' | 'AA |

## Name-Value Pair Arguments

Optional comma-separated pairs of Name, Value arguments, where Name is the argument name and Value is the corresponding value. Name must

## transprob

appear inside single quotes (' ' ). You can specify several name-value pair arguments in any order as Name1, Value1, , NameN, ValueN.
algorithm
Estimation algorithm, in string format. Valid values are duration or cohort.

Default: duration
endDate
End date of the estimation time window, in string or numeric format.

Default: Latest date in data

## labels

Cell array of size nRatings-by-1, or 1-by-nRatings, containing the credit-rating scale. It must be consistent with the ratings labels used in the third column of data.

Default: \{'AAA','AA','A','BBB','BB','B','CCC','D'\} snapsPerYear

Integer indicating the number of credit-rating snapshots per year to be considered for the estimation. Valid values are $1,2,3,4,6$, 12 . This parameter is only used with the cohort algorithm.

Default: 1 - One snapshot per year
startDate
Start date of the estimation time window, in string or numeric format.

Default: Earliest date in data

## transInterval

Length of the transition interval, in years.
Default: 1 - One year transition probabilities

## Output Arguments

## transMat

Matrix of transition probabilities in percent. The size of the transition matrix is nRatings-by-nRatings.

## sampleTotals

Structure of totals for the whole sample. This contains a matrix of total transitions between ratings, and a vector of total time spent in each rating (for the duration approach), or total number of companies starting at each rating (for the cohort approach).

## idTotals

Struct array of size nIDs-by- 1 containing the totals per ID. Each element has a sparse matrix with the total transitions between ratings for the corresponding ID, and a sparse vector of total time spent in each rating (for duration approach), or total number of periods starting at each rating (for cohort approach).

## Definitions Cohort Estimation

The cohort algorithm estimates the transition probabilities based on a sequence of snapshots of credit ratings at regularly spaced points in time. If the credit rating of a company changes twice between two snapshot dates, the intermediate rating is overlooked and only the initial and final ratings influence the estimates.

## Duration Estimation

Unlike the cohort method, the duration algorithm estimates the transition probabilities based on the full credit ratings history, looking at the exact dates on which the credit rating migrations occur. There is no concept of snapshots in this method, and all credit rating migrations
influence the estimates, even when a company's rating changes twice within a short time.

## Examples

Using historical credit rating input data from Data_TransProb.mat display the first ten rows and compute the transition matrix:

```
load Data_TransProb
data(1:10,:)
% Estimate transition probabilities with default settings
transMat = transprob(data)
ans =
    '00010283' '10-Nov-1984' 'CCC'
    '00010283' '12-May-1986' 'B'
    '00010283' '29-Jun-1988' 'CCC'
    '00010283' '12-Dec-1991' 'D'
    00013326' '09-Feb-1985' 'A'
    00013326' '24-Feb-1994' 'AA'
    00013326' '10-Nov-2000' 'BBB'
    '00014413' '23-Dec-1982' 'B'
    '00014413' '20-Apr-1988' 'BB'
    '00014413' '16-Jan-1998' 'B'
transMat =
\begin{tabular}{rrrrrrrr}
93.1170 & 5.8428 & 0.8232 & 0.1763 & 0.0376 & 0.0012 & 0.0001 & 0.0017 \\
1.6166 & 93.1518 & 4.3632 & 0.6602 & 0.1626 & 0.0055 & 0.0004 & 0.0396 \\
0.1237 & 2.9003 & 92.2197 & 4.0756 & 0.5365 & 0.0661 & 0.0028 & 0.0753 \\
0.0236 & 0.2312 & 5.0059 & 90.1846 & 3.7979 & 0.4733 & 0.0642 & 0.2193 \\
0.0216 & 0.1134 & 0.6357 & 5.7960 & 88.9866 & 3.4497 & 0.2919 & 0.7050 \\
0.0010 & 0.0062 & 0.1081 & 0.8697 & 7.3366 & 86.7215 & 2.5169 & 2.4399 \\
0.0002 & 0.0011 & 0.0120 & 0.2582 & 1.4294 & 4.2898 & 81.2927 & 12.7167 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100.0000
\end{tabular}
```

Using historical credit rating input data from Data_TransProb.mat, compute the transition matrix using the cohort algorithm:

| load Data_TransProb |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \%Estimate transition probabilities with 'cohort' algorithm transMatCoh = transprob(data,'algorithm','cohort') |  |  |  |  |  |  |  |
| transMatCoh = |  |  |  |  |  |  |  |
| 93.1345 | 5.9335 | 0.7456 | 0.1553 | 0.0311 | 0 | 0 | 0 |
| 1.7359 | 92.9198 | 4.5446 | 0.6046 | 0.1560 | 0 | 0 | 0.0390 |
| 0.1268 | 2.9716 | 91.9913 | 4.3124 | 0.4711 | 0.0544 | 0 | 0.0725 |
| 0.0210 | 0.3785 | 5.0683 | 89.7792 | 4.0379 | 0.4627 | 0.0421 | 0.2103 |
| 0.0221 | 0.1105 | 0.6851 | 6.2320 | 88.3757 | 3.6464 | 0.2873 | 0.6409 |
| 0 | 0 | 0.0761 | 0.7230 | 7.9909 | 86.1872 | 2.7397 | 2.2831 |
| 0 | 0 | 0 | 0.3094 | 1.8561 | 4.5630 | 80.8971 | 12.3743 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Algorithms Cohort Estimation

The algorithm first determines a sequence $t_{0}, \ldots, t_{K}$ of snapshot dates. The elapsed time, in years, between two consecutive snapshot dates $t_{k-1}$ and $t_{k}$ is equal to $1 / n s$, where $n s$ is the number of snapshots per year. These $K+1$ dates determine $K$ transition periods.

The algorithm computes $N_{i}^{n}$, the number of transition periods in which obligor $n$ starts at rating $i$. These are added up over all obligors to get $N_{i}$, the number of obligors in the sample that start a period at rating $i$. The number periods in which obligor $n$ starts at rating $i$ and ends at rating $j$, or migrates from $i$ to $j$, denoted by $N_{i j}^{n}$, is also computed. These are also added up to get $N_{i j}$, the total number of migrations from $i$ to $j$ in the sample.

## transprob

The estimate of the transition probability from $i$ to $j$ in one period, denoted by $P_{i j}$, is given by:

$$
P_{i j}=\frac{N_{i j}}{N_{i}}
$$

These probabilities are arranged in a one-period transition matrix $P_{0}$, where the $i, j$ entry in $P_{0}$ is $P_{i j}$.

If the number of snapshots per year $n s$ is 4 (quarterly snapshots), the probabilities in $P_{0}$ are 3 -month (or 0.25 -year) transition probabilities. You may, however, be interested in 1-year or 2 -year transition probabilities. The latter time interval is called the transition interval, $\Delta t$, and it is used to convert $P_{o}$ into the final transition matrix, $P$, according to the formula:

$$
P=P_{0}^{n s \square t}
$$

For example, if $n s=4$ and $\Delta t=2, P$ contains the 2 -year transition probabilities estimated from quarterly snapshots.

Note For the cohort algorithm, optional output arguments idTotals and sampleTotals from transprob contain the following information:

- idTotals(n).totalsVec $=\left(N_{i}^{n}\right) \forall i$
- idTotals(n).totalsMat $=\left(N_{i, j}^{n}\right) \forall i j$
- sampleTotals.totalsVec $=\left(N_{i}\right) \forall i$
- sampleTotals.totalsMat $=\left(N_{i, j}\right) \forall i j$

For efficiency, the vectors and matrices in idTotals are stored as sparse arrays. If a cohort-based idTotals or sampleTotals is used as input to transprobbytotals, set the transprobbytotals argument algorithm to cohort.

## Duration Estimation

The algorithm computes $T_{i}^{n}$, the total time that obligor $n$ spends in rating $i$ within the estimation time window. These quantities are
added up over all obligors to get $T_{i}$, the total time spent in rating $i$, collectively, by all obligors in the sample. The algorithm also computes
$T_{i j}^{n}$, the number times that obligor $n$ migrates from rating $i$ to rating $j$, with $i$ not equal to $j$, within the estimation time window. And it also
adds them up to get $T_{i j}$, the total number of migrations, by all obligors in the sample, from the rating $i$ to $j$, with $i$ not equal to $j$.

To estimate the transition probabilities, the duration algorithm first needs to compute a generator matrix $\Lambda$. Each off-diagonal entry of this matrix is an estimate of the transition rate out of rating $i$ into rating $j$, and is given by:

## transprob

$$
\lambda_{i j}=\frac{T_{i j}}{T_{i}}, i \neq j
$$

The diagonal entries are computed as:

$$
\lambda_{i i}=-\sum_{j \neq i} \lambda_{i j}
$$

With the generator matrix and the transition interval $\Delta t$ (e.g., $\Delta t=2$ corresponds to 2 -year transition probabilities), the transition matrix is obtained as $P=\exp (\Delta t \Lambda)$, where $\exp$ denotes matrix exponentiation (expm in MATLAB).

Note For the duration algorithm, optional output arguments idTotals and sampleTotals from transprob contain the following information:

- idTotals(n).totalsVec $=\left(T_{i}^{n}\right) \forall i$
- idTotals(n).totalsMat $=\left(T_{i, j}^{n}\right) \forall i j$
- sampleTotals.totalsVec $=\left(T_{i}\right) \forall i$
- sampleTotals.totalsMat $=\left(T_{i, j}\right) \forall i j$

For efficiency, the vectors and matrices in idTotals are stored as sparse arrays. If a duration-based idTotals or sampleTotals is used as input to transprobbytotals, set the transprobbytotals argument algorithm to duration.

References<br>Hanson, S., T. Schuermann, "Confidence Intervals for Probabilities of Default," Journal of Banking \& Finance, Elsevier, vol. 30(8), pages 2281-2301, August 2006.

Löffler, G., P. N. Posch, Credit Risk Modeling Using Excel and VBA, West Sussex, England: Wiley Finance, 2007.
Schuermann, T., "Credit Migration Matrices," in E. Melnick, B. Everitt (eds.), Encyclopedia of Quantitative Risk Analysis and Assessment, Wiley, 2008.

## See Also <br> transprobbytotals

How To

- "Estimation of Transition Probabilities" on page 6-3


## transprobbytotals

| Purpose | Estimation of transition probabilities from preprocessed credit ratings data |
| :---: | :---: |
| Syntax | ```[transMat, sampleTotals] = transprobbytotals(totals) [transMat, sampleTotals] = transprobbytotals(totals, Name, Value)``` |
| Description | [transMat, sampleTotals] = transprobbytotals(totals) constructs transition probabilities by reusing preprocessed data. <br> [transMat, sampleTotals] = transprobbytotals(totals, Name, Value) constructs transition probabilities by reusing preprocessed data with additional options specified by one or more Name, Value pair arguments. |
| Input Arguments | totals <br> This can be either: <br> - An idTotals struct array usually obtained by previously using transprob. <br> - A sampleTotals structure usually obtained by previously using transprob. |
|  | Name-Value Pair Arguments |
|  | Optional comma-separated pairs of Name, Value arguments, where Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name-value pair arguments in any order as Name1, Value1, , NameN, ValueN. <br> algorithm |
|  | Estimation algorithm, in string format. Valid values are duration or cohort. |
|  | Default: duration |

## snapsPerYear

Integer indicating the number of credit-rating snapshots per year to be considered for the estimation. Valid values are $1,2,3,4,6$, or 12. This parameter is only used when using the cohort algorithm.

Default: 1 - One snapshot per year

## transInterval

Length of the transition interval, in years.
Default: 1 - One year transition probabilities

## Output Arguments

Matrix of transition probabilities in percent. The size of the transition matrix is nRatings-by-nRatings.

```
sampleTotals
```

Structure of totals for the whole sample. This contains a matrix of total transitions between ratings and a vector of total time spent in each rating (for the duration approach), or total number of companies ever starting at each rating (for the cohort approach).

## Definitions Cohort Estimation

The cohort algorithm estimates the transition probabilities based on a sequence of snapshots of credit ratings at regularly spaced points in time. If the credit rating of a company changes twice between two snapshot dates, the intermediate rating is overlooked and only the initial and final ratings influence the estimates.

## Duration Estimation

Unlike the cohort method, the duration algorithm estimates the transition probabilities based on the full credit ratings history, looking at the exact dates on which the credit rating migrations occur. There is no concept of snapshots in this method, and all credit rating migrations
influence the estimates, even when a company's rating changes twice within a short time.

Examples

Use historical credit rating input data from Data_TransProb.mat and transprob to generate input for transprobbytotals:

| [transMat, <br> transMat <br> transMat $=$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 93.1170 | 5.8428 | 0.8232 | 0.1763 | 0.0376 | 0.0012 | 0.0001 | 0.0017 |
| 1.6166 | 93.1518 | 4.3632 | 0.6602 | 0.1626 | 0.0055 | 0.0004 | 0.0396 |
| 0.1237 | 2.9003 | 92.2197 | 4.0756 | 0.5365 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2312 | 5.0059 | 90.1846 | 3.7979 | 0.4733 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7960 | 88.9866 | 3.4497 | 0.2919 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3366 | 86.7215 | 2.5169 | 2.4399 |
| 0.0002 | 0.0011 | 0.0120 | 0.2582 | 1.4294 | 4.2898 | 81.2927 | 12.7167 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

Suppose companies 4 and 27 are outliers; remove them from the preprocessed idTotals struct array and estimate the new transition probabilities:

| ```[transMat1, sampleTotals1] = transprobbytotals(idTotals) transMat1 transMat1 =``` |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| 93.1172 | 5.8427 | 0.8231 | 0.1763 | 0.0377 | 0.0012 | 0.0001 | 0.0017 |
| 1.6213 | 93.1501 | 4.3584 | 0.6614 | 0.1631 | 0.0055 | 0.0004 | 0.0397 |
| 0.1239 | 2.9027 | 92.2297 | 4.0628 | 0.5367 | 0.0661 | 0.0028 | 0.0753 |
| 0.0236 | 0.2313 | 5.0070 | 90.1825 | 3.7986 | 0.4734 | 0.0642 | 0.2193 |
| 0.0216 | 0.1134 | 0.6357 | 5.7959 | 88.9866 | 3.4497 | 0.2920 | 0.7050 |
| 0.0010 | 0.0062 | 0.1081 | 0.8697 | 7.3367 | 86.7217 | 2.5171 | 2.4395 |


| 0.0002 | 0.0011 | 0.0120 | 0.2591 | 1.4340 | 4.3034 | 81.3027 | 12.6875 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100.0000 |

## Algorithms <br> Cohort Estimation

The algorithm first determines a sequence $t_{0, \ldots, t_{K}}$ of snapshot dates. The elapsed time, in years, between two consecutive snapshot dates $t_{k-1}$ and $t_{k}$ is equal to $1 / n s$, where $n s$ is the number of snapshots per year. These $K+1$ dates determine $K$ transition periods.

The algorithm computes $N_{i}^{n}$, the number of transition periods in which obligor $n$ starts at rating $i$. These are added up over all obligors to get $N_{i}$, the number of obligors in the sample that start a period at rating $i$. The number periods in which obligor $n$ starts at rating $i$ and ends at
rating $j$, or migrates from $i$ to $j$, denoted by $N_{i j}^{n}$, is also computed. These
are also added up to get $N_{i j}$, the total number of migrations from $i$ to $j$ in the sample.

The estimate of the transition probability from $i$ to $j$ in one period, denoted by $P_{i j}$, is given by:

$$
P_{i j}=\frac{N_{i j}}{N_{i}}
$$

These probabilities are arranged in a one-period transition matrix $P_{0}$, where the $i, j$ entry in $P_{0}$ is $P_{i j}$.
If the number of snapshots per year $n s$ is 4 (quarterly snapshots), the probabilities in $P_{0}$ are 3-month (or 0.25 -year) transition probabilities. You may, however, be interested in 1 -year or 2 -year transition probabilities. The latter time interval is called the transition interval, $\Delta t$, and it is used to convert $P_{0}$ into the final transition matrix, $P$, according to the formula:

$$
P=P_{0}^{n s \square t}
$$

## transprobbytotals

For example, if $n s=4$ and $\Delta t=2, P$ contains the 2 -year transition probabilities estimated from quarterly snapshots.

Note For the cohort algorithm, optional output arguments idTotals and sampleTotals from transprob contain the following information:

- idTotals(n).totalsVec $=\left(N_{i}^{n}\right) \forall i$
- idTotals(n).totalsMat $=\left(N_{i, j}^{n}\right) \forall i j$
- sampleTotals.totalsVec $=\left(N_{i}\right) \forall i$
- sampleTotals.totalsMat $=\left(N_{i, j}\right) \forall i j$

For efficiency, the vectors and matrices in idTotals are stored as sparse arrays. If a cohort-based idTotals or sampleTotals is used as input to transprobbytotals, set the transprobbytotals argument algorithm to cohort.

## Duration Estimation

The algorithm computes $T_{i}^{n}$, the total time that obligor $n$ spends in rating $i$ within the estimation time window. These quantities are
added up over all obligors to get $T_{i}$, the total time spent in rating $i$, collectively, by all obligors in the sample. The algorithm also computes
$T_{i j}^{n}$, the number times that obligor $n$ migrates from rating $i$ to rating $j$, with $i$ not equal to $j$, within the estimation time window. And it also
adds them up to get $T_{i j}$, the total number of migrations, by all obligors in the sample, from the rating $i$ to $j$, with $i$ not equal to $j$.

To estimate the transition probabilities, the duration algorithm first needs to compute a generator matrix $\Lambda_{\Lambda}$. Each off-diagonal entry of this
matrix is an estimate of the transition rate out of rating $i$ into rating $j$, and is given by:

$$
\lambda_{i j}=\frac{T_{i j}}{T_{i}}, i \neq j
$$

The diagonal entries are computed as:

$$
\lambda_{i i}=-\sum_{j \neq i} \lambda_{i j}
$$

With the generator matrix and the transition interval $\Delta t$ (e.g., $\Delta t=2$ corresponds to 2 -year transition probabilities), the transition matrix is obtained as $P=\exp (\Delta t \Lambda)$, where $\exp$ denotes matrix exponentiation (expm in MATLAB).

Note For the duration algorithm, optional output arguments idTotals and sampleTotals from transprob contain the following information:

- idTotals(n).totalsVec $=\left(T_{i}^{n}\right) \forall i$
- idTotals(n).totalsMat $=\left(T_{i, j}^{n}\right) \forall i j$
- sampleTotals.totalsVec $=\left(T_{i}\right) \forall i$
- sampleTotals.totalsMat $=\left(T_{i, j}\right) \forall i j$

For efficiency, the vectors and matrices in idTotals are stored as sparse arrays. If a duration-based idTotals or sampleTotals is used as input to transprobbytotals, set the transprobbytotals argument algorithm to duration.

## transprobbytotals

References Hanson, S., T. Schuermann, "Confidence Intervals for Probabilities ofDefault," Journal of Banking \& Finance, Elsevier, vol. 30(8), pages2281-2301, August 2006.Löffler, G., P. N. Posch, Credit Risk Modeling Using Excel and VBA,West Sussex, England: Wiley Finance, 2007.Schuermann, T., "Credit Migration Matrices," in E. Melnick, B. Everitt(eds.), Encyclopedia of Quantitative Risk Analysis and Assessment,Wiley, 2008.
See Also | transprob
How To - "Estimation of Transition Probabilities" on page 6-3

## Purpose Acceleration between times

Syntax acc = tsaccel(data, nTimes, datatype) accts $=$ tsaccel(tsobj, nTimes, datatype)

## Arguments

| data | Data series. |
| :--- | :--- |
| nTimes | (Optional) Number of times. Default = 12. |
| datatype | (Optional) Indicates whether data contains the data <br> itself or the momentum of the data: |
|  | $0=$ Data contains the data itself (default). |
|  | $1=$ Data contains the momentum of the data. |
| tsobj | Name of an existing financial time series object. |

## Description

Acceleration is the difference of two momentums separated by some number of periods.
acc = tsaccel(data, nTimes, datatype) calculates the acceleration of a data series, essentially the difference of the current momentum with the momentum some number of periods ago. If nTimes is specified, tsaccel calculates the acceleration of a data series data with time distance of nTimes times.
accts $=$ tsaccel(tsobj, nTimes, datatype) calculates the acceleration of the data series in the financial time series object tsobj, essentially the difference of the current momentum with the momentum some number of periods ago. Each data series in tsobj is treated individually. accts is a financial time series object with similar dates and data series names as tsobj.

Note, to compute a quantity over $n$ periods, you must specify $n+1$ for $n$ Times. If you specify nTimes $=0$, the function returns your original time series.

## Examples Compute the acceleration for Disney stock and plot the results:

```
        load disney.mat
        dis = rmfield(dis,'VOLUME') % remove VOLUME field
        dis_Accel = tsaccel(dis);
        plot(dis_Accel)
        title('Acceleration for Disney')
```



## References

See Also

Kaufman, P. J., The New Commodity Trading Systems and Methods, New York: John Wiley \& Sons, 1987.

## Purpose Momentum between times

Syntax mom = tsmom(data, nTimes)
momts = tsmom(tsobj, nTimes)

## Arguments

| data | Data series. Column-oriented vector or matrix. |
| :--- | :--- |
| nTimes | (Optional) Number of times. Default $=12$. |
| tsobj | Financial time series object. |

## Description

Examples Compute the momentum for Disney stock and plot the results:

```
load disney.mat
dis = rmfield(dis,'VOLUME') % remove VOLUME field
dis_Mom = tsmom(dis);
plot(dis_Mom)
title('Momentum for Disney')
```



See Also
tsaccel

## Purpose

Moving average

```
output = tsmovavg(tsobj, 's', lag)
(Simple)
output = tsmovavg(vector, 's', lag, dim)
output = tsmovavg(tsobj, 'e', timeperiod) (Exponential)
output = tsmovavg(vector, 'e', timeperiod, dim)
output = tsmovavg(tsobj, 't', numperiod) (Triangular)
output = tsmovavg(vector, 't', numperiod, dim)
output = tsmovavg(tsobj, 'w', weights) (Weighted)
output = tsmovavg(vector, 'w', weights, dim)
output = tsmovavg(tsobj, 'm', numperiod) (Modified)
output = tsmovavg(vector, 'm', numperiod, dim)
```


## Arguments

| tsobj | Financial time series object. |
| :--- | :--- |
| lag | Number of previous data points. |
| vector | Row vector or row-oriented matrix. Each row is a <br> set of observations. |
| dim | (Optional) Specifies dimension when input is a <br> vector or matrix. Default = 2 (row-oriented matrix: <br> each row is a variable, and each column is an <br> observation). If dim = 1, input is assumed to be <br> a column vector or column-oriented matrix (each <br> column is a variable and each row an observation). <br> output is identical in format to input. |
| timeperiod | Length of time period. |
| numperiod | Number of periods considered. <br> weights |

output = tsmovavg(tsobj, 's', lag) and output = tsmovavg(vector, 's', lag, dim) compute the simple moving average. lag indicates the number of previous data points used with the current data point when calculating the moving average.
output = tsmovavg(tsobj, 'e', timeperiod) and output = tsmovavg(vector, 'e', timeperiod, dim) compute the exponential weighted moving average. The exponential moving average is a weighted moving average, where timeperiod specifies the time period. Exponential moving averages reduce the lag by applying more weight to recent prices. For example, a 10 -period exponential moving average weights the most recent price by $18.18 \%$. (2/(timeperiod + 1)).
output = tsmovavg(tsobj, 't', numperiod) and output = tsmovavg(vector, 't', numperiod, dim) compute the triangular moving average. The triangular moving average double-smooths the data. tsmovavg calculates the first simple moving average with window width of ceil(numperiod + 1)/2. Then it calculates a second simple moving average on the first moving average with the same window size.
output = tsmovavg(tsobj, 'w', weights) and output = tsmovavg(vector, 'w', weights, dim) calculate the weighted moving average by supplying weights for each element in the moving window. The length of the weight vector determines the size of the window. If larger weight factors are used for more recent prices and smaller factors for previous prices, the trend is more responsive to recent changes.
output = tsmovavg(tsobj, 'm', numperiod) and output = tsmovavg(vector, 'm', numperiod, dim) calculate the modified moving average. The modified moving average is similar to the simple moving average. Consider the argument numperiod to be the lag of the simple moving average. The first modified moving average is calculated like a simple moving average. Subsequent values are calculated by adding the new price and subtracting the last average from the resulting sum.

# References Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 184-192. 

See Also mean | peravg

## typprice

## Purpose Typical price

```
Syntax tprc = typprice(highp, lowp, closep)
tprc = typprice([highp lowp closep])
tprcts = typprice(tsobj)
tprcts = typprice(tsobj, ParameterName, ParameterValue, ...)
```


## Arguments

| highp | High price (vector). |
| :--- | :--- |
| lowp | Low price (vector). |
| closep | Closing price (vector). |
| tsobj | Financial time series object. |

## Description

tprc = typprice(highp, lowp, closep) calculates the typical prices tprc from the high (highp), low (lowp), and closing (closep) prices. The typical price is the average of the high, low, and closing prices for each period.
tprc = typprice([highp lowp closep]) accepts a three-column matrix as the input rather than two individual vectors. The columns of the matrix represent the high, low, and closing prices, in that order.
tprcts = typprice(tsobj) calculates the typical prices from the stock data contained in the financial time series object tsobj. The object must contain, at least, the High, Low, and Close data series. The typical price is the average of the closing price plus the high and low prices. tprcts is a financial time series object of the same dates as tsobj containing the data series TypPrice.
tprcts = typprice(tsobj, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- HighName: high prices series name
- LowName: low prices series name
- CloseName: closing prices series name

Parameter values are the strings that represent the valid parameter names.

## Examples

Compute the typical price for Disney stock and plot the results:

```
load disney.mat
dis_Typ = typprice(dis);
plot(dis_Typ)
title('Typical Price for Disney')
```



## typprice

References Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 291-292.

## See Also medprice | wclose

# Purpose Univariate GARCH(P,Q) parameter estimation with Gaussian innovations <br> ```Syntax \\ [Kappa, Alpha, Beta] = ugarch(U, P, Q)``` 

## Arguments

## Description

[Kappa, Alpha, Beta] = ugarch(U, P, Q) computes estimated univariate $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ parameters with Gaussian innovations.

Kappa is the estimated scalar constant term ([[KAPPA]]) of the GARCH process.

Alpha is a P-by- 1 vector of estimated coefficients, where $P$ is the number of lags of the conditional variance included in the GARCH process.

Beta is a $Q$-by- 1 vector of estimated coefficients, where $Q$ is the number of lags of the squared innovations included in the GARCH process.

The time-conditional variance, $\sigma_{t}^{2}$, of a $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ process is modeled as

$$
\sigma_{t}^{2}=K+\sum_{i=1}^{P} \alpha_{i} \sigma_{t-i}^{2}+\sum_{j=1}^{Q} \beta_{j} \varepsilon_{t-j}^{2}
$$

where $\alpha$ represents the argument Alpha, $\beta$ represents Beta, and the $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ coefficients $\{K, a, \beta\}$ are subject to the following constraints.

$$
\begin{aligned}
& \sum_{i=1}^{P} \alpha_{i}+\sum_{j=1}^{Q} \beta_{j}<1 \\
& K>0 \\
& \alpha_{i} \geq 0 \quad i=1,2, \ldots, P \\
& \beta_{j} \geq 0 \quad j=1,2, \ldots, Q .
\end{aligned}
$$

Note that $U$ is a vector of residuals or innovations $\left(\varepsilon_{t}\right)$ of an econometric model, representing a mean-zero, discrete-time stochastic process.

Although $\sigma_{t}^{2}$ is generated using the equation above, $\varepsilon_{t}$ and $\sigma_{t}^{2}$ are related as

$$
\varepsilon_{t}=\sigma_{t} v_{t}
$$

where $\left\{v_{t}\right\}$ is an independent, identically distributed (iid) sequence $\sim$ $\mathrm{N}(0,1)$.

# Note ugarch corresponds generally to the Econometrics Toolbox function garchfit. The Econometrics Toolbox software provides a comprehensive and integrated computing environment for the analysis of volatility in time series. For information, see the Econometrics Toolbox User's Guide documentation or the financial products Web page at http://www.mathworks.com/products/finprod/. 

Examples See ugarchsim for an example of a $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ process.<br>References James D. Hamilton, Time Series Analysis, Princeton University Press, 1994

See Also<br>ugarchpred | ugarchsim | garchfit

## ugarchllf

Purpose Log-likelihood objective function of univariate $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ processes with Gaussian innovations

Syntax<br>LogLikelihood = ugarchllf(Parameters, U, P, Q)

## Arguments

| Parameters | $(1+P+Q)$-by-1 column vector of GARCH(P,Q) <br> process parameters. The first element is the scalar <br> constant term [[KAPPA]] of the GARCH process; the |
| :--- | :--- |
| next $P$ elements are coefficients associated with the |  |
| P lags of the conditional variance terms; the next $Q$ |  |
| elements are coefficients associated with the $Q$ lags |  |
| of the squared innovations terms. |  |

Note The latest value of residuals is the last element of vector $U$.

P Nonnegative, scalar integer representing a model order of the GARCH process. $P$ is the number of lags of the conditional variance. $P$ can be zero; when $P=$ 0 , a $\operatorname{GARCH}(0, Q)$ process is actually an $\operatorname{ARCH}(\mathrm{Q})$ process.

Q Positive, scalar integer representing a model order of the GARCH process. $Q$ is the number of lags of the squared innovations.

## Description

LogLikelihood = ugarchllf(Parameters, U, P, Q) computes the log-likelihood objective function of univariate $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ processes with Gaussian innovations.

LogLikelinood is a scalar value of the $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ log-likelihood objective function given the input arguments. This function is meant to be optimized via the fmincon function of the Optimization Toolbox software.
fmincon is a minimization routine. To maximize the log-likelihood function, the LogLikelihood output parameter is actually the negative of what is formally presented in most time series or econometrics references.

The time-conditional variance, $\sigma_{t}^{2}$, of a $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ process is modeled as

$$
\sigma_{t}^{2}=K+\sum_{i=1}^{P} \alpha_{i} \sigma_{t-i}^{2}+\sum_{j=1}^{Q} \beta_{j} \varepsilon_{t-j}^{2}
$$

where $\alpha$ represents the argument Alpha, and $\beta$ represents Beta. U is a vector of residuals or innovations $\left(\varepsilon_{t}\right)$ representing a mean-zero, discrete time stochastic process. Although $\sigma_{t}^{2}$ is generated via the equation above, $\varepsilon_{t}$ and $\sigma_{t}^{2}$ are related as

$$
\varepsilon_{t}=\sigma_{t} v_{t}
$$

where $\left\{v_{t}\right\}$ is an independent, identically distributed (iid) sequence $\sim$ $\mathrm{N}(0,1)$.

Since ugarchllf is really just a helper function, no argument checking is performed. This function is not meant to be called directly from the command line.

> Note The Econometrics Toolbox software provides a comprehensive and integrated computing environment for the analysis of volatility in time series. For information, see the Econometrics Toolbox User's Guide documentation or the financial products Web page at http://www.mathworks.com/products/finprod/.

See Also
ugarch | ugarchpred | ugarchsim

## Purpose Forecast conditional variance of univariate GARCH $(\mathrm{P}, \mathrm{Q})$ processes <br> Syntax [VarianceForecast, H] = ugarchpred(U, Kappa, Alpha, Beta, NumPeriods)

## Arguments

## ugarchpred

Description
[VarianceForecast, H] = ugarchpred(U, Kappa, Alpha, Beta, NumPeriods) forecasts the conditional variance of univariate GARCH(P,Q) processes.

VarianceForecast is a number of periods (NUMPERIODS)-by-1 vector of the minimum mean-square error forecast of the conditional variance of the innovations time series vector $U$ (that is, $\varepsilon_{\mathrm{t}}$ ). The first element contains the 1-period-ahead forecast, the second element contains the 2 -period-ahead forecast, and so on. Thus, if a forecast horizon greater than 1 is specified (NUMPERIODS > 1), the forecasts of all intermediate horizons are returned as well. In this case, the last element contains the variance forecast of the specified horizon, NumPeriods from the most recent observation in $U$.

H is a vector of the conditional variances $\left(\sigma_{\mathrm{t}}{ }^{2}\right)$ corresponding to the innovations vector $U$. It is inferred from the innovations $U$, and is a reconstruction of the "past" conditional variances, whereas the VarianceForecast output represents the projection of conditional variances into the "future." This sequence is based on setting pre-sample values of $\sigma_{t}^{2}$ to the unconditional variance of the $\left\{\varepsilon_{t}\right\}$ process. H is a single column vector of the same length as the input innovations vector $U$.

The time-conditional variance, $\sigma_{t}^{2}$, of a $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ process is modeled as

$$
\sigma_{t}^{2}=K+\sum_{i=1}^{P} \alpha_{i} \sigma_{t-i}^{2}+\sum_{j=1}^{Q} \beta_{j} \varepsilon_{t-j}^{2},
$$

where $\alpha$ represents the argument Alpha, $\beta$ represents Beta, and the $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ coefficients $\{K, \alpha, \beta\}$ are subject to the following constraints.

$$
\begin{aligned}
& \sum_{i=1}^{P} \alpha_{i}+\sum_{j=1}^{Q} \beta_{j}<1 \\
& K>0 \\
& \alpha_{i} \geq 0 \quad i=1,2, \ldots, P \\
& \beta_{j} \geq 0 \quad j=1,2, \ldots, Q .
\end{aligned}
$$

Note that $U$ is a vector of residuals or innovations $\left(\varepsilon_{t}\right)$ of an econometric model, representing a mean-zero, discrete-time stochastic process.

Although $\sigma_{t}^{2}$ is generated using the equation above, $\varepsilon_{t}$ and $\sigma_{t}^{2}$ are related as

$$
\varepsilon_{t}=\sigma_{t} v_{t}
$$

where $\left\{v_{t}\right\}$ is an independent, identically distributed (iid) sequence $\sim$ $\mathrm{N}(0,1)$.

Note ugarchpred corresponds generally to the Econometrics Toolbox function garchpred. The Econometrics Toolbox software provides a comprehensive and integrated computing environment for the analysis of volatility in time series. For information, see the Econometrics Toolbox User's Guide documentation or the financial products Web page at http://www.mathworks.com/products/finprod/.

## Examples See ugarchsim for an example of forecasting the conditional variance of a univariate GARCH $(\mathrm{P}, \mathrm{Q})$ process.

See Also ugarch | ugarchsim | garchpred

Purpose
Syntax

Arguments

| Kappa | Scalar constant term [[KAPPA]] of the GARCH <br> process. |
| :--- | :--- |
| Alpha | P-by-1 vector of coefficients, where $P$ is the number <br> of lags of the conditional variance included in the <br> GARCH process. Alpha can be an empty matrix, <br> in which case $P$ is assumed 0; when $P=0, ~ a$ <br> GARCH $(0, Q)$ process is actually an ARCH(Q) <br> process. |
| Beta | Q-by-1 vector of coefficients, where $Q$ is the number <br> of lags of the squared innovations included in the <br> GARCH process. |
| NumSamples | Positive, scalar integer indicating the number <br> of samples of the innovations $U$ and conditional <br> variance $H$ (see below) to simulate. |

## Description

Simulate univariate $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ process with Gaussian innovations
[U, H] = ugarchsim(Kappa, Alpha, Beta, NumSamples)

Beta $Q$-by- 1 vector of coefficients, where $Q$ is the number of lags of the squared innovations included in the GARCH process.
NumSamples Positive, scalar integer indicating the number of samples of the innovations $U$ and conditional variance H (see below) to simulate.
[U, H] = ugarchsim(Kappa, Alpha, Beta, NumSamples) simulates a univariate GARCH $(\mathrm{P}, \mathrm{Q})$ process with Gaussian innovations.

U is a number of samples (NUMSAMPLES)-by-1 vector of innovations $\left(\varepsilon_{t}\right)$, representing a mean-zero, discrete-time stochastic process. The innovations time series $U$ is designed to follow the $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ process specified by the inputs Kappa, Alpha, and Beta.

H is a NUMSAMPLES-by- 1 vector of the conditional variances $\left(\sigma_{t}^{2}\right)$ corresponding to the innovations vector $U$. Note that $U$ and $H$ are the same length, and form a "matching" pair of vectors. As shown in
the following equation, $\sigma_{t}^{2}$ (that is, $\left.H(t)\right)$ represents the time series inferred from the innovations time series $\left\{\varepsilon_{t}\right\}$ (that is, $U$ ).

The time-conditional variance, $\sigma_{t}^{2}$, of a $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ process is modeled as

$$
\sigma_{t}^{2}=K+\sum_{i=1}^{P} \alpha_{i} \sigma_{t-i}^{2}+\sum_{j=1}^{Q} \beta_{j} \varepsilon_{t-j}^{2}
$$

where a represents the argument Alpha, $B$ represents Beta, and the $\operatorname{GARCH}(\mathrm{P}, \mathrm{Q})$ coefficients $\{K, \alpha, \beta\}$ are subject to the following constraints.

$$
\begin{aligned}
& \sum_{i=1}^{P} \alpha_{i}+\sum_{j=1}^{Q} \beta_{j}<1 \\
& K>0 \\
& \alpha_{i} \geq 0 \quad i=1,2, \ldots, P \\
& \beta_{j} \geq 0 \quad j=1,2, \ldots, Q
\end{aligned}
$$

Note that $U$ is a vector of residuals or innovations $\left(\varepsilon_{t}\right)$ of an econometric model, representing a mean-zero, discrete-time stochastic process.

Although $\sigma_{t}^{2}$ is generated using the equation above, $\varepsilon_{t}$ and $\sigma_{t}^{2}$ are related as

$$
\varepsilon_{t}=\sigma_{t} v_{t}
$$

where $\left\{v_{t}\right\}$ is an independent, identically distributed (iid) sequence $\sim$ $\mathrm{N}(0,1)$.

The output vectors $U$ and $H$ are designed to be steady-state sequences in which transients have arbitrarily small effect. The (arbitrary) metric used by ugarchsim strips the first $N$ samples of $U$ and $H$ such that the

## ugarchsim

sum of the GARCH coefficients, excluding Kappa, raised to the Nth power, does not exceed 0.01 .

$$
0.01=\left(\text { sum (Alpha) }+\operatorname{sum}(\text { Beta) })^{\wedge} N\right.
$$

Thus

$$
N=\log (0.01) / \log ((\operatorname{sum}(A l p h a)+\operatorname{sum}(\text { Beta })))
$$

Note ugarchsim corresponds generally to the Econometrics Toolbox function garchsim. The Econometrics Toolbox software provides a comprehensive and integrated computing environment for the analysis of volatility in time series. For information see the Econometrics Toolbox User's Guide documentation or the financial products Web page at http://www.mathworks.com/products/finprod/.

## Examples

This example simulates a $\operatorname{GARCH}(P, Q)$ process with $P=2$ and $Q=1$.

```
% Set the random number generator seed for reproducability.
randn('seed', 10)
% Set the simulation parameters of GARCH(P,Q) = GARCH(2,1) process.
Kappa = 0.25; %a positive scalar.
Alpha = [0.2 0.1]'; %a column vector of nonnegative numbers (P = 2).
Beta = 0.4; % Q = 1.
NumSamples = 500; % number of samples to simulate.
% Now simulate the process.
[U , H] = ugarchsim(Kappa, Alpha, Beta, NumSamples);
% Estimate the process parameters.
```

```
P = 2; % Model order P (P = length of Alpha).
Q = 1; % Model order Q (Q = length of Beta).
[k, a, b] = ugarch(U , P , Q);
disp(' ')
disp(' Estimated Coefficients:')
disp(' ----------------------')
disp([k; a; b])
disp(' ')
% Forecast the conditional variance using the estimated
%coefficients.
NumPeriods = 10; % Forecast out to 10 periods.
[VarianceForecast, H1] = ugarchpred(U, k, a, b, NumPeriods);
disp(' Variance Forecasts:')
disp(' ----------------'')
disp(VarianceForecast)
disp(' ')
```

When the above code is executed, the screen output looks like the display shown.
$\% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \%$
Diagnostic Information

Number of variables: 4

Functions

| Objective: | ugarchllf |
| :--- | :--- |
| Gradient: | finite-differencing |
| Hessian: | finite-differencing (or Quasi-Newton) |

Constraints
Nonlinear constraints: do not exist
Number of linear inequality constraints: 1
Number of linear equality constraints: 0
Number of lower bound constraints: 4

Number of upper bound constraints:
Algorithm selected
medium-scale

## $\% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \% \%$

End diagnostic information

|  |  |  |  | max | Directional |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Iter | F-count | $f(x)$ | constraint | Step-size | derivative | Procedure |
| 1 | 5 | 699.185 | -0.125 | 1 | $-2.97 e+006$ |  |
| 2 | 22 | 658.224 | -0.1249 | 0.000488 | -64.6 |  |
| 3 | 28 | 610.181 | 0 | 1 | -49.4 |  |
| 4 | 35 | 590.888 | 0 | 0.5 | -38.9 |  |
| 5 | 42 | 583.961 | -0.03317 | 0.5 | -29.8 |  |
| 6 | 49 | 583.224 | -0.02756 | 0.5 | -31.8 |  |
| 7 | 57 | 582.947 | -0.02067 | 0.25 | -7.28 |  |
| 8 | 63 | 578.182 | 0 | 1 | -2.43 |  |
| 9 | 71 | 578.138 | -0.09145 | 0.25 | -0.55 |  |
| 10 | 77 | 577.898 | -0.04452 | 1 | -0.148 |  |
| 11 | 84 | 577.882 | -0.06128 | 0.5 | -0.0488 |  |
| 12 | 90 | 577.859 | -0.07117 | 1 | -0.000758 |  |
| 13 | 96 | 577.858 | -0.07033 | 1 | -0.000305 | Hessian modified |
| 14 | 102 | 577.858 | -0.07042 | 1 | -3.32e-005 | Hessian modified |
| 15 | 108 | 577.858 | -0.0707 | 1 | -1.29e-006 | Hessian modified |
| 16 | 114 | 577.858 | -0.07077 | 1 | -1.29e-007 | Hessian modified |
| 17 | 120 | 577.858 | -0.07081 | 1 | -1.97e-007 | Hessian modified |

[^11]0.4000

Variance Forecasts:
1.3243
0.9594
0.9186
0.8402
0.7966
0.7634
0.7407
0.7246
0.7133
0.7054

References<br>James D. Hamilton, Time Series Analysis, Princeton University Press, 1994

See Also ugarch | ugarchpred | garchsim

## uicalendar

## Purpose Graphical calendar

```
Syntax uicalendar('PARAM1', VALUE1, 'PARAM2', VALUE2', ...)
```

Arguments
'BusDays ' Values are:

- 0 - (Default) Standard calendar without nonbusiness day indicators.
- 1 - Marks NYSE nonbusiness days in red.
'BusDaySelect' Values are:
- 0 - Only allow selection of business days. Nonbusiness days are determined from the following parameters:
- 'BusDays'
- 'Holiday'
- 'Weekend'
- 1 - (Default) Allows selections of business and nonbusiness days.
'DateBoxColor' [date R G B] : Sets the color of the date squares to the specified [R G B] color.
'DateStrColor' [date R G B] : Sets the color of the numeric date number in the date square to the specified $[R G$ B] color.


## 'DestinationUI' Values are:

- H - Scalar or vector of the destination object's handles. The default UI property that is populated with the date(s) is 'string'.
- $\{\mathrm{H},\{\operatorname{Prop}\}\}$ - Cell array of handles and the destination object's UI properties. H must be a scalar or vector and Prop must be a single property string or a cell array of property strings.
'Holiday ' Sets the specified holiday dates into the calendar. The corresponding date string of the holiday will appear Red. The Date(s) must be a scalar or vector of datenums.
'InitDate ' Values are:
- Datenum - Numeric date value specifying the initial start date when the calendar is initialized. The default date is TODAY.
- Datestr - Date string value specifying the initial start date when the calendar is initialized. Datestr must include a Year, Month, and Day (for example, 01-Jan-2006).
'InputDateFormat ' Format - Sets the format of initial start date, InitDate. See 'help datestr' for date format values.
'OutputDateFormatFormat - Sets the format of output date string. See 'help datestr' for date format values.


## uicalendar

'OutputDateStyle ' Values are:

- 0 - (Default) Returns a single date string or a cell array (row) of date string. For example, \{'01-Jan-2001, 02-Jan-2001, ...'\}.
- 1 - Returns a single date string or a cell (column) array of date strings. For example, \{'01-Jan-2001; 02-Jan-2001; ...'\}.
- 2 - Returns a string representation of a row vector of datenums. For example, ' [732758, 732759, 732760, 732761]'.
- 3 - Returns a string representation of a column vector of datenums. For example, '[732758; 732759; 732760; 732761]'.
'SelectionType' Values are:
- 0 - (Default) Allows multiple date selections.
- 1 - Allows only a single date selection.
'Weekend ' DayOfWeek - Sets the specified days of the week as weekend days. Weekend days are marked in red. DayOfWeek can be a vector containing the following numeric values:
- 1 - Sunday
- 2 - Monday
- 3 - Tuesday
- 4 - Wednesday
- 5 - Thursday
- 6 - Friday

| $\bullet$ | 7 - Saturday |
| :--- | :--- |
|  | Also this value can be a vector of length 7 <br> containing 0 's and 1's. The value 1 indicates a <br> weekend day. The first element of this vector <br> corresponds to Sunday. For example, when |
|  | $\left.\begin{array}{l}\text { Saturday and Sunday are weekend days then } \\ \text { WEEKEND }=\left[\begin{array}{lllll}1 & 0 & 0 & 0 & 0\end{array} 0\right.\end{array}\right]$. |
| 'WindowStyle' | Values are: |

- Normal - (Default) Standard figure properties.
- Modal - Modal figures remain stacked above all normal figures and the MATLAB Command Window.

[^12]
## uicalendar

| - ${ }^{\text {UI }}$ | Cale | ndar |  |  |  | - $x$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 20 | 07 | $\checkmark$ | - |
| Su | Mo | Tu | ave | Th | Fr | Sa |
| 29 | 30 | 1 | 2 | 3 | 4 | 5 |
| 6 | 7 | $\bigcirc$ | 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 27 | 28 | 29 | 30 | 31 |  | 2 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Clear all |  |  |  | OK |  | ncel |

For more information on using uicalendar with an application, see "Example of Using UICalendar with an Application" on page 13-5.

## See Also <br> holidays

Purpose Unary minus of financial time series object
Syntax uminus
Description uminus implements unary minus for a financial time series object.
See Also ..... uplus

Purpose Unary plus of financial time series object
Syntax uplus

Description
uplus implements unary plus for a financial time series object.
See Also uminus

## Purpose Variance

Syntax $\quad y=\operatorname{var}(X)$
$\mathrm{y}=\operatorname{var}(\mathrm{X}, 1)$
$\mathrm{y}=\operatorname{var}(\mathrm{X}, \mathrm{W})$
$\mathrm{y}=\operatorname{var}(\mathrm{X}, \mathrm{W}, \mathrm{DIM})$

## Arguments

| X | Financial times series object. |
| :--- | :--- |
| W | Weight vector used in calculating variance. |
| DIM | Dimension of X used in calculating variance. |

## Description

var supports financial time series objects based on the MATLAB var function. See var in the MATLAB documentation.
$y=\operatorname{var}(X)$, if $X$ is a financial time series object and returns the variance of each series.
var normalizes y by $\mathrm{N}-1$ if $\mathrm{N}>1$, where N is the sample size. This is an unbiased estimator of the variance of the population from which $X$ is drawn, as long as X consists of independent, identically distributed samples. For $N=1$, $y$ is normalized by $N$.
$y=\operatorname{var}(X, 1)$ normalizes by $N$ and produces the second moment of the sample about its mean. $\operatorname{var}(X, 0)$ is the same as $\operatorname{var}(X)$.
$y=\operatorname{var}(X, W)$ computes the variance using the weight vector $W$. The length of $W$ must equal the length of the dimension over which var operates, and its elements must be nonnegative. var normalizes $W$ to sum to 1 . Use a value of 0 for $W$ to use the default normalization by $N$ -1 , or use a value of 1 to use $N$.
$y=\operatorname{var}(X, W$, DIM $)$ takes the variance along the dimension DIM of $x$.
Examples The variance is the square of the standard deviation. Consider if$\mathrm{f}=\mathrm{fints}(($ today:today+1)', $[4-21 ; 9$ 7])then
$\operatorname{var}(f, 0,1)$
is
$\left[\begin{array}{lll}12.5 & 24.5 & 18.0\end{array}\right]$
and
$\operatorname{var}(f, 0,2)$
is
[9.0; 4.0]
See Also corrcoef | cov | mean | std

## Purpose <br> Description

## Examples

Create two financial time series objects with daily frequencies:

```
myfts = fints((today:today+4)', (1:5)', 'DataSeries', 'd');
yourfts = fints((today+5:today+9)', (11:15)', 'DataSeries', 'd');
```

Use vertcat to concatenate them vertically:

```
newfts1 = [myfts; yourfts]
newfts1 =
    desc: ||
    freq: Daily (1)
    'dates: (10)' 'DataSeries: (10)'
    '11-Dec-2001' [ 1]
    '12-Dec-2001' [ 2]
    '13-Dec-2001' [ 3]
    '14-Dec-2001' [ 4]
    '15-Dec-2001' [ 5]
    '16-Dec-2001' [ 11]
    '17-Dec-2001' [ 12]
    '18-Dec-2001' [ 13]
    '19-Dec-2001' [ 14]
    '20-Dec-2001' [ 15]
```

Create two financial time series objects with different frequencies:

```
myfts = fints((today:today+4)', (1:5)', 'DataSeries', 'd');
```

```
hisfts = fints((today+5:7:today+34)', (11:15)', 'DataSeries',...
'w');
```

Concatenate these two objects vertically:

```
newfts2 = [myfts; hisfts]
newfts2 =
    desc: ||
    freq: Unknown (0)
    'dates: (10)' 'DataSeries: (10)'
    '11-Dec-2001' [ 1]
    '12-Dec-2001' [ 2]
    '13-Dec-2001' [ 3]
    '14-Dec-2001' [ 4]
    '15-Dec-2001' [ 5]
    '16-Dec-2001' [ 11]
    '23-Dec-2001' [ 12]
    '30-Dec-2001' [ 13]
    '06-Jan-2002' [ 14]
    '13-Jan-2002' [ 15]
```

If all frequency indicators are the same, the new object has the same frequency indicator. However, if one of the concatenated objects has a different freq from the other(s), the frequency of the resulting object is set to Unknown (0). In these examples, newfts1 has Daily frequency, while newfts2 has Unknown (0) frequency.

## See Also <br> horzcat

## Purpose Price and volume chart <br> Syntax volarea(X) <br> Arguments

## Description

## Examples

M-by-3 matrix where the first column contains date numbers, the second column is the asset price, and the third column is the volume.
volarea( X ) plots asset date, price, and volume on a single axis.

If asset $X$ is an $M$-by- 3 matrix of date numbers, asset price, and volume:

$$
x=[\ldots
$$

| 733299.00 | 41.99 | $15045445.00 ; \ldots$ |
| ---: | ---: | ---: |
| 733300.00 | 42.14 | $15346658.00 ; \ldots$ |
| 733303.00 | 41.93 | $9034397.00 ; \ldots$ |
| 733304.00 | 41.98 | $14486275.00 ; \ldots$ |
| 733305.00 | 41.75 | $16389872.00 ; \ldots$ |
| 733306.00 | 41.61 | $20475208.00 ; \ldots$ |
| 733307.00 | 42.29 | $14833200.00 ; \ldots$ |
| 733310.00 | 42.19 | $18945176.00 ; \ldots$ |
| 733311.00 | 41.82 | $25188101.00 ; \ldots$ |
| 733312.00 | 41.93 | $22689878.00 ; \ldots$ |
| 733313.00 | 41.81 | $21084723.00 ; \ldots$ |
| 733314.00 | 41.37 | $27963619.00 ; \ldots$ |
| 733317.00 | 41.17 | $20385033.00 ; \ldots$ |
| 733318.00 | 42.02 | $27783775.00]$ |

then the price volume chart is

```
volarea(X)
```

which plots the asset prices with respect to date and volume as follows.

## volarea



See Also
bolling | candle | highlow | kagi | linebreak | movavg | pointfig | priceandvol | renko

## Purpose Volume rate of change

Syntax

```
vroc = volroc(tvolume nTimes)
vrocts = volroc(tsobj, nTimes)
vrocts = volroc(tsobj, nTimes, ParameterName, ParameterValue)
```


## Arguments

| tvolume | Volume traded. |
| :--- | :--- |
| nTimes | (Optional) Time difference. Default $=12$. |
| tsobj | Financial time series object. |

Description
vroc $=$ volroc(tvolume nTimes) calculates the volume rate of change, vroc, from the volume traded data tvolume. If nTimes is specified, the volume rate of change is calculated between the current volume and the volume nTimes ago.
vrocts $=$ volroc(tsobj, nTimes) calculates the volume rate of change, vrocts, from the financial time series object tsobj. The vrocts output is a financial time series object with similar dates as tsobj and a data series named VolumeROC. If nTimes is specified, the volume rate of change is calculated between the current volume and the volume nTimes ago.
vrocts = volroc(tsobj, nTimes, ParameterName, ParameterValue) specifies the name for the required data series when it is different from the default name. The valid parameter name is

- VolumeName: volume traded series name

The parameter value is a string that represents the valid parameter name.

Note, to compute a quantity over $n$ periods, you must specify $n+1$ for $n$ Times. If you specify $n T i m e s=0$, the function returns your original time series.

## Examples <br> Compute the volume rate of change for Disney stock and plot the results:

```
load disney.mat
dis_VolRoc = volroc(dis)
plot(dis_VolRoc)
title('Volume Rate of Change for Disney')
```



## References

Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 310-311.

## See Also

proroc

## Purpose Weighted close

Syntax

```
wcls = wclose(highp, lowp, closep)
wcls = wclose([highp lowp closep])
wclsts = wclose(tsobj)
wclsts = wclose(tsobj, ParameterName, ParameterValue, ...)
```


## Arguments

| highp | High price (vector). |
| :--- | :--- |
| lowp | Low price (vector). |
| closep | Closing price (vector). |
| tsobj | Financial time series object. |

## Description

The weighted close price is the average of twice the closing price plus the high and low prices.
wcls = wclose(highp, lowp, closep) calculates the weighted close prices wcls based on the high (highp), low (lowp), and closing (closep) prices per period.
wcls = wclose([highp lowp closep]) accepts a three-column matrix consisting of the high, low, and closing prices, in that order.
wclsts = wclose(tsobj) computes the weighted close prices for a set of stock price data contained in the financial time series object tsobj. The object must contain the high, low, and closing prices needed for this function. The function assumes that the series are named High, Low, and Close. All three are required. wclsts is a financial time series object of the same dates as tsobj and contains the data series named WClose.
wclsts = wclose(tsobj, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs
specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- HighName: high prices series name
- LowName: low prices series name
- CloseName: closing prices series name

Parameter values are the strings that represent the valid parameter names.

## Examples

Compute the weighted closing prices for Disney stock and plot the results:

```
load disney.mat
dis_Wclose = wclose(dis)
plot(dis_Wclose)
title('Weighted Closing Prices for Disney')
```



## References

Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 312-313.

## See Also

medprice | typprice

Purpose Day of week
Syntax
[ $N, S]=$ weekday $(D)$
[N, S] = weekday(D, form)
[N, S] = weekday(D, locale)
[N, S] = weekday(D, form, locale)

## Description

[ $\mathrm{N}, \mathrm{S}$ ] = weekday (D) returns the day of the week in numeric ( N ) and string (S) form for a given serial date number or date string D. Input argument $D$ can represent more than one date in an array of serial date numbers or a cell array of date strings.
[ $N, S]$ = weekday (D, form) returns the day of the week in numeric ( N ) and string (S) form, where the content of $S$ depends on the form argument. If form is 'long', then $S$ contains the full name of the weekday (for example, Tuesday). If form is 'short', then $S$ contains an abbreviated name (for example, Tues) from this table.
The days of the week are assigned these numbers and abbreviations.

| $\mathbf{N}$ | S (short) | S (long) |
| :--- | :--- | :--- |
| 1 | Sun | Sunday |
| 2 | Mon | Monday |
| 3 | Tue | Tuesday |
| 4 | Wed | Wednesday |
| 5 | Thu | Thursday |
| 6 | Fri | Friday |
| 7 | Sat | Saturday |

[ $N, S$ ] = weekday ( D, locale) returns the day of the week in numeric (N) and string (S) form, where the format of the output depends on the locale argument. If locale is 'local', then weekday uses local format for its output. If locale is 'en_US', then weekday uses US English.
[ $\mathrm{N}, \mathrm{S}]=$ weekday (D, form, locale) returns the day of the week using the formats described above for form and locale.

## Examples Either

[n, s] = weekday(728647)
or
[ $\mathrm{n}, \mathrm{s}$ ] = weekday('19-Dec-1994')
returns $\mathrm{n}=2$ and $\mathrm{s}=$ Mon.
See Also datenum | datestr | datevec | day

## Purpose Week in a year

## Syntax <br> [ N$]=$ weeknum( D ) <br> [N] = weeknum(D,W)

## Arguments

## Description

[ $N$ ] = weeknum(D) returns the week of the year given $D$, a serial date number or a date string.
[ $N$ ] = weeknum( $D, W$ ) returns the week of the year given $D$, a serial date number or a date string, and W , a numeric representation of the day a week begins.

The weeknum function considers the week containing January 1 to be the first week of the year.

## Examples <br> You can determine the week of the year using a serial date number

$$
\mathrm{N}=\text { weeknum(728647) }
$$

## $\mathrm{N}=$

52
or a date string
$\mathrm{N}=$ weeknum('19-Dec-1994')
$\mathrm{N}=$

52
The first week of the year must have at least 4 days in it. For example, January 8, 2004 was a Thursday.

```
weeknum('08-jan-2004')
ans =
```


## 2

You can use weeknum with datenum:

```
weeknum(datenum('01-Jan-2004'):datenum('08-Jan-2004'))
ans =
```



The default start day of the week is Sunday. Every day after, and including the first Sunday of the year (04-Jan-2004), returns 2 denoting the second week. In this case, the first of week of the year started before January 1, 2004.

You can also use weeknum with datenum and specify a W value of 5 to indicate that the weeks start on Thursday:

```
weeknum(datenum('01-Jan-2004'):datenum('08-Jan-2004'),5 (
ans =
```

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

The first week of the year that has 4 or more days, based on the specified start day, is considered week one (even if this isn't the first week in the calendar). Any day falling in (or before) this week is given a week number of 1 .

## See Also

datenum | datestr | datevec | day

| Purpose | Portfolio values and weights into holdings |
| :--- | :--- |
| Syntax | Holdings $=$ weights2holdings (Values, Weights, Prices) |

Arguments

| Values | Scalar or number of portfolios (NPORTS) vector <br> containing portfolio values. |
| :--- | :--- |
| Weights | NPORTS by number of assets (NASSETS) matrix with <br> portfolio weights. The weights sum to the value <br> of a Budget constraint, which is usually 1. (See <br> holdings2weights for information about budget <br> constraints.) |
| Prices | NASSETS vector of prices. |

## Description

Holdings = weights2holdings(Values, Weights, Prices) converts portfolio values and weights into portfolio holdings.

Holdings is a NPORTS-by-NASSETS matrix containing the holdings of NPORTS portfolios that contain NASSETS assets.

Note This function does not create round-lot positions. Holdings are floating-point values.

See Also
holdings2weights

Purpose Williams Accumulation/Distribution line

```
Syntax wadl = willad(highp, lowp, closep)
wadl = willad([highp lowp closep])
wadlts = willad(tsobj)
wadlts = willad(tsobj, ParameterName, ParameterValue, ...)
```


## Arguments

| highp | High price (vector) |
| :--- | :--- |
| lowp | Low price (vector) |
| closep | Closing price (vector) |
| tsobj | Time series object |

## Description

wadl = willad(highp, lowp, closep) computes the Williams Accumulation/Distribution line for a set of stock price data. The prices needed for this function are the high (highp), low (lowp), and closing (closep) prices. All three are required.
wadl $=$ willad([highp lowp closep]) accepts a three-column matrix of prices as input. The first column contains the high prices, the second contains the low prices, and the third contains the closing prices.
wadlts = willad(tsobj) computes the Williams
Accumulation/Distribution line for a set of stock price data contained in the financial time series object tsobj. The object must contain the high, low, and closing prices needed for this function. The function assumes that the series are named High, Low, and Close. All three are required. wadlts is a financial time series object with the same dates as tsobj and a single data series named WillAD.
wadlts = willad(tsobj, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs
specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- HighName: high prices series name
- LowName: low prices series name
- CloseName: closing prices series name

Parameter values are the strings that represent the valid parameter names.

Examples Compute the Williams A/D line for Disney stock and plot the results:

```
load disney.mat
dis_Willad = willad(dis)
plot(dis_Willad)
title('Williams A/D Line for Disney')
```



## References <br> Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 314-315.

See Also<br>adline | adosc | willpctr

## Purpose

Williams \%R

## Syntax

```
wpctr = willpctr(highp, lowp, closep, nperiods)
wpctr = willpctr([highp, lowp, closep], nperiods)
wpctrts = willpctr(tsobj)
wpctrts = willpctr(tsobj, nperiods)
wpctrts = willpctr(tsobj, nperiods, ParameterName, ParameterValue,
... )
```


## Arguments

| highp | High price (vector) |
| :--- | :--- |
| lowp | Low price (vector) |
| closep | Closing price (vector) |
| nperiods | Number of periods (scalar). Default = 14. |
| tsobj | Financial time series object |

## Description

wpctr = willpctr(highp, lowp, closep, nperiods) calculates the Williams $\% \mathrm{R}$ values for the given set of stock prices for a specified number of periods nperiods. The stock prices needed are the high (highp), low (lowp), and closing (closep) prices. wpctr is a vector that represents the Williams \%R values from the stock data.
wpctr = willpctr([highp, lowp, closep], nperiods) accepts the price input as a three-column matrix representing the high, low, and closing prices, in that order.
wpctrts = willpctr(tsobj) calculates the Williams \%R values for the financial time series object tsobj. The object must contain at least three data series named High (high prices), Low (low prices), and Close (closing prices). wpctrts is a financial time series object with the same dates as tsobj and a single data series named WillPctR.
wpctrts = willpctr(tsobj, nperiods) calculates the Williams \%R values for the financial time series object tsobj for nperiods periods.
wpctrts = willpctr(tsobj, nperiods, ParameterName, ParameterValue, ...) accepts parameter name/parameter value pairs as input. These pairs specify the name(s) for the required data series if it is different from the expected default name(s). Valid parameter names are

- HighName: high prices series name
- LowName: low prices series name
- CloseName: closing prices series name

Parameter values are the strings that represent the valid parameter names.

## Examples Compute the Williams \%R values for Disney stock and plot the results:

```
load disney.mat
dis_Wpctr = willpctr(dis)
plot(dis_Wpctr)
title('Williams %R for Disney')
```



Achelis, Steven B., Technical Analysis from A to Z, Second printing, McGraw-Hill, 1995, pp. 316-317.

## See Also

## wrkdydif

Purpose Number of working days between dates

```
Syntax Days = wrkdydif(StartDate, EndDate, Holidays)
```


## Arguments

StartDate Enter as serial date numbers or date strings.
EndDate Enter as serial date numbers or date strings.
Holidays A vector containing values for the number of Holidays between the two dates.

## Description

Examples

```
Days = wrkdydif('9/1/2000', '9/11/2000', 1)
```

or
Days = wrkdydif(730730, 730740, 1)
returns
Days =

6

## See Also busdate | datewrkdy | days360 | days365 | daysact | daysdif |

 holidays | yearfrac> Purpose
> Excel serial date number to MATLAB serial date number
> Syntax MATLABDate = x2mdate(ExcelDateNumber, Convention)

## Arguments

| ExcelDateNumber | A vector or scalar of Excel serial date numbers. <br> (Optional) Excel date system. A vector or <br> scalar. When Convention = 0 (default), the <br> Excel 1900 date system is in effect. When <br> Convention = 1, the Excel 1904 date system <br> is used. |
| :--- | :--- |
|  | In the Excel 1900 date system, the Excel serial |
| date number 1 corresponds to January 1, |  |
|  | 1900 A.D. In the Excel 1904 date system, date |
|  | number 0 is January 1, 1904 A.D. |
|  | Due to a software limitation in Excel software, |
| the year 1900 is considered a leap year. As a |  |
| result, all DATEVALUEs reported by Excel |  |
| software between Jan. 1, 1900 and Feb. 28, |  |
|  | 1900 (inclusive) differ from the values reported |
| by 1. For example: |  |

- In Excel software, Jan. 1, $1900=1$
- In MATLAB, Jan. 1, $1900=2$

Vector arguments must have consistent dimensions.

## Description

DateNumber = x2mdate(ExcelDateNumber, Convention) converts Excel serial date numbers to MATLAB serial date numbers. MATLAB date numbers start with $1=$ January 1, 0000 A.D., hence there is a

## x2mdate

difference of 693960 relative to the 1900 date system, or 695422 relative to the 1904 date system. This function is useful with Spreadsheet Link EX software.

```
Examples Given Excel date numbers in the 1904 system
    ExDates = [[35423 35788 36153];
    convert them to MATLAB date numbers
    MATLABDate = x2mdate(ExDates, 1)
    MATLABDate =
        730845 731210 731575
        and then to date strings.
    datestr(MATLABDate)
    ans =
    25-Dec-2000
    25-Dec-2001
    25-Dec-2002
See Also datenum | datestr | m2xdate
```


## Purpose

Internal rate of return for nonperiodic cash flow
Syntax
Description
Return $=$ xirr(CashFlow, CashFlowDates)
Return $=$ xirr(CashFlow, CashFlowDates, Guess, MaxIterations, Basis)

Return $=$ xirr(CashFlow, CashFlowDates) returns the internal rate of return for a schedule of nonperiodic cash flows.

```
Return = xirr(CashFlow, CashFlowDates, Guess,
``` MaxIterations, Basis) returns the internal rate of return for a schedule of nonperiodic cash flows with optional inputs.

\section*{Input Arguments}

CashFlow
A vector or matrix of cash flows. If CashFlow is a matrix, each
column represents a separate stream of cash flows whose internal rate of return is calculated. The first cash flow of each stream is the initial investment, entered as a negative number.

\section*{CashFlowDates}
(Required) A vector or matrix of serial date numbers the same size as CashFlow, or a cell array of date strings the same size as CashFlow. Each column of CashFlowDate represents the dates of the corresponding column of CashFlow.

Guess
The initial estimate of the internal rate of return. Guess is a scalar applied to all streams, or a vector the same length as the number of streams.

Default: 0.1 (10\%)

\section*{MaxIterations}

The positive integer number of iterations used by Newton's method to solve the internal rate of return. MaxIterations is a
scalar applied to all streams, or a vector the same length as the number of streams.

Default: 50

\section*{Basis}

Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.
Default: 0

\section*{Output Arguments}

\section*{Examples}

Return
Vector of the annualized internal rate of return of each cash flow stream. A NaN indicates that a solution is not found.

Find the internal rate of return for an investment of \$10,000 that returns the following nonperiodic cash flow. The original investment is the first cash flow and is a negative number.

\section*{Cash Flow}
(\$10000)
\(\$ 2500\)
\(\$ 2000\)
\(\$ 3000\)
\(\$ 4000\)

\section*{Dates}

January 12, 2007
February 14, 2008
March 3, 2008
June 14, 2008
December 1, 2008

Calculate the internal rate of return for this nonperiodic cash flow:
```

CashFlow = [-10000, 2500, 2000, 3000, 4000];
CashFlowDates = ['01/12/2007'
'02/14/2008'
'03/03/2008'
'06/14/2008'
'12/01/2008'];
Return = xirr(CashFlow, CashFlowDates)

```

This returns:
```

Return =
0.1006 (or 10.0644% per annum)

```

Brealey and Myers, Principles of Corporate Finance, McGraw-Hill Higher Education, Chapter 5, 2003.

\section*{xirr}

Sharpe, William F., and Gordon J. Alexander, Investments. Englewood Cliffs, NJ: Prentice-Hall. 4th ed., 1990.

\section*{See Also fvvar | irr | mirr \| pvvar}
Purpose Year of date
Syntax Year = year(Date)
Year = year(Date, F)
Description Year = year(Date) returns the year of a serial date number or a datestring.Year = year(Date, F) returns the day of the of the month, given aserial date number or date string, in a specified date format.
Examples Year = year(731798.776)
or
Year = year('05-Aug-2003')returns
Year =2003
You can also use the F argument to designate a country-specific date format:
```

    Year = year('2003/08/05','yyyy/mm/dd')
    returns Year = 2003

```
See Also
datevec | day | month | yeardays

Purpose Number of days in year
Syntax Days = yeardays(Year, Basis)

\section*{Arguments}
\[
\begin{array}{ll}
\text { Year } \\
\text { Basis } & \begin{array}{l}
\text { Enter as a four-digit integer. } \\
\text { (Optional) Day-count basis of the instrument. A } \\
\text { vector of integers. }
\end{array} \\
\text { - } 0=\text { actual/actual (default) } \\
& \text { - } 1=30 / 360 \text { (SIA) } \\
& \text { - } 2=\text { actual/360 } \\
& \text { - } 3=\text { actual/365 } \\
\text { - } 4=30 / 360 ~(B M A) ~ \\
& \text { - } 5=30 / 360 \text { (ISDA) } \\
\text { - } 6=30 / 360 \text { (European) } \\
\text { - } 7=\text { actual/365 (Japanese) } \\
\text { - } 8=\text { actual/actual (ICMA) } \\
\text { - } 9=\text { actual/360 (ICMA) } \\
\text { - } 10=\text { actual/365 (ICMA) } \\
\text { - } 11=30 / 360 E ~(I C M A) ~ \\
& \text { - } 12=\text { actual/actual (ISDA) } \\
\text { - } 13=\mathrm{BUS} / 252
\end{array}
\]

For more information, seebasis on page Glossary-1.

Description Days = yeardays(Year, Basis) returns the number of days in the given year, based upon the day-count basis.

\author{
Examples \\ Days = yeardays(2000) \\ Days = \\ 366 \\ Days = yeardays(2000, 1) \\ Days = \\ 360
}

See Also days360 | days365 | daysact | year | yearfrac

Purpose Fraction of year between dates
```

Syntax YearFraction = yearfrac(StartDate, EndDate, Basis)

```

\section*{Arguments}

StartDate Enter as serial date numbers or date strings.
EndDate Enter as serial date numbers or date strings.
Basis (Optional) Day-count basis of the instrument. A vector of integers.
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- 7 = actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\mathrm{actual} / 365\) (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

All specified arguments must be number of instruments (NUMINST)-by-1 or 1-by-NUMINST conforming vectors or scalar arguments.

\section*{Description YearFraction = yearfrac(StartDate, EndDate, Basis) returns a fraction based on the number of days between dates StartDate and EndDate using the given day-count basis. If EndDate is earlier than StartDate, Fraction is negative.}
```

Examples
YearFraction = yearfrac('14 mar 01', '14 sep 01', 0)
YearFraction =
0.5041
YearFraction = yearfrac('14 mar 01', '14 sep 01', 1)
YearFraction =
0.5000

```
See Also days360|days365 | daysact | daysdif | months | wrkdydif | year
    | yeardays

Purpose Yield of discounted security
```

Syntax
Yield = ylddisc(Settle, Maturity, Face, Price, Basis)

```

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. Enter as serial date number or date \\
string. Settle must be earlier than Maturity.
\end{tabular} \\
Maturity & \begin{tabular}{l} 
Maturity date. Enter as serial date number or date \\
string.
\end{tabular} \\
Face & \begin{tabular}{l} 
Redemption (par, face) value.
\end{tabular} \\
Price & \begin{tabular}{l} 
Discounted price of the security. \\
Basis
\end{tabular} \\
\begin{tabular}{l} 
(Optional) Day-count basis of the instrument. A \\
vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- \(2=\) actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

Purpose Yield with interest at maturity
```

Syntax Yield = yldmat(Settle, Maturity, Issue, Face, Price,
CouponRate, Basis)

```

\section*{Arguments}
\begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Settlement date. Enter as serial date number or date \\
string. Settle must be earlier than Maturity. \\
Maturity
\end{tabular} \\
Issue & \begin{tabular}{l} 
Maturity date. Enter as serial date number or date \\
string.
\end{tabular} \\
Face & \begin{tabular}{l} 
Issue date. Enter as serial date number or date \\
string.
\end{tabular} \\
Price & \begin{tabular}{l} 
Redemption (par, face) value.
\end{tabular} \\
CouponRate & \begin{tabular}{l} 
Price of the security.
\end{tabular} \\
Coupon rate. Enter as decimal fraction. \\
Basis & \begin{tabular}{l} 
(Optional) Day-count basis of the instrument. A \\
vector of integers.
\end{tabular}
\end{tabular}
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

\section*{Description}

Examples
Using the data
```

    Settle = '02/07/2000';
    ```
    Maturity = '04/13/2000';
    Issue = '10/11/1999';
    Face = 100;
    Price = 99.98;
    CouponRate = 0.0608;
    Basis = 1;
    Yield = yldmat(Settle, Maturity, Issue, Face, Price,...
    CouponRate, Basis)
returns
Yield = 0.0607 (or 6.07\%)

References Mayle, Standard Securities Calculation Methods, Volumes I-II, 3rd edition. Formula 3.

See Also acrubond | bndprice | bndyield | prmat | ylddisc | yldtbill
\begin{tabular}{ll} 
Purpose & Yield of Treasury bill \\
Syntax & Yield \(=\) yldtbill(Settle, Maturity, Face, Price)
\end{tabular}

Arguments

\section*{Description}

Examples The settlement date of a Treasury bill is February 10, 2000, the maturity date is August 6,2000 , the par value is \(\$ 1000\), and the price is \(\$ 981.36\). Using this data
```

    Yield = yldtbill('2/10/2000', '8/6/2000', 1000, 981.36)
    returns
Yield =
0.0384 (or 3.84%)

```
References Bodie, Kane, and Marcus, Investments, pages 41-43.
See Also beytbill | bndyield | prtbill | yldmat

\section*{zbtprice}

Purpose Zero curve bootstrapping from coupon bond data given price
```

Syntax [ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle, OutputCompounding)

```

\section*{Arguments}

Bonds Coupon bond information used to generate the zero curve. An n-by-2 to n-by-6 matrix where each row describes a bond. The first two columns are required; the rest are optional but must be added in order. All rows in Bonds must have the same number of columns.
\(\left.\begin{array}{l}\begin{array}{l}\text { Columns are } \\ \text { [Maturity CouponRate Face Period Basis } \\ \text { EndMonthRule] where }\end{array} \\ \text { Maturity } \\ \\ \\ \text { Maturity date of the bond, } \\ \text { as a serial date number. } \\ \text { Use datenum to convert date } \\ \text { strings to serial date numbers. }\end{array}\right\}\)
(Optional) Day-count basis of the bond:
- \(0=\) actual/actual (default)
- \(1=30 / 360\) (SIA)
- 2 = actual/360
- \(3=\) actual \(/ 365\)
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

\section*{zbtprice}
\begin{tabular}{ll} 
EndMonthRule & \begin{tabular}{l} 
(Optional) End-of-month flag. \\
\\
This flag applies only when \\
Maturity is an end-of-month \\
date for a month having 30 or
\end{tabular} \\
& fewer days. 0 = ignore flag, \\
& meaning that a bond's coupon \\
payment date is always the \\
& same day of the month. \(1=\) set \\
flag (default), meaning that a \\
bond's coupon payment date \\
is always the last day of the \\
month.
\end{tabular}
Prices \begin{tabular}{ll} 
Settle & \begin{tabular}{l} 
Column vector containing the clean price (price \\
without accrued interest) of each bond in \\
Bonds, respectively. The number of rows (n) \\
must match the number of rows in Bonds.
\end{tabular} \\
OutputCompounding & \begin{tabular}{l} 
Settlement date, as a scalar serial date number. \\
This represents time zero for deriving the \\
zero curve, and it is normally the common \\
settlement date for all the bonds. \\
(Optional) Scalar that sets the compounding \\
frequency per year for the output zero rates in \\
ZeroRates. Allowed values are:
\end{tabular} \\
1 & \begin{tabular}{l} 
Annual compounding
\end{tabular} \\
2 & \begin{tabular}{l} 
Semiannual compounding \\
(default)
\end{tabular} \\
3 & \begin{tabular}{l} 
Compounding three times per \\
year \\
Quarterly compounding
\end{tabular} \\
4 & Bimonthly compounding
\end{tabular}

Monthly compounding
Continuous compounding

\section*{Description}
[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle, OutputCompounding) uses the bootstrap method to return a zero curve given a portfolio of coupon bonds and their prices. A zero curve consists of the yields to maturity for a portfolio of theoretical zero-coupon bonds that are derived from the input Bonds portfolio. The bootstrap method that this function uses does not require alignment among the cash-flow dates of the bonds in the input portfolio. It uses theoretical par bond arbitrage and yield interpolation to derive all zero rates; specifically, the interest rates for cash flows are determined using linear interpolation. For best results, use a portfolio of at least 30 bonds evenly spaced across the investment horizon.
ZeroRates
CurveDates
An m-by-1 vector of decimal fractions that
are the implied zero rates for each point
along the investment horizon represented
by CurveDates; \(m\) is the number of bonds of
unique maturity dates. In aggregate, the rates
in ZeroRates constitute a zero curve.
If more than one bond has the same maturity
date, zbtprice returns the mean zero rate for
that maturity. \(\quad\)\begin{tabular}{l} 
An m-by-1 vector of unique maturity dates (as \\
serial date numbers) that correspond to the \\
zero rates in ZeroRates; \(m\) is the number of \\
bonds of different maturity dates. These dates \\
begin with the earliest maturity date and end \\
with the latest maturity date Maturity in the \\
Bonds matrix.
\end{tabular}

\section*{zbtprice}

Examples Given data and prices for 12 coupon bonds, two with the same maturity date; and given the common settlement date


Prices = [99.375;
99.875;
105.75 ;
96.875;
103.625 ;
101.125;
103.125;
99.375 ;
101.0 ;
101.25 ;
96.375 ;
102.75 ];

Settle = datenum('12/18/1997');
Set semiannual compounding for the zero curve.
\[
\text { OutputCompounding = } 2 \text {; }
\]

Execute the function
```

[ZeroRates, CurveDates] = zbtprice(Bonds, Prices, Settle,...

```

\section*{OutputCompounding)}
which returns the zero curve at the maturity dates. Note the mean zero rate for the two bonds with the same maturity date.
```

ZeroRates =
0.0616
0.0609
0.0658
0.0590
0.0648
0.0655*
0.0606
0.0601
0.0642
0.0621
0.0627
CurveDates =
729907 (serial date number for 01-Jun-1998)
730364 (01-Sep-1999)
730439 (15-Nov-1999)
730500 (15-Jan-2000)
730667 (30-Jun-2000)

```

```

    730971 (30-Apr-2001)
    731032 (30-Jun-2001)
    731033 (01-Jul-2001)
    731321 (15-Apr-2002)
    731336 (30-Apr-2002)
    ```

\author{
References Fabozzi, Frank J. "The Structure of Interest Rates." Ch. 6 in Fabozzi,
} Frank J. and T. Dessa Fabozzi, eds. The Handbook of Fixed Income Securities. 4th ed. New York: Irwin Professional Publishing. 1995.

\section*{zbtprice}

McEnally, Richard W. and James V. Jordan. "The Term Structure of Interest Rates." Ch. 37 in Fabozzi and Fabozzi, ibid.

Das, Satyajit. "Calculating Zero Coupon Rates." Swap and Derivative Financing. Appendix to Ch. 8, pp. 219-225. New York: Irwin Professional Publishing. 1994.

See Also
zbtyield

Purpose
Syntax
Zero curve bootstrapping from coupon bond data given yield
[ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle, OutputCompounding)

\section*{Arguments}
\begin{tabular}{|c|c|c|}
\hline \multirow[t]{5}{*}{Bonds} & \multicolumn{2}{|l|}{Coupon bond information used to generate the zero curve. An n-by-2 to n-by- 6 matrix where each row describes a bond. The first two columns are required; the rest are optional but must be added in order. All rows in Bonds must have the same number of columns. Columns are [Maturity CouponRate Face Period Basis EndMonthRule] where} \\
\hline & Maturity & Maturity date of the bond, as a serial date number. Use datenum to convert date strings to serial date numbers. \\
\hline & CouponRate & Coupon rate of the bond, as a decimal fraction. \\
\hline & Face & (Optional) Redemption or face value of the bond. Default = 100. \\
\hline & Period & (Optional) Coupons per year of the bond, as an integer. Allowed values are \(0,1,2\) (default), 3, 4, 6, and 12 . \\
\hline
\end{tabular}

Basis (Optional) Day-count basis of the bond.
- 0 = actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- \(4=30 / 360\) (BMA)
- \(5=30 / 360\) (ISDA)
- \(6=30 / 360\) (European)
- \(7=\) actual/365 (Japanese)
- \(8=\) actual/actual (ICMA)
- \(9=\) actual/360 (ICMA)
- \(10=\) actual/365 (ICMA)
- \(11=30 / 360 \mathrm{E}\) (ICMA)
- \(12=\) actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

EndMonthRule (Optional) End-of-month flag This flag applies only when Maturity is an end-of-month date for a month having 30 or fewer days. \(0=\) ignore flag, meaning that a bond's coupon payment date is always the same day of the month. \(1=\) set flag (default), meaning that a bond's coupon payment date is always the last day of the month.
Yields Column vector containing the yield to maturity of each bond in Bonds, respectively. The number of rows ( n ) must match the number of rows in Bonds. Yield to maturity must be compounded semiannually.
\begin{tabular}{ll} 
Settle & Settlement date, as a scalar serial date \\
& number. This represents time zero for deriving
\end{tabular} the zero curve, and it is normally the common settlement date for all the bonds.
OutputCompounding (Optional) Scalar that sets the compounding frequency per year for the output zero rates in ZeroRates. Allowed values are:

1
2

3

4
6

Annual compounding
Semiannual compounding (default)

Compounding three times per year
Quarterly compounding
Bimonthly compounding

\section*{zbtyield}

Monthly compounding
Continuous compounding

Description [ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle, OutputCompounding) uses the bootstrap method to return a zero curve given a portfolio of coupon bonds and their yields. A zero curve consists of the yields to maturity for a portfolio of theoretical zero-coupon bonds that are derived from the input Bonds portfolio. The bootstrap method that this function uses does not require alignment among the cash-flow dates of the bonds in the input portfolio. It uses theoretical par bond arbitrage and yield interpolation to derive all zero rates; specifically, the interest rates for cash flows are determined using linear interpolation. For best results, use a portfolio of at least 30 bonds evenly spaced across the investment horizon.

ZeroRates An m-by-1 vector of decimal fractions that are the implied zero rates for each point along the investment horizon represented by CurveDates; \(m\) is the number of bonds of different maturity dates. In aggregate, the rates in ZeroRates constitute a zero curve.

If more than one bond has the same maturity date, zbtyield returns the mean zero rate for that maturity.
CurveDates
An m-by-1 vector of unique maturity dates (as serial date numbers) that correspond to the zero rates in ZeroRates; \(m\) is the number of bonds of different maturity dates. These dates begin with the earliest maturity date and end with the latest maturity date Maturity in the Bonds matrix. Use datestr to convert serial date numbers to date strings.

Examples

Given data and yields to maturity for 12 coupon bonds, two with the same maturity date; and given the common settlement date
```

Bonds = [datenum('6/1/1998') 0.0475 100 2 0 0;
datenum('7/1/2000') 0.06 100 2 0 0;
datenum('7/1/2000') 0.09375 100 6 1 0;
datenum('6/30/2001') 0.05125 100 1 3 1;
datenum('4/15/2002') 0.07125 100 4 1 0;
datenum('1/15/2000') 0.065 100 2 0 0;
datenum('9/1/1999') 0.08 100 3 3 0;
datenum('4/30/2001') 0.05875 100 2 0 0;
datenum('11/15/1999') 0.07125 100 2 0 0;
datenum('6/30/2000') 0.07 100 2 3 1;
datenum('7/1/2001') 0.0525 100 2 3 0;
datenum('4/30/2002') 0.07 100 2 0 0];

```
Yields = [0.0616
    0.0605
    0.0687
    0.0612
    0.0615
    0.0591
    0.0603
    0.0608
    0.0655
    0.0646
    0.0641
    0.0627];
Settle = datenum('12/18/1997');

Set semiannual compounding for the zero curve.
```

OutputCompounding = 2;

```

\section*{zbtyield}

Execute the function
[ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle,...
OutputCompounding)
which returns the zero curve at the maturity dates. Note the mean zero rate for the two bonds with the same maturity date.

ZeroRates =
\[
0.0616
\]
0.0603
0.0657
0.0590
0.0649
0.0650
0.0606
0.0611
0.0643
0.0614
0.0627

CurveDates =
```

729907 (serial date number for 01-Jun-1998)
730364 (01-Sep-1999)
730439 (15-Nov-1999)
730500 (15-Jan-2000)
730667 (30-Jun-2000)
730668 (01-Jul-2000)
730971 (30-Apr-2001)
731032 (30-Jun-2001)
731033 (01-Jul-2001)
731321 (15-Apr-2002)
731336 (30-Apr-2002)

```

\section*{zbtyield}
References Fabozzi, Frank J. "The Structure of Interest Rates." Ch. 6 in Fabozzi,Frank J. and T. Dessa Fabozzi, eds. The Handbook of Fixed IncomeSecurities. 4th ed. New York: Irwin Professional Publishing. 1995.McEnally, Richard W. and James V. Jordan. "The Term Structure ofInterest Rates." Ch. 37 in Fabozzi and Fabozzi, ibid.Das, Satyajit. "Calculating Zero Coupon Rates." Swap and DerivativeFinancing. Appendix to Ch. 8, pp. 219-225. New York: IrwinProfessional Publishing. 1994.
See Also ..... zbtprice
How To - "Term Structure of Interest Rates" on page 2-36

\section*{zero2disc}

\author{
Purpose Discount curve given zero curve \\ ```
Syntax [DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates, Settle, \\ Compounding, Basis)
```

}

## Arguments

| ZeroRates | Number of bonds (NUMBONDS)-by-1 vector of annualized zero rates, as decimal fractions. In aggregate, the rates constitute an implied zero curve for the investment horizon represented by CurveDates. |
| :---: | :---: |
| CurveDates | NUMBONDS-by- 1 vector of maturity dates (as serial date numbers) that correspond to the zero rates. |
| Settle | Serial date number that is the common settlement date for the zero rates; that is, the settlement date for the bonds from which the zero curve was bootstrapped. |
| Compounding | (Optional) Scalar that indicates the compounding frequency per year used for annualizing the input zero rates in ZeroRates. Allowed values are: |
|  | 1 Annual compounding |
|  | 2 Semiannual compounding (default) |
|  | 3 Compounding three times per year |
|  | 4 Quarterly compounding |
|  | 6 Bimonthly compounding |
|  | 12 Monthly compounding |
|  | 365 Daily compounding |



For more information, seebasis on page Glossary-1.

## Description

[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates, Settle, Compounding, Basis) returns a discount curve given a zero curve and its maturity dates.

## zero2disc

| DiscRates | A NUMBONDS-by-1 vector of discount factors, as <br> decimal fractions. In aggregate, the factors in <br> constitute a discount curve for the investment <br> horizon represented by CurveDates. |
| :--- | :--- |
| CurveDates | A NUMBONDS-by-1 vector of maturity dates (as serial <br> date numbers) that correspond to the discount <br> rates. This vector is the same as the input vector <br> CurveDates. |

## Examples

Given a zero curve over a set of maturity dates and a settlement date

```
ZeroRates = [0.0464
    0.0509
    0.0524
    0.0525
    0.0531
    0.0525
    0.0530
    0.0531
    0.0549
    0.0536];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
```

The zero curve was compounded daily on an actual/365 basis.

```
Compounding = 365;
Basis = 3;
```

Execute the function
[DiscRates, CurveDates] = zero2disc(ZeroRates, CurveDates,... Settle, Compounding, Basis)
which returns the discount curve DiscRates at the maturity dates CurveDates.

DiscRates =
0.9996
0.9947
0.9896
0.9866
0.9826
0.9787
0.9745
0.9665
0.9552
0.9466

CurveDates =

730796
730831
730866
730887
730914
730943
730971
731027
731098
731167

## zero2disc

For readability, ZeroRates and DiscRates are shown here only to the basis point. However, MATLAB software computed them at full precision. If you enter ZeroRates as shown, DiscRates may differ due to rounding.

## See Also <br> disc2zero

How To . "Term Structure of Interest Rates" on page 2-36

## Purpose

Forward curve given zero curve

Syntax<br>[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, Settle, Compounding, Basis)

## Arguments

```
-1 Continuous compounding
Basis (Optional) Day-count basis used to construct the
    input zero and output implied forward rate curves.
    - 0 = actual/actual (default)
    - 1 = 30/360 (SIA)
    - 2 = actual/360
    - 3 = actual/365
    - 4 = 30/360 (BMA)
    - 5 = 30/360 (ISDA)
    - 6 = 30/360 (European)
    - 7 = actual/365 (Japanese)
    - 8 = actual/actual (ICMA)
    - 9 = actual/360 (ICMA)
    - 10 = actual/365 (ICMA)
    - 11 = 30/360E (ICMA)
    - 12 = actual/actual (ISDA)
    - 13 = BUS/252
```

For more information, seebasis on page Glossary-1.

[^13]| ForwardRates | A NUMBONDS-by-1 vector of decimal fractions. In <br> aggregate, the rates in ForwardRates constitute a <br> forward curve over the dates in CurveDates. |
| :--- | :--- |
| CurveDates | A NUMBONDS-by-1 vector of maturity dates (as serial <br> date numbers) that correspond to the forward rates <br> in. This vector is the same as the input vector <br> CurveDates. |

## Examples

Given a zero curve over a set of maturity dates, a settlement date, and a compounding rate, compute the forward rate curve.

```
ZeroRates = [0.0458
    0.0502
    0.0518
    0.0519
    0.0524
    0.0519
    0.0523
    0.0525
    0.0541
    0.0529];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
    datenum('15-Jan-2001')
    datenum('05-Feb-2001')
    datenum('04-Mar-2001')
    datenum('02-Apr-2001')
    datenum('30-Apr-2001')
    datenum('25-Jun-2001')
    datenum('04-Sep-2001')
    datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
Compounding = 1;
```

Execute the function
[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates,... Settle, Compounding)
which returns the forward rate curve ForwardRates at the maturity dates CurveDates.

ForwardRates =
0.0458
0.0506
0.0535
0.0522
0.0541
0.0498
0.0544
0.0531
0.0594
0.0476

CurveDates =
730796
730831
730866
730887
730914
730943
730971
731027
731098
731167
For readability, ZeroRates and ForwardRates are shown here only to the basis point. However, MATLAB software computed them at full
precision. If you enter ZeroRates as shown, ForwardRates may differ due to rounding.

## See Also

fwd2zero
How To

- "Term Structure of Interest Rates" on page 2-36

Purpose Par yield curve given zero curve

Syntax [ParRates, CurveDates] = zero2pyld(ZeroRates, CurveDates, Settle, Compounding, Basis, InputCompounding)

## Arguments

| ZeroRates | A number of bonds (NUMBONDS)-by-1 vector of <br> annualized zero rates, as decimal fractions. In <br> aggregate, the rates constitute an implied zero <br> curve for the investment horizon represented <br> by CurveDates. |
| :--- | :--- |
| CurveDates | A NUMBONDS-by-1 vector of maturity dates (as <br> serial date numbers) that correspond to the <br> zero rates. |
| Settle | A serial date number that is the common <br> settlement date for the zero rates. <br> (Optional) Scalar value representing the <br> periodicity in which the output par rates are <br> compounded when annualized. Allowed values <br> are: <br> 1 |
| 2 Annual compounding |  |
| 3 | Semiannual compounding (default) |

Basis (Optional) Day-count basis used to annualize the implied zero rates.

- 0 = actual/actual (default)
- $1=30 / 360$ (SIA)
- 2 = actual/360
- $3=$ actual $/ 365$
- $4=30 / 360$ (BMA)
- $5=30 / 360$ (ISDA)
- $6=30 / 360$ (European)
- 7 = actual/365 (Japanese)
- $8=$ actual/actual (ICMA)
- $9=$ actual/360 (ICMA)
- $10=$ actual/365 (ICMA)
- $11=30 / 360 \mathrm{E}$ (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, seebasis on page Glossary-1.

InputCompounding (Optional) Scalar value representing the periodicity in which the input zero rates were compounded when annualized. The default is the value for Compounding.
[ParRates, CurveDates] = zero2pyld(ZeroRates, CurveDates, Settle, Compounding, Basis, InputCompounding) returns a par yield curve given a zero curve and its maturity dates.

| ParRates | A NUMBONDS-by-1 vector of annualized par yields, as <br> decimal fractions. (Par yields = coupon rates.) In <br> aggregate, the yield rates in ParRates constitute <br> a par yield curve for the investment horizon <br> represented by CurveDates. |
| :--- | :--- |
| CurveDates $\quad$A NUMBONDS-by-1 vector of maturity dates (as serial <br> date numbers) that correspond to the par yield <br> rates. This vector is the same as the input vector <br> CurveDates. |  |

## Examples Given

- A zero curve over a set of maturity dates and
- A settlement date
- Annual compounding for the input zero curve and monthly compounding for the output par rates
compute a par yield curve.

```
ZeroRates = [0.0457
    0.0487
    0.0506
    0.0507
    0.0505
    0.0504
    0.0506
    0.0516
    0.0539
    0.0530];
CurveDates = [datenum('06-Nov-2000')
    datenum('11-Dec-2000')
```

```
datenum('15-Jan-2001')
datenum('05-Feb-2001')
datenum('04-Mar-2001')
datenum('02-Apr-2001')
datenum('30-Apr-2001')
datenum('25-Jun-2001')
datenum('04-Sep-2001')
datenum('12-Nov-2001')];
Settle = datenum('03-Nov-2000');
InputCompounding = 1;
Compounding = 12;
[ParRates, CurveDates] = zero2pyld(ZeroRates, CurveDates,...
Settle, Compounding, [] , InputCompounding)
ParRates =
    0.0479
    0.0511
    0.0530
    0.0531
    0.0526
    0.0524
    0.0525
    0.0534
    0.0555
    0.0543
CurveDates =
730796
730831
730866
730887
730914
730943
```


## zero2pyld

730971
731027
731098
731167

For readability, ZeroRates and ParRates are shown only to the basis point. However, MATLAB software computed them at full precision. If you enter ZeroRates as shown, ParRates may differ due to rounding.

## See Also <br> pyld2zero

How To . "Term Structure of Interest Rates" on page 2-36

## Bibliography

- "Bond Pricing and Yields" on page A-2
- "Term Structure of Interest Rates" on page A-3
- "Derivatives Pricing and Yields" on page A-4
- "Portfolio Analysis" on page A-5
- "Investment Performance Metrics" on page A-6
- "Financial Statistics" on page A-8
- "Standard References" on page A-9
- "Credit Risk Analysis" on page A-11
- "Portfolio Optimization" on page A-12

Note For the well-known algorithms and formulas used in Financial Toolbox software (such as how to compute a loan payment given principal, interest rate, and length of the loan), no references are given here. The references here pertain to less common formulas.

## Bond Pricing and Yields

The pricing and yield formulas for fixed-income securities come from:
[1] Golub, B.W. and L.M. Tilman, Risk Management: Approaches for Fixed Income Markets Wiley, 2000.
[2] Martellini, L., P. Priaulet, and S. Priaulet Fixed Income Securities Wiley, 2003.
[3] Mayle, Jan, Standard Securities Calculation Methods New York: Securities Industry Association, Inc. Vol. 1, 3rd ed., 1993, ISBN 1-882936-01-9. Vol. 2, 1994, ISBN 1-882936-02-7.
[4] Tuckman, B. Fixed Income Securities: Tools for Today's Markets Wiley, 2002.

In many cases these formulas compute the price of a security given yield, dates, rates, and other data. These formulas are nonlinear, however; so when solving for an independent variable within a formula, Financial Toolbox software uses Newton's method. See any elementary numerical methods textbook for the mathematics underlying Newton's method.

## Term Structure of Interest Rates

The formulas and methodology for term structure functions come from:
[5] Fabozzi, Frank J., "The Structure of Interest Rates." Ch. 6 in Fabozzi, Frank J. and T. Dessa Fabozzi, eds. The Handbook of Fixed Income Securities. 4th ed. New York: Irwin Professional Publishing, 1995, ISBN 0-7863-0001-9.
[6] McEnally, Richard W. and James V. Jordan, "The Term Structure of Interest Rates." Ch. 37 in Fabozzi and Fabozzi, ibid.
[7] Das, Satyajit, "Calculating Zero Coupon Rates." Swap and Derivative Financing. Appendix to Ch. 8, pp. 219-225, New York: Irwin Professional Publishing., 1994, ISBN 1-55738-542-4.

## Derivatives Pricing and Yields

The pricing and yield formulas for derivative securities come from:
[8] Chriss, Neil A., "Black-Scholes and Beyond: Option Pricing Models," Chicago: Irwin Professional Publishing, 1997, ISBN 0-7863-1025-1.
[9] Cox, J., S. Ross, and M. Rubenstein, "Option Pricing: A Simplified Approach", Journal of Financial Economics 7, Sept. 1979, pp. 229-263.
[10] Hull, John C., Options, Futures, and Other Derivatives, Prentice Hall, 5th edition, 2003, ISBN 0-13-009056-5.

## Portfolio Analysis

The Markowitz model is used for portfolio analysis computations. For a discussion of this model see Chapter 7 of:
[11] Bodie, Zvi, Alex Kane, and Alan J. Marcus, Investments, Burr Ridge, IL: Irwin. 2nd. ed., 1993, ISBN 0-256-08342-8.

To solve the quadratic minimization problem associated with finding the efficient frontier, the toolbox uses the fmincon function (finds the constrained minimum of a function of several variables) in the Optimization Toolbox documentation. See that toolbox documentation for more details.

## Investment Performance Metrics

The risk and ratio formulas for investment performance metrics come from:
[12] Daniel Bernoulli, "Exposition of a New Theory on the Measurement of Risk," Econometrica, Vol. 22, No 1, January 1954, pp. 23-36 (English translation of "Specimen Theoriae Novae de Mensura Sortis," Commentarii Academiae Scientiarum Imperialis Petropolitanae, Tomus V, 1738, pp. 175-192).
[13] Martin Eling and Frank Schuhmacher, Does the Choice of Performance Measure Influence the Evaluation of Hedge Funds?, Working Paper, November 2005.
[14] John Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stocks Portfolios and Capital Budgets," Review of Economics and Statistics, Vol. 47, No. 1, February 1965, pp. 13-37.
[15] Malik Magdon-Ismail, Amir F. Atiya, Amrit Pratap, and Yaser S. Abu-Mostafa, "On the Maximum Drawdown of a Brownian Motion," Journal of Applied Probability, Volume 41, Number 1, March 2004, pp. 147-161.
[16] Malik Magdon-Ismail and Amir Atiya, "Maximum Drawdown," www.risk.net, October 2004.
[17] Harry Markowitz, "Portfolio Selection," Journal of Finance, Vol. 7, No. 1, March 1952, pp. 77-91.
[18] Harry Markowitz, Portfolio Selection: Efficient Diversification of Investments, John Wiley \& Sons, 1959.
[19] Jan Mossin, "Equilibrium in a Capital Asset Market," Econometrica, Vol. 34, No. 4, October 1966, pp. 768-783.
[20] Christian S. Pedersen and Ted Rudholm-Alfvin, "Selecting a Risk-Adjusted Shareholder Performance Measure," Journal of Asset Management, Vol. 4, No. 3, 2003, pp. 152-172.
[21] William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," Journal of Finance, Vol. 19, No. 3, September 1964, pp. 425-442.
[22] Katerina Simons, "Risk-Adjusted Performance of Mutual Funds," New England Economic Review, September/October 1998, pp. 34-48.

## Financial Statistics

The discussion of computing statistical values for portfolios containing missing data elements derives from the following references:
[23] Little, Roderick J.A. and Donald B. Rubin, Statistical Analysis with Missing Data, 2nd ed., John Wiley \& Sons, Inc., 2002.
[24] Meng, Xiao-Li, and Donald B. Rubin, "Maximum Likelihood Estimation via the ECM Algorithm," Biometrika, Vol. 80, No. 2, 1993, pp. 267-278.
[25] Sexton, Joe and Anders Rygh Swensen, "ECM Algorithms That Converge at the Rate of EM," Biometrika, Vol. 87, No. 3, 2000, pp. 651-662.
[26] Dempster, A.P., N.M. Laird, and Donald B. Rubin, "Maximum Likelihood from Incomplete Data via the EM Algorithm," Journal of the Royal Statistical Society, Series B, Vol. 39, No. 1, 1977, pp. 1-37.

## Standard References

Standard references include:
[27] Addendum to Securities Industry Association, Standard Securities Calculation Methods: Fixed Income Securities Formulas for Analytic Measures, Vol. 2, Spring 1995. This addendum explains and clarifies the end-of-month rule.
[28] Brealey, Richard A. and Stewart C. Myers, Principles of Corporate Finance, New York: McGraw-Hill. 4th ed., 1991, ISBN 0-07-007405-4.
[29] Daigler, Robert T., Advanced Options Trading. Chicago: Probus Publishing Co., 1994, ISBN 1-55738-552-1.
[30] A Dictionary of Finance. Oxford: Oxford University Press., 1993, ISBN 0-19-285279-5.
[31] Fabozzi, Frank J. and T. Dessa Fabozzi, eds. The Handbook of Fixed-Income Securities. Burr Ridge, IL: Irwin. 4th ed., 1995, ISBN 0-7863-0001-9.
[32] Fitch, Thomas P., Dictionary of Banking Terms. Hauppauge, NY: Barron's. 2nd ed., 1993, ISBN 0-8120-1530-4.
[33] Hill, Richard O., Jr., Elementary Linear Algebra. Orlando, FL: Academic Press. 1986, ISBN 0-12-348460-X.
[34] Luenberger, David G., Investment Science, Oxford University Press, 1998. ISBN 0195108094.
[35] Marshall, John F. and Vipul K. Bansal, Financial Engineering: A Complete Guide to Financial Innovation. New York: New York Institute of Finance. 1992, ISBN 0-13-312588-2.
[36] Sharpe, William F., Macro-Investment Analysis. An "electronic work-in-progress" published on the World Wide Web, 1995, at http://www.stanford.edu/~wfsharpe/mia/mia.htm.
[37] Sharpe, William F. and Gordon J. Alexander, Investments. Englewood Cliffs, NJ: Prentice-Hall. 4th ed., 1990, ISBN 0-13-504382-4.
[38] Stigum, Marcia, with Franklin Robinson, Money Market and Bond Calculations. Richard D. Irwin., 1996, ISBN 1-55623-476-7.

## Credit Risk Analysis

The credit rating and estimation transition probabilities come from:
[39] Altman, E., "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," Journal of Finance, Vol. 23, No. 4, (Sep., 1968), pp. 589-609.
[40] Basel Committee on Banking Supervision, International Convergence of Capital Measurement and Capital Standards: A Revised Framework, Bank for International Settlements (BIS), comprehensive version, June 2006.
[41] Hanson, S. and T. Schuermann, "Confidence Intervals for Probabilities of Default," Journal of Banking \& Finance, Elsevier, vol. 30(8), August 2006, pp. 2281-2301.
[42] Jafry, Y. and T. Schuermann, "Measurement, Estimation and Comparison of Credit Migration Matrices," Journal of Banking \& Finance, Elsevier, vol. 28(11), November 2004, pp. 2603-2639.
[43] Löffler, G. and P. N. Posch, Credit Risk Modeling Using Excel and VBA,West Sussex, England: Wiley Finance, 2007.
[44] Schuermann, T., "Credit Migration Matrices," in E. Melnick and B. Everitt (eds.), Encyclopedia of Quantitative Risk Analysis and Assessment, Wiley, 2008.

## Portfolio Optimization

The Markowitz model is used for portfolio optimization computations.
[45] Markowitz, H., "Portfolio Selection,"Journal of Finance, Vol. 7, No. 1, March 1952, pp. 77-91.
[46] Markowitz, H. M., Portfolio Selection: Efficient Diversification of Investments, John Wiley \& Sons, Inc., 1959.

## Examples

Use this list to find examples in the documentation.

## Bond Examples

"Single Bond Example" on page 2-27
"Bond Portfolio Example" on page 2-29

## Portfolio Examples

"Efficient Frontier Example" on page 3-5
"Optimal Risky Portfolio Example" on page 3-9
"Constraint Specification" on page 3-12

## Portfolio Object Examples

"Asset Allocation Example" on page 4-100

## Estimation of Transition Probabilities

"Estimating Transition Probabilities" on page 6-4
"Estimating $t$-year Default Probabilities and Confidence Intervals" on page 6-7
"Removing Outliers, Estimating Subgroup Probabilities, and Aggregating Datasets" on page 6-9

## Financial Statistics

"Example of Portfolios with Missing Data" on page 7-26
"Capital Asset Pricing Model" on page 7-34

## Sample Programs

"Sensitivity of Bond Prices to Changes in Interest Rates" on page 8-3
"Constructing a Bond Portfolio to Hedge Against Duration and Convexity" on page 8-6
"Sensitivity of Bond Prices to Parallel Shifts in the Yield Curve" on page 8-9
"Sensitivity of Bond Prices to Nonparallel Shifts in the Yield Curve" on page 8-12
"Constructing Greek-Neutral Portfolios of European Stock Options" on page 8-14
"Term Structure Analysis and Interest Rate Swap Pricing" on page 8-18

## Graphics Programs

"Plotting an Efficient Frontier" on page 8-21
"Plotting Sensitivities of an Option" on page 8-24
"Plotting Sensitivities of a Portfolio of Options" on page 8-26

## Charting Financial Time Series

"Using chartfts" on page 9-18

## Indexing Financial Time Series

"Indexing with Date Strings" on page 10-8
"Indexing with Date String Range" on page 10-10
"Indexing with Integers" on page 10-11
"Indexing When Time-of-Day Data Is Present" on page 10-13

## Financial Time Series Demonstration Program

"Demonstration Program" on page 10-25

# Financial Time Series Graphical User Interface Examples 

"Fill Missing Data" on page 12-10
"Frequency Conversion" on page 12-12
"Analysis Menu" on page 12-13
"Graphs Menu" on page 12-15

## Technical Analysis

"Moving Average Convergence/Divergence (MACD)" on page 14-4 "Williams \%R" on page 14-6<br>"Relative Strength Index (RSI)" on page 14-7<br>"On-Balance Volume (OBV)" on page 14-8

## active return

Amount of return achieved in excess of the return produced by an appropriate benchmark (for example, an index portfolio).

## active risk

Standard deviation of the active return. Also known as the tracking error on page Glossary-15.

## American option

An option that can be exercised any time until its expiration date. Contrast with European option.

## amortization

Reduction in value of an asset over some period for accounting purposes. Generally used with intangible assets. Depreciation is the term used with fixed or tangible assets.

## annuity

A series of payments over a period of time. The payments are usually in equal amounts and usually at regular intervals such as quarterly, semiannually, or annually.

## arbitrage

The purchase of securities on one market for immediate resale on another market to profit from a price or currency discrepancy.

## basis point

One hundredth of one percentage point, or 0.0001.

## basis

Day count basis determines how interest accrues over time for various instruments and the amount transferred on interest payment dates. The calculation of accrued interest for dates between payments also uses day count basis. Day count basis is a fraction of Number of interest accrual days/Days in the relevant coupon period. Supported day count conventions and basis values are:

| Basis <br> Value | Day Count Convention |
| :--- | :--- |
| 0 | actual/actual (default) - Number of days in both a period <br> and a year is the actual number of days. |
| 1 | $30 / 360$ SIA - Year fraction is calculated based on a 360 <br> day year with 30-day months, after applying the following |
| 2 | actual/360 - Number of days in a period is equal to the <br> actual number of days, however the number of days in a year <br> is 360. |
| $\mathbf{3}$ | actual/365 - Number of days in a period is equal to the <br> actual number of days, however the number of days in a year <br> is 365 (even in a leap year). |
| 4 | 30/360 PSA - Number of days in every month is set to 30 <br> (including February). If the start date of the period is either <br> the 31st of a month or the last day of February, the start <br> date is set to the 30th, while if the start date is the 30th of a <br> month and the end date is the 31st, the end date is set to the <br> 30th. The number of days in a year is 360. |
| 5 | 30/360 ISDA - Number of days in every month is set to 30, <br> except for February where it is the actual number of days. If <br> the start date of the period is the 31st of a month, the start <br> date is set to the 30th while if the start date is the 30th of a <br> month and the end date is the 31st, the end date is set to the <br> 30th. The number of days in a year is 360. |
| 6 | 30E /360 - Number of days in every month is set to 30 <br> except for February where it is equal to the actual number <br> of days. If the start date or the end date of the period is the <br> 31st of a month, that date is set to the 30th. The number of <br> days in a year is 360. |
| 7 | actual/365 Japanese - Number of days in a period is equal <br> to the actual number of days, except for leap days (29th <br> February) which are ignored. The number of days in a year <br> is 365 (even in a leap year). |

Glossary-2

| Basis <br> Value | Day Count Convention |
| :--- | :--- | $\mathbf{~}$| $\mathbf{8}$ | actual/actual ISMA - Number of days in both a period and <br> a year is the actual number of days and the compounding <br> frequency is annual. |
| :--- | :--- |
| 9 | actual/360 ISMA - Number of days in a period is equal to <br> the actual number of days, however the number of days in a <br> year is 360 and the compounding frequency is annual. |
| 10 | actual/365 ISMA - Number of days in a period is equal to <br> the actual number of days, however the number of days in <br> a year is 365 (even in a leap year) and the compounding <br> frequency is annual. |
| 11 | $30 / 360$ ISMA - Number of days in every month is set to 30, <br> except for February where it is equal to the actual number of <br> days. If the start date or the end date of the period is the 31st <br> of a month, that date is set to the 30th. The number of days <br> in a year is 360 and the compounding frequency is annual. |
| 12 | actual/365 ISDA - The day count fraction is calculated using <br> the following formula: (Actual number of days in period <br> that fall in a leap year / 366) + (Actual number of <br> days in period that fall in a normal year / 365 ). |
| 13 | bus/252 - The number of days in a period is equal to the <br> actual number of days however the number of days in a year <br> is 252. |

## beta

The price volatility of a financial instrument relative to the price volatility of a market or index as a whole. Beta is commonly used with respect to equities. A high-beta instrument is riskier than a low-beta instrument.

## binomial model

A method of pricing options or other equity derivatives in which the probability over time of each possible price follows a binomial distribution. The basic assumption is that prices can move to only two values (one higher and one lower) over any short time period.

## Black-Scholes model

The first complete mathematical model for pricing options, developed by Fischer Black and Myron Scholes. It examines market price, strike price, volatility, time to expiration, and interest rates. It is limited to only certain kinds of options.

## Bollinger band chart

A financial chart that plots actual asset data along with three other bands of data: the upper band is two standard deviations above a user-specified moving average; the lower band is two standard deviations below that moving average; and the middle band is the moving average itself.

## bootstrapping, bootstrap method

An arithmetic method for backing an implied zero curve out of the par yield curve.

## building a binomial tree

For a binomial option model: plotting the two possible short-term price-changes values, and then the subsequent two values each, and then the subsequent two values each, and so on over time, is known as "building a binomial tree." See also binomial model on page Glossary-3.
call
a. An option to buy a certain quantity of a stock or commodity for a specified price within a specified time. See also put on page Glossary-12. b. A demand to submit bonds to the issuer for redemption before the maturity date. c. A demand for payment of a debt. d. A demand for payment due on stock bought on margin.

## callable bond

A bond that allows the issuer to buy back the bond at a predetermined price at specified future dates. The bond contains an embedded call option; that is, the holder has sold a call option to the issuer. See also puttable bond on page Glossary-12.

## candlestick chart

A financial chart usually used to plot the high, low, open, and close price of a security over time. The body of the "candle" is the region between
the open and close price of the security. Thin vertical lines extend up to the high and down to the low, respectively. If the open price is greater than the close price, the body is empty. If the close price is greater than the open price, the body is filled. See also high-low-close chart on page Glossary-9.

## cap

Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain level.

## cash flow

Cash received and paid over time.

## clean price

The price of a bond excluding any interest that has accrued since issue or the most recent coupon payment.

## collar

Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain upper level nor fall below a lower level. It is designed to protect an investor against wide fluctuations in interest rates.

## convexity

A measure of the rate of change in duration; measured in time. The greater the rate of change, the more the duration changes as yield changes.

## correlation

The simultaneous change in value of two random numeric variables.

## correlation coefficient

A statistic in which the covariance is scaled to a value between minus one (perfect negative correlation) and plus one (perfect positive correlation).

## coupon

Detachable certificate attached to a bond that shows the amount of interest payable at regular intervals, usually semiannually. Originally
coupons were actually attached to the bonds and had to be cut off or "clipped" to redeem them and receive the interest payment.

## coupon dates

The dates when the coupons are paid. Typically a bond pays coupons annually or semiannually.

## coupon rate

The nominal interest rate that the issuer promises to pay the buyer of a bond.

## covariance

A measure of the degree to which returns on two assets move in tandem.
A positive covariance means that asset returns move together; a negative covariance means they vary inversely.

## day count convention

A convention used to determine the number of days between two coupon dates, which is important in calculating accrued interest and present value when the next coupon payment is less than a full coupon period away. See also basis on page Glossary-1

## delta

The rate of change of the price of a derivative security relative to the price of the underlying asset; that is, the first derivative of the curve that relates the price of the derivative to the price of the underlying security.

## depreciation

Reduction in value of fixed or tangible assets over some period for accounting purposes. See also amortization on page Glossary-1.

## derivative

A financial instrument that is based on some underlying asset. For example, an option is a derivative instrument based on the right to buy or sell an underlying instrument.

## dirty price

The price of a bond including the accrued interest.

## discount curve

The curve of discount rates versus maturity dates for bonds.

## drawdown

The peak to trough decline during a specific record period of an investment or fund.

## duration

The expected life of a fixed-income security considering its coupon yield, interest payments, maturity, and call features. As market interest rates rise, the duration of a financial instrument decreases. See also Macaulay duration on page Glossary-10.

## efficient frontier

A graph representing a set of portfolios that maximizes expected return at each level of portfolio risk. See also Markowitz model on page Glossary-10.

## efficient portfolio

Portfolios satisfying the criteria of minimum risk for a given level of return and maximum return for a given level of risk. See also Markowitz model on page Glossary-10.
elasticity
See Lambda on page Glossary-10.

## European option

An option that can be exercised only on its expiration date. Contrast with American option.

## ex-ante

Referring to future events, such as the future price of a stock.

## ex-post

Referring to past events, when uncertainty of the result has been eliminated.

## exercise price

The price set for buying an asset (call) or selling an asset (put). The strike price.

## face value

The maturity value of a security. Also known as par value, principal value, or redemption value.

## fixed-income security

A security that pays a specified cash flow over a specific period. Bonds are typical fixed-income securities.

## floor

Interest-rate option that guarantees that the rate on a floating-rate loan will not fall below a certain level.

## forward curve

The curve of forward interest rates versus maturity dates for bonds.

## forward rate

The future interest rate of a bond inferred from the term structure, especially from the yield curve of zero-coupon bonds, calculated from the growth factor of an investment in a zero held until maturity.

## future value

The value that a sum of money (the present value) earning compound interest will have in the future.

## gamma

The rate of change of delta for a derivative security relative to the price of the underlying asset; that is, the second derivative of the option price relative to the security price.

## greeks

Collectively, "greeks" refer to the financial measures beta, delta, gamma, lambda, rho, theta, and vega, which are sensitivity measures used in evaluating derivatives.

## ISDA

International Swaps and Derivatives Association.

## ISMA

International Securities Market Association.

## hedge

A securities transaction that reduces or offsets the risk on an existing investment position.

## high-low-close chart

A financial chart usually used to plot the high, low, open, and close price of a security over time. Plots are vertical lines whose top is the high, bottom is the low, open is a short horizontal tick to the left, and close is a short horizontal tick to the right.

## implied volatility

For an option, the variance that makes a call option price equal to the market price. Given the option price, strike price, and other factors, the Black-Scholes model computes implied volatility.

## information ratio

The ratio of relative return to relative risk.

## internal rate of return

a. The average annual yield earned by an investment during the period held. b. The effective rate of interest on a loan. c. The discount rate in discounted cash flow analysis. d. The rate that adjusts the value of future cash receipts earned by an investment so that interest earned equals the original cost. See also yield on page Glossary-16.

## issue date

The date a security is first offered for sale. That date usually determines when interest payments, known as coupons, are made.

## Ito process

Statistical assumptions about the behavior of security prices. For details, see the book by Hull in "Derivatives Pricing and Yields" on page A-4.

## key rate duration

Key rate duration measures the sensitivity of a portfolio's (or security's) value in relation to changes in specific maturities of the zero or spot curve.

## Lambda

The percentage change in the price of an option relative to a $1 \%$ change in the price of the underlying security. Also known as elasticity.

## long position

Outright ownership of a security or financial instrument. The owner expects the price to rise in order to make a profit on some future sale.

## long rate

The yield on a zero-coupon Treasury bond.

## lower partial moment

A model for the moments of asset returns that fall below a minimum acceptable level of return.

## Macaulay duration

A widely used measure of price sensitivity to yield changes developed by Frederick Macaulay in 1938. It is measured in years and is a weighted average-time-to-maturity of an instrument. The Macaulay duration of an income stream, such as a coupon bond, measures how long, on average, the owner waits before receiving a payment. It is the weighted average of the times payments are made, with the weights at time T equal to the present value of the money received at time $T$.

## Markowitz model

A model for selecting an optimum investment portfolio, devised by H . M. Markowitz. It uses a discrete-time, continuous-outcome approach for modeling investment problems, often called the mean-variance paradigm. See also efficient portfolio on page Glossary-7 and efficient frontier on page Glossary-7.

## maturity date

The date when the issuer returns the final face value of a bond to the buyer.

## mean

a. A number that typifies a set of numbers, such as a geometric mean or an arithmetic mean. $\mathbf{b}$. The average value of a set of numbers.

## modified duration

The Macaulay duration discounted by the per-period interest rate; that is, divided by (1+rate/frequency).

## Monte-Carlo simulation

A mathematical modeling process. For a model that has several parameters with statistical properties, pick a set of random values for the parameters and run a simulation. Then pick another set of values, and run it again. Run it many times (often 10,000 times) and build up a statistical distribution of outcomes of the simulation. This distribution of outcomes is then used to answer whatever question you are asking.

## moving average

A price average that is adjusted by adding other parametrically determined prices over some time period.

## moving-averages chart

A financial chart that plots leading and lagging moving averages for prices or values of an asset.

## normal (bell-shaped) distribution

In statistics, a theoretical frequency distribution for a set of variable data, usually represented by a bell-shaped curve symmetrical about the mean.

## odd first or last period

Fixed-income securities may be purchased on dates that do not coincide with coupon or payment dates. The length of the first and last periods may differ from the regular period between coupons, and thus the bond owner is not entitled to the full value of the coupon for that period. Instead, the coupon is prorated according to how long the bond is held during that period.

## on-the-run treasury bonds

The most recently auctioned issue of a U.S. Treasury bond or note of a particular maturity.

## option

A right to buy or sell specific securities or commodities at a stated price (exercise or strike price) within a specified time. An option is a type of derivative.

## par value

The maturity or face value of a security or other financial instrument.

## par yield curve

The yield curve of bonds selling at par, or face, value.

## point and figure chart

A financial chart usually used to plot asset price data. Upward price movements are plotted as X's and downward price movements are plotted as O's.

## present value

Today's value of an investment that yields some future value when invested to earn compounded interest at a known interest rate; that is, the future value at a known period in time discounted by the interest rate over that time period.

## principal value

See par value on page Glossary-12.

## PSA

Public Securities Association.

## purchase price

Price actually paid for a security. Typically the purchase price of a bond is not the same as the redemption value.
put
An option to sell a stipulated amount of stock or securities within a specified time and at a fixed exercise price. See also call on page Glossary-4.

## puttable bond

A bond that allows the holder to redeem the bond at a predetermined price at specified future dates. The bond contains an embedded put
option; that is, the holder has bought a put option. See also callable bond on page Glossary-4.

## Quant

A quantitative analyst; someone who does numerical analysis of financial information in order to detect relationships, disparities, or patterns that can lead to making money.

## redemption value

See par value on page Glossary-12.

## regression analysis

Statistical analysis techniques that quantify the relationship between two or more variables. The intent is quantitative prediction or forecasting, particularly using a small population to forecast the behavior of a large population.

## rho

The rate of change in a derivative's price relative to the underlying security's risk-free interest rate.

## return proxy

The proxy for return is a function that characterizes either the gross benefits or net benefits associated with portfolio choices.

## risk proxy

The proxy for risk is a function that characterizes either the variability or losses associated with portfolio choices.

## sensitivity

The "what if" relationship between variables; the degree to which changes in one variable cause changes in another variable. A specific synonym is volatility.

## settlement date

The date when money first changes hands; that is, when a buyer actually pays for a security. It need not coincide with the issue date.

## Sharpe ratio

The ratio of the excess return of an asset divided by the asset's standard deviation of returns.

## short rate

The annualized one-period interest rate.

## short sale, short position

The sale of a security or financial instrument not owned, in anticipation of a price decline and making a profit by purchasing the instrument later at a lower price, and then delivering the instrument to complete the sale. See also long position on page Glossary-10.

## SIA

Securities Industry Association.

## spot curve, spot yield curve

See zero curve, zero-coupon yield curve on page Glossary-16.

## spot rate

The current interest rate appropriate for discounting a cash flow of some given maturity.

## spread

For options, a combination of call or put options on the same stock with differing exercise prices or maturity dates.

## standard deviation

A measure of the variation in a distribution, equal to the square root of the arithmetic mean of the squares of the deviations from the arithmetic mean; the square root of the variance.

## stochastic

Involving or containing a random variable or variables; involving chance or probability.

## straddle

A strategy used in trading options or futures. It involves simultaneously purchasing put and call options with the same exercise price and
expiration date, and it is most profitable when the price of the underlying security is very volatile.

## strike

Exercise a put or call option.

## strike price

See exercise price on page Glossary-7.

## swap

A contract between two parties to exchange cash flows in the future according to some formula.

## swaption

A swap option; an option on an interest-rate swap. The option gives the holder the right to enter into a contracted interest-rate swap at a specified future date. See also swap on page Glossary-15.

## term structure

The relationship between the yields on fixed-interest securities and their maturity dates. Expectation of changes in interest rates affects term structure, as do liquidity preferences and hedging pressure. A yield curve is one representation in the term structure.

## theta

The rate of change in the price of a derivative security relative to time. Theta is usually very small or negative since the value of an option tends to drop as it approaches maturity.

## tracking error

See active risk on page Glossary-1.

## Treasury bill

Short-term U.S. government security issued at a discount from the face value and paying the face value at maturity.

## Treasury bond

Long-term debt obligation of the U.S. government that makes coupon payments semiannually and is sold at or near par value in $\$ 1000$ denominations or higher. Face value is paid at maturity.

## variance

The dispersion of a variable. The square of the standard deviation.

## vega

The rate of change in the price of a derivative security relative to the volatility of the underlying security. When vega is large, the security is sensitive to small changes in volatility.

## volatility

a. Another general term for sensitivity. b. The standard deviation of the annualized continuously compounded rate of return of an asset. c. A measure of uncertainty or risk.

## yield

a. Measure of return on an investment, stated as a percentage of price. Yield can be computed by dividing return by purchase price, current market value, or other measure of value. b. Income from a bond expressed as an annualized percentage rate. c. The nominal annual interest rate that gives a future value of the purchase price equal to the redemption value of the security. Any coupon payments determine part of that yield.

## yield curve

Graph of yields (vertical axis) of a particular type of security versus the time to maturity (horizontal axis). This curve usually slopes upward, indicating that investors usually expect to receive a premium for securities that have a longer time to maturity. The benchmark yield curve is for U.S. Treasury securities with maturities ranging from three months to 30 years. See also term structure on page Glossary-15.

## yield to maturity

A measure of the average rate of return that will be earned on a bond if held to maturity.

## zero curve, zero-coupon yield curve

A yield curve for zero-coupon bonds; zero rates versus maturity dates. Since the maturity and duration (Macaulay duration) are identical for zeros, the zero curve is a pure depiction of supply/demand conditions for loanable funds across a continuum of durations and maturities. Also known as spot curve or spot yield curve.

## zero-coupon bond, or zero

A bond that, instead of carrying a coupon, is sold at a discount from its face value, pays no interest during its life, and pays the principal only at maturity.

## Symbols and Numerics

1900 date system 17-511 17-908
1904 date system 17-511 17-908
360-day year 17-266
365-day year 17-274

## A

abs2active 17-2
AbstractPortfolio 17-6
acceleration 17-851
accrfrac 17-14
accrued interest 2-22 17-17 17-19
computing fractional period 17-14
acrubond 17-17
acrudisc 17-19
active return 3-20
active risk $3-20$
active2abs 17-21
actual days
between dates 17-276
addEquality 17-23
addGroupRatio 17-26
addGroups 17-30
addInequality 17-33
adding a scalar and a matrix 1-9
adding matrices 1-8
adline 17-36
adosc 17-39
advance payments, periodic payment given 17-583
after-tax rate of return 17-779
algebra, linear 1-9 1-14
American options 2-3 2-42
amortization 1-22 2-19 to 2-20 17-42
amortize 17-42
analysis models for equity derivatives $2-40$
analysis, technical 14-2
analyzing
and computing cash flows 2-17
equity derivatives 2-39
portfolios 2-43
annuity 2-19
payment of with odd first period 17-585
periodic interest rate of $17-45$
periodic payment of loan or 17-586
annurate $17-45$
annuterm 17-46
apostrophe or prime character ( $\backslash$ q) 1-7
arguments
function return 1-21
interest rate 1-22
matrices as, limitations 1-22
vectors as, limitations 1-22
arith2geom 17-47
arithmetic 10-16
array operations 1-18
ASCII character 1-21
ascii2fts 17-50
creating object with 9-14
asset covariance matrix with exponential
weighting 17-375
asset life 1-22
axes
combining 9-24
axis labels, converting 17-236

## B

bank format 17-232
bar 17-55
bar3 17-58
bar3h 17-58
barh 17-55
basis 2-23
basis, day-count 17-281
beytbill 17-61
binomial
functions 2-3
model 2-42
put and call pricing 17-62
tree, building 2-42
binprice 17-62
Black-Scholes
elasticity 17-76
functions 2-3
implied volatility 17-73
model 2-41
options 8-24 8-26
put and call pricing 17-78
sensitivity to
interest rate change $17-80$
time-until-maturity change 17-82
underlying delta change 17-71
underlying price change 17-69
underlying price volatility 17-84
Black's option pricing 17-67
blkimpv 17-65
blkprice 17-67
blsdelta 17-69
blsgamma 17-71
blsimpv 17-73
blslambda 17-76
blsprice 17-78
blsrho 17-80
blstheta 17-82
blsvega 17-84
bndconvp 17-86
bndconvy 17-91
bnddurp 17-96
bnddury 17-102
bndkrdur 17-108
bndprice 17-113
bndspread 17-120
bndyield 17-127
bolling 17-134
bollinger 17-136
Bollinger band chart 2-15
bond
convexity 8-3
duration 8-3
equivalent yield for Treasury bill 17-61
portfolio
constructing based on key rate duration 8-12
constructing to hedge against duration and convexity 8-6
visualizing sensitivity of price to parallel shifts in the yield curve 8-9
sensitivity of prices to changes in interest rates 8-3
zero-coupon 17-927
bootstrapping 2-37 17-831 17-924 17-931
boxcox 17-138
example 10-20
building a binomial tree 2-42
busdate 17-140
busdays 17-142
business date
last of month 17-494
business day
next 2-10 17-140
previous $17-140$
business days 17-484

## C

call and put pricing
Black-Scholes 17-78
candle 17-144
candle (time series) 17-147
candlestick chart 17-144
capital allocation line 3-3
cash flow
analyzing and computing 2-17
convexity 17-160
dates 2-10 17-161
duration 17-165
future value of varying 17-431
internal rate of return 17-480
internal rate of return for nonperiodic 17-909 irregular 17-431
modified internal rate of return 17-528
negative 2-17
portfolio form of amounts 17-166
present value of varying 17-690
sensitivity of 2-19
uniform payment equal to varying 17-587
cell array 8-19
cfamounts 17-150
cfconv 17-160
cfdates 17-161
cfdur 17-165
cfport 17-166
cftimes 17-170
chaikosc 17-175
chaikvolat 17-178
character array
strings stored as 1-21
character, ASCII 1-21
chart
Bollinger band 2-15
candlestick 17-144
high, low, open, close 17-468
leading and lagging moving averages 17-534
point and figure 17-612
chartfts 17-181
combine axes feature 9-24
purpose 9-18
using 9-18
chartfts zoom feature 9-21
charting 14-2
charting financial data 2-12
checkFeasibility 17-184
chfield 17-186
colon (:) 1-7
Combine Axes tool 9-24
commutative law 1-9 1-14
compatible time series $10-16$
component 10-3
computing
cash flows 2-17
dot products of vectors 1-11
yields for fixed-income securities 2-21
constraint functions 3-15
constraint matrix 3-17
constructing
a bond portfolio to hedge against duration and convexity 8-6
greek-neutral portfolios of European stock options 8-14
conventions
SIA 2-21
conversions
currency 2-12
date input $2-5$
date output 2-7
convert2sur 17-187
converting
and handling dates 2-4
axis labels 17-236
convertto 17-189
convexity 8-3
cash flow 17-160
constructing a bond portfolio to hedge against 8-6
portfolio 8-5 8-7
corr2cov 17-192
corr2cov function 17-192
corrcoef 17-190
coupon bond
prices to zero curve $17-924$
yields to zero curve 17-931
coupon date
after settlement date 17-200
days between 17-218 17-221
coupon dates 2-30
coupon payments remaining until
maturity 17-197
coupon period
containing settlement date 17-224
fraction of 17-14
coupons payable between dates 17-197
cov 17-193
cov2corr 17-195
covariance matrix 3-5
covariance matrix with exponential
weighting 17-375
cpncount 17-197
cpndaten 17-200
cpndatenq 17-204
cpndatep 17-209
cpndatepq 17-213
cpndaysn 17-218
cpndaysp 17-221
cpnpersz 17-224
createholidays 17-227
graphical user interface 13-2
Credit rating 6-2
cumsum 17-229
cur2frac 17-231
cur2str 17-232
currency
converting 2-12
decimal 17-405
formatting 2-12
fractional 17-231 17-405
values 17-231
current date 17-805
and time 2-8 17-572

## D

data extraction 10-4
data series vector 10-4
data transformation 10-19
date 2-8
components 17-259
conversions 2-5
current 2-8 17-572 17-805
end of month 17-342
first business, of month 17-379
formats 2-4
hour of 17-477
input conversions 2-5
last date of month 17-342
last weekday in month 17-507
maturity 2-22
minute of 17-527
number 2-4 17-247
displaying as string 17-239
Excel to MATLAB 17-907
indices of in matrix 17-241
MATLAB to Excel 17-510
of day in future or past month 17-243
of future or past workday 17-263
output conversions 2-7
seconds of 17-712
starting, add month to 17-243
string 2-4 17-251
year of 17-913
date and time functions 17-344
date of specific weekday in month 17-573
date string $10-8$
indexing 10-8
range 10-10
date system
1900 17-511 17-908
1904 17-511 17-908
date vector 10-4 17-260
date2time 17-233
dateaxis $17-236$
datedisp 17-239
datefind 17-241
datemnth 17-243
datenum 17-247
dates
actual days between 17-276
business days 17-484
cash-flow 2-10 17-161
coupon 2-30
days between 17-266 17-274 17-276 17-278 17-281
determining 2-9
first coupon 2-22
fraction of year between 17-916
handling and converting 2-4
investment horizon 2-37
issue 2-21
last coupon 2-22
number of months between 17-532
quasi-coupon 2-22
settlement 2-21
vector of 1-21
working days between 17-906
datestr 10-8 17-251
datevec 17-259
datewrkdy 17-263
day 17-265
date of specific weekday in month 17-573
of month 17-265
of month, last 17-344
of the week 17-894
day-count basis 17-281
day-count convention 2-23
days
between
coupon date and settlement date 17-221
dates 17-266 17-274 17-276 17-278
17-281 17-906
settlement date and next coupon
date 17-218
business 17-484
holidays 17-473
in coupon period containing settlement date 17-224
last business date of month 17-494
last weekday in month 17-507
nontrading 17-473
number of, in year 17-914
days360 17-266
days360e 17-268
days360isda 17-270
days360psa 17-272
days365 17-274
daysact 17-276
daysadd 17-278
daysdif 17-281
dec2thirtytwo 17-283
decimal currency 17-405
to fractional currency 17-231
declining-balance depreciation
fixed 2-19 17-285
general 2-19 17-286
default values 10-3
definitions 1-5
delta 2-39
change, Black-Scholes sensitivity to underlying 17-71
demonstration program 10-25
depfixdb 17-285
depgendb 17-286
deprdv 17-287
depreciable value, remaining 17-287
depreciation 2-19
fixed declining-balance 2-19 17-285
general declining-balance 2-19 17-286
straight-line 2-19 17-290
sum of years' digits 2-19 17-288
depsoyd 17-288
depstln 17-290
derivatives
equity, pricing and analyzing 2-39
sensitivity measures for 2-39
description field
component name 10-3
setting 9-14
determining dates 2-9
diff 17-291
disc2zero 17-292
discount curve
from zero curve 17-938
to zero curve 17-292
discount rate of a security 17-297
discount security 17-19
future value of 17-428
price of 17-679
yield of 17-918
discrate 17-297
dividing matrices 1-14
dot products of vectors 1-11
double-colon operator 10-10
duration
cash-flow and modified 17-165
constructing a bond portfolio to hedge against 8-6
for fixed-income securities 2-33
Macaulay 2-33
modified 2-33
portfolio 8-5 8-7

## E

ECM (expectation conditional
maximization) 17-327
ecmlsrmle 17-299
ecmlsrobj 17-305
ecmmvnrfish 17-307
ecmmvnrmle 17-310
ecmmvnrobj 17-315
ecmmvnrstd 17-317
ecmnfish 17-320
ecmnhess 17-322
ecmninit 17-324
ecmnmle 17-326
ecmnobj 17-332
ecmnstd 17-333
effective rate of return 17-335
efficient frontier 3-5
plotting an 8-21
tracking error 3-20
effrr 17-335
elasticity
Black-Scholes 17-76
element by element
operating 1-18
element-by-element 1-8
elements, referencing matrix 1-6
elpm 17-336
emaxdrawdown 17-338
end $17-340$
MATLAB variable 10-13
end-of-month rule 2-26
enlarging matrices 1-6
eomdate 17-342
eomday 17-344
eq (time series) 17-345
equal time series $10-16$
equations
solving simultaneous linear 1-15
equity derivatives 2-39
analysis models for 2-40
estimateAssetMoments 17-346
estimateBounds 17-353
estimateFrontier 17-356
estimateFrontierByReturn 17-360
estimateFrontierByRisk 17-363
estimateFrontierLimits 17-366
estimatePortMoments 17-369
estimatePortReturn 17-371
estimatePortRisk 17-373
European options 2-3
constructing greek-neutral portfolios of 8-14
ewstats 17-375
Excel date number
from MATLAB date number 17-510
to MATLAB date number 17-907
$\exp$ 17-377
expectation conditional maximization 17-327
expected lower partial moments 5-14
expected maximum drawdown 5-17
exponential weighting of covariance
matrix 17-375
extfield 17-378
extracting data 10-4

## F

fbusdate 17-379
fetch 17-381
fieldnames 17-386
fillts 17-387
example 12-10
filter 17-393
financial data charting 2-12
Financial Time Series Tool 11-2
loading data 11-5
supported tasks 11-10
using with other Financial Time Series
GUIs 11-18
fints 17-394
syntaxes 9-3
first business date of month 17-379
first coupon date 2-22
fixed declining-balance depreciation 2-19 17-285
fixed periodic payments
future value with 17-430
fixed-income securities
cash-flow dates 17-161
Macaulay and modified durations for 2-33
pricing 2-31
pricing and computing yields for 2-21
terminology 2-21
yield functions for $2-32$
fixed-income sensitivities 2-33
formats
bank 17-232
date 2-4
formatting currency and charting financial data 2-12
forward curve
from zero curve 17-943
to zero curve 17-434
fpctkd 17-402
frac2cur 17-405
fraction of
coupon period 17-14
year between dates 17-916
fractional currency 17-231 17-405
freqnum 17-406
freqstr 17-408
frequency
indicator field 10-3
indicators 9-13
setting 9-13
frequency conversion functions
Data menu 12-12
table 10-19
frontcon 3-5 17-410
frontier 17-414
plotting an efficient 8-21
frontier, efficient 3-5
fts2ascii 17-416
fts2mat 17-418
ftsbound 17-420
displaying dates with 10-11
ftsdata subdirectory 9-15
ftsgui 17-421
command 12-2
ftsinfo 17-422
ftstool 11-2 17-425
ftsuniq 17-427
function
return arguments 1-21
future month, date of day in 17-243
future value 2-18 17-46
of discounted security 17-428
of varying cash flow 17-431
with fixed periodic payments 17-430
fvdisc 17-428
fvfix 17-430
fvvar 17-431
fwd2zero 17-434

## G

gamma 2-39
general declining-balance depreciation 2-19
17-286
generating and referencing matrix elements 1-7 geom2arith 17-439
getAssetMoments 17-442
getBounds 17-444
getBudget 17-446
getCosts 17-448
getEquality 17-450
getfield 17-456
getGroupRatio 17-452
getGroups 17-454
getInequality 17-459
getnameidx 17-461
graphical user interface 12-2
graphics
producing 8-21
three-dimensional 8-12
greek-neutral portfolios, constructing 8-14
greeks 2-39
neutrality 8-14
GUI 12-2
starting with ftsgui 17-421
starting with ftstool 17-425

## H

handling and converting dates 2-4
hedging 8-3
a bond portfolio against duration and convexity 8-6
hhigh 17-463
high, low, open, close chart 17-468
highlow 17-468
highlow (time series) 17-465
hist 17-470
holdings2weights 17-472
holidays 2-10 17-473
holidays and nontrading days 17-473
horzcat 17-475
hour 17-477
hour of date or time 17-477

## I

identity matrix 1-14
iid (independent identically-distributed data) 17-325
implied volatility 2-40
Black-Scholes 17-73
independent identically-distributed data 17-325
indexing
date range $10-10$
date string 10-8
integer 10-11
with time-of-day data 10-13
indices
of date numbers in matrix 17-241
of nonrepeating integers in matrix 17-241
indifference curve 3-8
inforatio 17-478
Information ratio 5-8
inner dimension rule 1-9
input
conversions 2-5
string 1-21
interest 17-42
accrued 17-17 17-19
on loan 2-19
interest rate swap 8-18
interest rates
arguments 1-22
Black-Scholes sensitivity to change 17-80
of annuity, periodic $17-45$
rate of return 2-17
risk-free 8-27
sensitivity of bond prices to changes in 8-3
term structure 2-2 2-36
internal rate of return 17-480
for nonperiodic cash flow 17-909
modified 17-528
inversion, matrix 1-14
investment horizon 2-37
irr 17-480
isbusday 17-484
iscompatible 17-486
isempty 17-488
isequal 17-487
isfield 17-489
issorted 17-490
issue date 2-21
Ito process 2-41

## K

Kagi chart 17-491
key rate duration
for bonds 2-34

## L

lagging and leading moving averages
chart 17-534
lagts 17-493
lambda 2-40
last
business date of month 17-494
date of month 17-342
day of month 17-344
weekday in month 17-507
last coupon date $2-22$
lbusdate 17-494
leading and lagging moving averages chart 17-534
leadts 17-496
left division 1-17
length 17-497
leverage of an option 17-76
Line break chart 17-498
linear algebra 1-9 1-14
linear equations 8-8
solving simultaneous 1-15
system of 1-15
llow 17-500
loan
interest on 2-19
payment with odd first period 17-585
periodic payment of $17-586$
log 17-502
log10 17-503
log2 17-504
lpm 17-505
lweekdate 17-507

## M

m2xdate 17-510
Macaulay duration 8-3
for fixed-income securities 2-33
macd 17-512
MACD signal line 17-512
main GUI window $12-2$
MATLAB
date number
from Excel date number 17-907
to Excel date number 17-510
matrices
adding and subtracting 1-8
as arguments, limitations 1-22
dividing 1-14
enlarging 1-6
multiplying 1-9 1-12
multiplying vectors and 1-12
of string input 1-21
singular 1-14
square 1-14
transposing 1-7
matrix 1-5
adding or subtracting a scalar 1-9
algebra refresher 1-8
covariance 17-375
elements
generating 1-7
referencing 1-6
identity 1-14
indices of date numbers 17-241
indices of integers in 17-241
inversion 1-14
multiplying by a scalar 1-14
numbers and strings in a 1-21
maturity
price with interest at 17-683
yield of a security paying interest at 17-921
maturity date 2-22
$\max$ 17-515
maxdrawdown 17-516
maximum drawdown 5-17
maximum likelihood estimate (MLE) 17-329
mean $17-518$
medprice 17-519
merge 17-521
min 17-525
minus $17-526$
minute 17-527
minute of date or time 17-527
mirr 17-528
MLE (maximum likelihood estimate) 17-329
modified duration 8-3 17-165
for fixed-income securities $2-33$
modified internal rate of return 17-528
momentum 17-853
month 17-530
add, to starting date $17-243$
date of specific weekday 17-573
day of $17-265$
first business date of 17-379
last business date 17-494
last date of $17-342$
last day of 17-344
months 17-532
last weekday in 17-507
number of months between dates 17-532
movavg 17-534
Moving Average Convergence/Divergence
(MACD) 17-512
moving averages chart 17-534
mrdivide 17-539
mtimes 17-541
multiplying
a matrix by a scalar 1-14
matrices 1-9
two matrices 1-12
vectors 1-10
vectors and matrices 1-12
mvnrfish 17-542
mvnrmle 17-545
mvnrobj 17-550
mvnrstd 17-552

## N

names
variable 1-8
NaN 2-28
nancov 17-555
nanmax 17-557
nanmean 17-559
nanmedian 17-560
nanmin 17-561
nanstd 17-563
nansum 17-565
nanvar 17-566
negative cash flows 2-17
negvolidx 17-568
Newton's method 2-32
next
business day $2-10$
coupon date after settlement date 17-200
or previous business day 17-140
nominal rate of return 17-571
nomrr 17-571
nontrading days 2-10 17-473
notation 1-5
row, column 1-6
now 17-572
number of
days in year 17-914
periods to obtain value 17-46
whole months between dates 17-532
numbers
and strings in a matrix 1-21
date 2-4
nweekdate 17-573
nyseclosures 17-576

## 0

object structure 9-3
observation 17-326
odd first period
payment of loan or annuity with 17-585
On-Balance Volume (OBV) 14-8
onbalvol 17-580
operating element by element 1-18
operations, array 1-18
opprofit 17-582
optimal portfolio 3-2
option
leverage of 17-76
plotting sensitivities of 8-24
plotting sensitivities of a portfolio of 8-26
pricing
Black's model 17-67
profit 17-582
output conversions, date 2-7
overloaded functions
most common 10-24
types of 10-15

## P

par value 2-22
par yield curve
from zero curve 17-948
to zero curve 17-693
past month, date of day in 17-243
payadv 17-583
payment
of loan or annuity with odd first period 17-585
periodic, given number of advance payments 17-583
periodic, of loan or annuity 17-586
uniform, equal to varying cash flow 17-587
payodd 17-585
payper 17-586
payuni 17-587
pcalims 17-589
pcgcomp 17-592
pcglims 17-595
pcpval 17-598
peravg 17-600
period 2-22
periodic interest rate of annuity 17-45
periodic payment
future value with fixed 17-430
given advance payments $17-583$
of loan or annuity $17-586$
present value with fixed 17-687
periodicreturns 17-603
plot 17-605
plotFrontier 17-607
plotting
efficient frontier 8-21
sensitivities of a portfolio of options 8-26
sensitivities of an option 8-24
plus 17-611
point and figure chart 17-612
pointfig 17-612
portalloc 3-9 to 3-10 17-613
portalpha 17-617
portcons 3-15 17-621
portfolio
convexity 8-5 8-7
duration 8-5 8-7
expected rate of return 17-662
of options, plotting sensitivities of 8-26
optimal 3-2
optimization 3-3
risks, returns, and weights randomized 17-647
selection 3-8
Portfolio 17-625
Portfolio object
asset allocation example 4-100
asset returns 4-36
common operations 4-29
constraints 4-52
estimating efficient frontier 4-88
estimating efficient portfolio 4-77
moments of asset returns 4-36
post-processing 4-95
transaction costs 4-49
troubleshooting 4-97
validating 4-73
Portfolio optimization
constructing portfolio object 4-22
portfolio object 4-12
portfolio object methods 4-12
portfolio object properties 4-12
problems 4-2
theory 4-2
portfolios
analyzing 2-43
of European stock options constructing greek-neutral 8-14
portopt 17-642
portrand 17-647
portror 17-650
portsim 17-651
portstats 17-662
portvar 17-664
portvrisk 17-665
posvolidx 17-667
power 17-670
prbyzero 17-671
prcroc 17-677
prdisc 17-679
present value 2-18
of varying cash flow 17-690
with fixed periodic payments 17-687
previous quasi coupon date 17-215
price
change, Black-Scholes sensitivity to underlying 17-69
of discounted security 17-679
of Treasury bill 17-686
volatility, Black-Scholes sensitivity to underlying 17-84
with interest at maturity 17-683
Price and volume chart 17-681 17-887
pricing
and analyzing equity derivatives 2-39
and computing yields for fixed-income securities 2-21
fixed-income securities 2-31
principal 17-42
prmat 17-683
profit, option 17-582
prtbill 17-686
purchase price 2-22
put and call pricing
binomial 17-62
Black-Scholes 17-78
pvfix 17-687
pvtrend 17-688
pvvar 17-690
pyld2zero 17-693

## Q

quasi coupon date
previous 17-215
quasi-coupon dates 2-22

## R

randomized portfolio risks, returns, and weights 17-647
rate of a security, discount 17-297
rate of return 2-17
after-tax 17-779
effective 17-335
internal 17-480
internal for nonperiodic cash flow 17-909
modified internal 17-528
nominal 17-571
portfolio expected 17-662
Ratio
information 5-8
Sharpe 5-6
rdivide 17-698
record 17-326
redemption value 2-22
reference date $2-30$
referencing matrix elements $1-6$ to 1-7
Relative Strength Index (RSI) 14-7
remaining depreciable value 2-19 17-287
Renko chart 17-700
resamplets 17-702
ret2tick 17-703
ret2tick (time series) 17-706 17-789
return arguments, function 1-21
rho 2-40
risk aversion 3-8
risk-adjusted return 5-11
risk-free interest rates 8-27
risks
returns, and weights
randomized portfolio 17-647
rmfield 17-709
row, column notation 1-6
row-by-column 1-5
rsindex 17-710

## S

sample lower partial moments 5-14
scalar 1-6
adding or subtracting 1-9
multiplying a matrix by 1-14
second 17-712
seconds of date or time 17-712
securities industry association 2-21
selectreturn 17-713
sensitivity
fixed-income 2-33
measures for derivatives 2-39
of a portfolio of options, plotting 8-26
of an option, plotting 8-24
of bond prices to changes in interest rates 8-3
of cash flow 2-19
to
interest rate change, Black-Scholes 17-80
to time-until-maturity change, Black-Scholes 17-82
to underlying delta change, Black-Scholes 17-71
to underlying price change, Black-Scholes 17-69
to underlying price volatility, Black-Scholes 17-84
visualizing to nonparallel shifts in the yield curve 8-12
visualizing to parallel shifts in the yield curve 8-9
serial dates $10-8$
setAssetList 17-714
setAssetMoments 17-717
setBounds 17-720
setBudget $17-723$
setCosts 17-726
setDefaultConstraints 17-730
setEquality 17-733
setfield 17-754
setGroupRatio 17-735
setGroups 17-739
setInequality 17-742
setInitPort 17-744
setOptions 17-747
setSolver 17-748
settlement date 2-21
coupon period containing 17-224
days between previous coupon date and 17-221
days between, and coupon date 17-218
next coupon date after 17-200
setTurnover 17-751
sharpe 17-756
Sharpe ratio 5-6
SIA 2-21
compatibility 2-21
order of precedence 2-30
SIA conventions 2-21
signal line 17-512
single quotes 1-21
singular matrices 1-14
size 17-758
smoothts 17-759
solving
sample problems with the toolbox 8-2
sortfts 17-761
spctkd $17-763$
spreadsheets 1-5
square matrices 1-14
std 17-766
stochosc 17-767
straight-line depreciation 2-19 17-290
strings
and numbers in a matrix 1-21
date 2-4 17-251
input, matrices of 1-21
stored as character array 1-21
structures 10-3
subsasgn 17-770
subsref 17-774
subtracting
a scalar and a matrix 1-9
matrices 1-8
sum of years' digits depreciation 2-19 17-288
swap 8-18
synch date 2-30
synchronization date 2-30
system of linear equations 1-15

## T

targetreturn 17-778
taxedrr 17-779
tbl2bond 17-780
technical analysis 14-2
term structure 2-2 2-36 8-3 17-292 17-434 17-693
17-780 17-924 17-931 17-938 17-943 17-948
parameters from Treasury bond parameters 17-831
terminology, fixed-income securities 2-21
text file transformation 9-14
theta 2-40
thirdwednesday 17-783
thirtytwo2dec 17-785
three-dimensional graphics 8-12
tick labels 17-236
tick2ret 17-786
time
current 2-8 17-572
hour of 17-477
minute of $17-527$
seconds of 17-712
time and date functions 17-344
time-until-maturity change
Black-Scholes sensitivity to $17-82$
time2date 17-791
times 17-795
toannual 17-796
todaily 17-801
today $17-805$
todecimal 17-806
tomonthly 17-807
toquarterly 17-812
toquoted 17-818
tosemi 17-819
totalreturnprice 17-825
toweekly 17-826
tr2bonds 17-831
tracking error 3-20 5-10
tracking error efficient frontier 3-20
Transition probabilities 6-3
transposing matrices 1-7
transprob 17-835
transprobbytotals 17-844
Treasury bill 2-36
bond equivalent yield for 17-61
parameters to Treasury bond parameters 17-780
price of 17-686
yield of 17-923
Treasury bond 2-36
parameters
from Treasury bill parameters 17-780
to term-structure parameters 17-831
tsaccel 17-851
tsmom 17-853
tsmovavg 17-855
typprice $17-858$

## U

ugarch 17-861
ugarchllf 17-864
ugarchpred 17-867
ugarchsim 17-870
uicalendar 17-876
graphical user interface 13-4
uminus 17-881
uniform payment equal to varying cash
flow 17-587
uplus 17-882

## V

value at risk (VaR)
portfolio 17-665
var 17-883
variable names $1-8$
vector 1-5
of dates 1-21
vectors
as arguments, limitations 1-22
computing dot products of 1-11
multiplying 1-10
multiplying matrices and 1-12
vega 2-40
vertcat 17-885
volatility
Black-Scholes implied 17-73
implied 2-40
volroc 17-889

## w

wclose 17-891
week, day of 17-894
week, in a year 17-896
weekday 17-894
date of specific, in month $17-573$
weeknum 17-896
weights2holdings 17-899
willad 17-900
Williams \%R 14-6
willpctr
example 14-6
willpctr function 17-903
workday, date of future or past 17-263
working days between dates 17-906
wrkdydif 17-906

## X

x2mdate 17-907
xirr 17-909

## $\mathbf{Y}$

year 17-913
fraction of between dates 17-916
number of days in 17-914
of date 17-913
yeardays $17-914$
yearfrac 17-916
yield
curve 8-4 8-6
visualizing sensitivity of bond portfolio's price to nonparallel shifts in 8-12
visualizing sensitivity of bond portfolio's price to parallel shifts in 8-9
for Treasury bill, bond equivalent 17-61
functions for fixed-income securities 2-32
of discounted security 17-918
of security paying interest at maturity 17-921
of Treasury bill 17-923
yield-to-maturity 2-22
yields
for fixed-income securities, pricing and computing 2-21
ylddisc 17-918
yldmat 17-920
yldtbill 17-923

## Z

zbtprice 17-924
zbtyield 17-931
zero curve 17-831 17-927 17-934
from coupon bond prices 17-924
from coupon bond yields 17-931
from discount curve 17-292
from forward curve 17-434
from par yield curve 17-693
to discount curve 17-938
to forward curve 17-943
to par yield curve 17-948
zero-coupon bond 17-294 17-927 17-934
zero2disc 17-938
zero2fwd 17-943
zero2pyld 17-948
Zoom tool 9-21


[^0]:    fprintf(1,'Seemingly Unrelated Regression Parameter

[^1]:    Note You must obtain a license for these products from MathWorks before you can use either of these toolboxes.

[^2]:    Note If the selected workspace variable that you are plotting is not a fints object, a fints object is created when you click Plot. The new fints object uses the name designated by the MATLAB workspace variable box on the Create tab.

[^3]:    See Also
    abs2active | pcalims | pcgcomp | pcglims | pcpval | portcons

[^4]:    Description fieldnames gets field names in a financial time series object.
    fnames $=$ fieldnames(tsobj) returns the field names associated with the financial time series object tsobj as a cell array of strings, including the common fields: desc, freq, dates (and times if present).
    fnames = fieldnames(tsobj, srsnameonly) returns field names depending upon the setting of srsnameonly. If srsnameonly is 0 , the function returns all field names, including the common fields: desc, freq, dates, and times. If srsnameonly is set to 1, fieldnames returns only the data series in fnames.

[^5]:    See Also
    chfield | getfield | isfield | rmfield | setfield

[^6]:    See Also
    minus | mrdivide | plus | times

[^7]:    Notes You can configure Design as a matrix if NUMSERIES = 1 or as a cell array if NUMSERIES $\geq 1$.

    - If Design is a cell array and NUMSERIES $=1$, each cell contains a NUMPARAMS row vector.
    - If Design is a cell array and NUMSERIES > 1, each cell contains a NUMSERIES-by-NUMPARAMS matrix.

    Examples

    See Also ecmmvnrmle | ecmmvnrstd | mvnrmle
    $\begin{array}{ll}\text { References } \quad \begin{array}{l}\text { Roderick J. A. Little and Donald B. Rubin, Statistical Analysis with } \\ \text { Missing Data, 2nd ed., John Wiley \& Sons, Inc., 2002. }\end{array} \\ & \end{array}$
    See "Multivariate Normal Regression" on page 7-17, "Least-Squares Regression" on page 7-18, "Covariance-Weighted Least Squares" on page 7-19, "Feasible Generalized Least Squares" on page 7-20, and "Seemingly Unrelated Regression" on page 7-21.

[^8]:    datestr(nyseclosures('1-jan-1899','31-dec-1899'),'dd-mmm-yyyy ddd')

[^9]:    Note plotFrontier handles multiple input formats as described above. Given an asset universe with NumAssets assets and an efficient frontier with NumPorts portfolios, remember that portfolio weights are NumAsset-by-NumPorts matrices and that portfolio risks and returns are NumPorts column vectors.

[^10]:    See Also
    estimateAssetMoments | Portfolio | estimateFrontierByRisk
    Tutorials . "Working with Asset Returns and Moments of Asset Returns" on page 4-36

[^11]:    Optimization Converged Successfully
    Magnitude of directional derivative in search direction
    less than 2*options. TolFun and maximum constraint violation is less than options.TolCon

    No Active Constraints

    Estimated Coefficients:
    $\qquad$
    0.2520
    0.0708
    0.1623

[^12]:    Description
    uicalendar('PARAM1', VALUE1, 'PARAM2', VALUE2', ...) supports a customizable graphical calendar that interfaces with uicontrols. uicalendar populates uicontrols with user-selected dates.

    Examples Create a uicontrol:

    ```
    textH1 = uicontrol('style', 'edit', 'position', [10 10 100 20]);
    ```

    Call UICalendar:

    ```
    uicalendar('DestinationUI', {textH1, 'string'})
    ```

    Select a date and click ' OK '.

[^13]:    Description [ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, Settle, Compounding, Basis) returns an implied forward rate curve given a zero curve and its maturity dates.

